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WAYS TO IMPROVE FOURIER SERIES CONVERGENCE AND ITS APPLICATION FOR LAPLACE NUMERICAL INVERSION

Tetyana Solyar

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine, Lviv, Ukraine

Summary. *The solution of a wide class of the problems of mathematical physics can be obtained in the form of Fourier series. When considering the problems, concerned with study of the localized actions, the quick-changing solutions are obtained, in this connection the series converge slowly. To solve complex problems the Fourier series method is used jointly with other approaches, in particular, when using in addition the boundary element method the series coefficients are determined by solving one- and two-dimensional integral equations that demands a large amount of calculations. The series coefficients are determined with certain errors what can cause the loss of calculation accuracy. In such cases the problem of improvement the series convergence with controlled accuracy of calculations will be of high priority. Below we propose method of improving the Fourier series convergence for functions which can be approximated with sufficiently high accuracy by the least squares method by means of the first degree piecewise-continuous polynomials on whole interval of series specifying.*

Keywords: *Fourier series, improvement of series convergence, piecewise-continuous polynomials, conformal mapping, numerical inversion of Laplace transform*

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Introduction. The solutions for wide range of applied problems can be found in Fourier series. Here often the series convergence is low that requires calculations of a large number of series terms. As it indicated in references [1], in such cases they predominantly use the formulas that are obtained from series summing-up as incorrect [2]. As the functions, being described by the series with improved convergence, are here calculated as certain averaging of basic value [1] then fast-variable values within localized operations will be determined with the largest error.

The article deals with the comprehensive research of the ways to improve Fourier series convergence for the functions in assumption that they can be approximated with given accuracy by piecewise-continuous polynomials of first degree. The constructed formula was used to improve the accuracy of numerical inversion of Laplace transform.

Formulation of problem. Let the function $f(x)$ at $0 \le x \le L$ is given by the series

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{i\lambda_n x} , \qquad (1)$$

where a_n are given coefficients, $\lambda_n = \dfrac{2\pi n}{L}$, $L = \text{const}$.

Let us look up the case when coefficients a_n slowly decrease during the increasing of n parameter due to which the series converge slowly. To define functions with given accuracy one has to consider a large number of series terms (around 1000). While finding solution of complicated problems in mathematical physics the series terms can be defined numerically (for example, by method integral equations [3, 4, 5]), that requires large number of calculations. Due to this fact there appears the relevant question how to find functions with controlled accuracy on the basis of relatively small number of coefficients in a_n series.

Approximation of functions with piecewise-continuous polynomials. Let us look at continuous along section $c \le x \le d$ function $f(x)$. We shall describe it along this section approximately with the following function

$$f(x) \cong f(x) = \sum_{j=0}^{N} A_j g_j(x),$$

where $g_j(x)$ are given linearly independent functions. Coefficients A_j are found with the method of least squares in order to minimize the formula

$$I = \int_a^b \varphi(x) \left[f(x) - \sum_{j=1}^{N} A_j g_j(x) \right]^2 dx,$$

where $\varphi(x)$ is a given function weighting. There comes system of equities to determine coefficients A_j

$$\sum_{j=0}^{N} \alpha_{ij} A_j = \beta_i, \quad i = 0, ..., N, \tag{2}$$

where

$$\alpha_{ij} = \int_a^b \varphi g_i g_j dx, \quad \beta_i = \int_a^b \varphi f g_i dx.$$

Let us study the approximation of functions with linear piecewise-continuous functions. The section $[c, d]$ is divided into N subsections with the step $h = \frac{d-c}{N}$ and put onto

$$g_j = \left(1 - \frac{|x - x_j|}{h}\right) S\left(\frac{x - x_j}{h}\right),$$

where $x_j = c + jh$, $S(z) = 1$ at $|z| \le 1$ and $S(z) = 0$ at $|z| > 1$.

It should be mentioned that in this case $f(x_j) = A_j$.

Then the equation system (2) at $\varphi = 1$ will be following

$$\begin{cases} aA_{j-1} + bA_j + aA_{j+1} = \beta_j, \quad j = 1, ..., N-1, \\ 0,5bA_0 + aA_1 = \beta_0, \\ aA_{N-1} + 0,5bA_N = \beta_N, \end{cases} \tag{3}$$

where $b = \frac{2}{3}h$, $a = \frac{1}{6}h$, $\beta_i = \int_{-h}^{h} f(x + x_i)\left(1 - \frac{|x|}{h}\right) dx$, $i = 1, ..., N-1$,

$$\beta_0 = \int_0^h f(x + c)\left(1 - \frac{x}{h}\right) dx, \quad \beta_N = \int_0^h f(d - x)\left(1 - \frac{x}{h}\right) dx.$$

Improvement of Fourier series convergence. Let us use the mentioned-above approach for the case when function $f(x)$ is determined by formula (1).

Coefficients β_j, that are constituents of equation system (3), in this case at $c=0$, $d=L$ are derived with formula

$$\beta_j = \int_0^L fg_j dx = \sum_{n=-\infty}^{\infty} a_n \int_{x_{i-1}}^{x_{i+1}} \left(1 - \frac{|x-x_j|}{h}\right) e^{i\lambda_n x} dx . \qquad (4)$$

Hence we find

$$\beta_j = h\sum_{n=-\infty}^{\infty} a_n \gamma(0.5\lambda_n h) e^{i\lambda_n x_j} \quad \text{at} \quad j \neq 0, N,$$

$$\beta_0 = -h\sum_{n=-\infty}^{\infty} a_n \frac{e^{i\lambda_n h} - 1 - i\lambda_n h}{(\lambda_n h)^2}, \quad \beta_N = -h\sum_{n=-\infty}^{\infty} a_n \frac{e^{-i\lambda_n h} - 1 + i\lambda_n h}{(\lambda_n h)^2},$$

where $\gamma(z) = \dfrac{\sin^2 z}{z^2}$.

Initially we accept that $f(x)$ is continuous periodic function (at $f(0) = f(L)$). Denote for it the coefficients A_j as C_j. For such functions the equation system (3) is recorded as

$$aC_{j-1} + bC_j + aC_{j+1} = \beta_j = h\sum_{n=-\infty}^{\infty} a_n \gamma(0.5\lambda_n h) e^{i\lambda_n x_j}, \quad j = 0,...,N, \qquad (5)$$

where $C_{-1} = C_{N-1}$, $\quad C_{N+1} = C_1$.

The solution for the system (5) is expressed as

$$C_j = h\sum_{n=-\infty}^{\infty} a_n B_n \gamma(0.5\lambda_n h) e^{i\lambda_n x_j}, \quad j = 0,...,N,$$

where B_n are unknown constants.

Having inserted this solution into system equation (5) we get

$$B_n \left[a\exp(i\lambda_n h) + b + a\exp(-i\lambda_n h) \right] = 1.$$

And there comes

$$B_n = \frac{3}{h\left(2 + \cos\lambda_n h\right)},$$

and

$$C_j = \sum_{n=-\infty}^{\infty} a_n \Gamma_n \exp(i\lambda_n x_j), \qquad (6)$$

where $\quad \Gamma_n = B_n \gamma(0,5\lambda_n h) h = g(\lambda_n h), \quad g(t) = \dfrac{3}{2+\cos t}\left(\dfrac{\sin(t/2)}{t/2}\right)^2.$

To find solution for equation system (3) in general case (when $f(0) \neq f(L)$) one has to additionally investigate relevant to it homogenous system which is recorded as

$$aA^*_{j-1} + bA^*_j + aA^*_{j+1} = 0, \quad j = 0, ..., N.$$

The general solution for this difference equation system is

$$A^*_j = U_1\gamma_1^j + U_2\gamma_2^j,$$

where $U_{1,2}$ are arbitrary constants, $\gamma_{1,2}$ are roots of equation

$$a + b\gamma + a\gamma^2 = 0,$$

that are equal to $\gamma_{1,2} = -2 \pm \sqrt{3}$.

Then the full solution for the equation system (3) is recorded as

$$A_j = C_j + U_1\gamma_1^j + \frac{U_2}{\gamma_2^N}\gamma_2^j.$$

The first system equation (3) is solved identically. Out of two last equations we obtain the following system

$$\begin{cases} U_1\left(0,5b + a\gamma_1\right) + U_2\left(0,5b + a\gamma_2\right)\gamma_2^{-N} = \beta_0 - 0,5bC_0 - aC_1, \\ U_1\left(0,5b\gamma_1 + a\right)\gamma_1^{N-1} + U_2\left(0,5b + a/\gamma_2\right) = \beta_N - 0,5bC_N - aC_{N-1}. \end{cases}$$

At $N > 10$ practically precise solution to this system can be recorded in simple manner

$$U_1 = \frac{\beta_0 - 0,5bC_0 - aC_1}{0,5b + a\gamma_1} = \frac{6\beta_0/h - 2C_0 - C_1}{\sqrt{3}},$$
$$U_2 = \frac{\beta_N - 0,5bC_N - aC_{N-1}}{0,5b + a/\gamma_2} = \frac{6\beta_N/h - 2C_N - C_{N-1}}{\sqrt{3}}.$$

Thus, we have

$$f(x_j) = \sum_{n=-\infty}^{\infty} a_n\Gamma_n \exp(i\lambda_n x_j) + U_1\gamma_1^j + U_2\gamma_2^{j-N}. \quad (7)$$

For periodic functions the joint points can be selected at random and we will get the formula for series calculation

$$f(x) = \sum_{n=-\infty}^{\infty} a_n\Gamma_n \exp(i\lambda_n x).$$

At large arguments values $g(t) = O(t^{-2})$, i.e. $\Gamma_n = O(n^{-2}) \to 0$ as $n \to \infty$, so the series (7) converges faster than initial series (4). The coefficients Γ_n are defined by means of function *g*. The graph of this function is displayed on Figure 1.

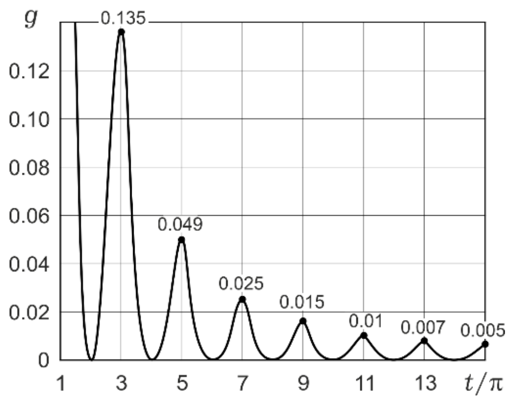

Figure 1. The graph of function g

Figure 1 shows that in order to improve the convergence of slow-converging series it is worth to limit terms in *M* series. They have to be selected in such way that $\lambda_M h \approx 2\pi, 4\pi$, i.e. accept $M \approx N$ or $M \approx 2N$. Here the first series terms will be least due to the close to zero values of Γ_n multipliers.

The next one is the example highlighting the peculiarities of the formula (6). We accept that $f(x) = \mathrm{sign}(x), -\pi < x < \pi$, i.e. the function is split. Fourier series distribution for this function is

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1} . \tag{8}$$

Having used the formulas to improve the convergence we get (function *f* is periodic)

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \Gamma_n \frac{\sin(2n-1)x}{2n-1} . \tag{9}$$

At $h = 2\pi / N$ in nodal points $x_j = hj$ the precise value of this function is

$$f(x_j) = \mathrm{sign}(x_j)(1+\gamma^{|j|}) \text{, where } \gamma = -2+\sqrt{3} . \tag{10}$$

Here N is an arbitrary integer number, $j = 0, \pm 1, \pm 2, \ldots$.

Out of formula (10) it is seen that relative error of the formula (9) in nodal points is equal to $\varepsilon_j = \gamma^{|j|}$. This error is alternating and decreases fast during counter-wise movement from point of discontinuity because $\gamma = -0.2679$. Maximal error value is reached in the first point. In the third point the relative error is less than 2 per cent.

Table 1 displays the function *f(x)* in nodal points at $N = 60$ and at series (8) there are 10000 terms (2nd column) and 60 terms ($f_N(x)$, 3rd column). The 4th and 5th columns display the precise *f* function value and approximate series value (9) when they retain 60 terms ($f_N(x)$).

Table № 1
Estimation of efficiency of the formula of convergence improvement (9) for the series (8)

x	$f(x)$	$f_N(x)$	$f(x)$	$f_N(x)$
0	-0,00009	0	0	0
0,1047	1,0003	0,90264	1,26302	1,26795
0,2094	1,00016	0,94957	0,93026	0,9282
0,3142	0,9998	0,96585	1,01893	1,01924
0,4189	1,00008	0,97399	0,99503	0,99484
0,5236	1,00006	0,97882	1,00139	1,00138
0,6283	0,99991	0,98197	0,99966	0,99963
0,7330	1,00005	0,98416	1,00011	1,0001
0,9425	0,99992	0,98689	1,00001	1,00001
1,0472	1,0001	0,98775	1	1
1,2566	0,99996	0,98885	1	1
1,3614	1,00003	0,98916	1	1
1,5708	0,99994	0,98939	1	1

Table 1 displays that to sum up directly the slowly converging series with high accuracy one has to calculate up to 10000 terms. The formula of convergence improvement allows finding of series value with accuracy higher than 1 per cent everywhere except for three nodal points being closest to point of discontinuity; the series in formula (6) is fast converging (data of 4th and 5th columns). The presented results illustrate inherent features of the selected formula of convergence improvement: high accuracy in the areas where the function is smooth; fast decreasing of errors during counter-wise movement from the function special points.

Let us use the obtained formula for convergence improvement of series that describe functions being used during conformable displaying of the exterior of the unit disk in given area

$$\omega(\zeta) = c\zeta + \sum_{n=0}^{\infty} c_n \zeta^{-n}, \quad |\zeta| \geq 1,$$

where c_j are given coefficients.

The closest convergence of this series appears on the line where

$$\omega(\sigma) = c\sigma + \sum_{n=0}^{\infty} c_n e^{-\mathrm{int}}, \quad \sigma = e^{it}, \quad 0 < t \leq 2\pi . \tag{11}$$

After convergence improvement in correlation (11) we get

$$\omega(\sigma) = c\sigma + \sum_{n=0}^{\infty} c_n \Gamma_n e^{-\mathrm{int}} . \tag{12}$$

Having extended analytically this series we obtain approximate formula for conformable displaying

$$\omega(\zeta) = c\zeta + \sum_{n=0}^{\infty} c_n \Gamma_n / \zeta^n . \tag{13}$$

Let us present some properties of formula (13). During application of this formula the ellipsis stays the ellipsis with the same correlation of semi-axis, but its dimensions grow into $g(h)$ times. Really, in general case for arbitrary allocation of ellipsis hole we get

$$\omega(\zeta) = a\zeta + b/\zeta + c,$$

where a, b, c are known constants.

Having applied the suggested convergence improvement we obtain

$$\omega(\zeta) = \Gamma_1\left(a\zeta + b/\zeta\right) + c .$$

As result it is seen that at $h \sim 0.1$ we have $\Gamma_1 \sim 1.001$, i.e. the dimensions remain practically unchangeable. It also means that selected transformation retains rectilinear crack the same crack. The boundaries fragments being ellipses, straight lines, and cracks retain their forms except for small junction areas between them.

Fourier series convergence improvement in Laplace inversion formula on Prudnikov main formulas.

Let us study the problem of function *f(t)* allocation on the basis of popular Laplace integral image $F(s) = \int_0^\infty f(t)\exp(-st)dt$. We mark via δ constant the fact that function $F(s)$ is analytical at $\mathrm{Re}(s) > \delta$. Then there exists the precise formula of original exterior through its image [6]

$$f(t) = \frac{1}{l}\exp(ct/l)\sum_{n=-\infty}^{\infty} F(s_n)\exp(2\pi nit/l) - R_1 , \tag{14}$$

where $s_n = (c + 2\pi ni)/l$; c is the constant to improve solution convergence ($\mathrm{Re}(c) > 0$); l is a certain constant, $0 < t < l$

$$R_1 = \sum_{n=1}^{\infty}\exp(-nc)\, f(t + nl). \tag{15}$$

As a rule, the series in formula (14) converges slowly due to the fact that for wide class of functions $F(s_n) = O(1/n)$, where at large n values the series is alternating. That is why during series calculations with formula (14) one has to keep a large number of terms in it. Accordingly, it is problematic to use the formula (14) directly. In certain list of issues [7 – 9] for the class of functions with known values of the original and its derivative at $t = 0$ and at large values of the independent variable the were obtained the specified inversion formulas that facilitate calculating the original with controlled accuracy via fast converging series.

Let us investigate the wide-spread case when the only known fact is original, which can be approximated with predetermined accuracy by the first degree piecewise-continuous polynomials.

To apply formula (14) it is necessary to sum-up slowly converging series with predetermined accuracy

$$S(t) = \sum_{n=-\infty}^{\infty} A_n \exp(2\pi nit/l), \quad 0 < t < l, \quad \text{where} \quad A_n = F(s_n).$$

We indicate that formula (7) can be efficiently applied with this purpose. As an example let us study the image predetermined with the formula $F(s) = 1\big/\sqrt{s^2 + a^2}$, where $a = \mathrm{const}$. It should be mentioned, that an original for this image is the function $f(t) = J_0(at)$, where $J_0(at)$ is the Bessel first class function. We accepted $c = 8$, $a = 1$, $l = 6$ and denied remainder term R_1, as it has small multiplier here $e^{-8} \approx 3 \cdot 10^{-4}$.

Table 2 displays values of precise function $y_T = f(t)$ and relative error in per cent

$$\varepsilon_1 = \frac{y_1 - y_T}{y_T}100 , \quad \varepsilon_2 = \frac{y_2 - y_T}{y_T}100 ,$$

where y_1 is the function obtained on the basis of formula (7) at $N = 60$ (there are 60 terms in series), y_2 is starting series with 1000 terms.

Table № 2.
Precise values of the originals and the error of Laplace inversion formula

	$y = J_0(x)$			$y = e^{-x}$		
T	y_T	ε_1	ε_2	y_T	ε_1	ε_2
0,000	1,000	-1,014	-50,0	1,000	-1,345	-50,0
0,500	0,938	-0,153	0,099	0,607	-0,448	0,140
1,000	0,765	-0,226	0,013	0,368	-0,450	0,000
1,500	0,512	-0,349	-0,212	0,223	-0,450	-0,526
2,000	0,224	-0,717	1,206	0,135	-0,450	1,953
2,500	-0,048	2,523	6,725	0,082	-0,449	-3,985
3,000	-0,260	0,272	0,007	0,050	-0,453	0,039
3,500	-0,380	0,044	-3,236	0,030	-0,490	40,91
4,000	-0,397	-0,070	9,601	0,018	-0,769	-207,7
4,500	-0,321	-0,122	-20,03	0,011	-3,078	579,2
5,000	-0,178	0,623	0,333	0,007	-29,2	-11,47

As it can be seen from Table 2 the suggested formula (7) facilitates higher accuracy at 60 terms in series than direct calculation of series with 1000 terms. The most relative errors of the formula (7) occur in points where function f*(x)* is small in terms of value.

Table 2 also displays the results of calculations for the function $f(t) = e^{-t}$, which image is $F(s) = 1/(s+1)$. We can see that original allocation accuracy due to formula (7) for this function is also high.

Conclusions. The author suggested Fourier series convergence improvement for the functions that can be approximated with sufficiently high accuracy with the method of least squares by piecewise – continuous polynomials of the first degree along the entire task section within a series. The obtained formulas are applied to improve the series convergences which appear at Laplace numerical transformation by means of Prudnikov formula. The basic problem in this method is reduced to finding the sum of slowly convergent Fourier series. Given examples illustrate the efficiency of given approach for numerical determination the originals on basis of their Laplace images.

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СПОСІБ ПОКРАЩЕННЯ ЗБІЖНОСТІ РЯДІВ ФУР'Є ТА ЙОГО ЗАСТОСУВАННЯ ДЛЯ ЧИСЛОВОГО ОБЕРНЕННЯ ПЕРЕТВОРЕННЯ ЛАПЛАСА

Тетяна Соляр

Інститут прикладних проблем механіки і математики ім. Я. С. Підстригача НАН України, Львів, Україна

Резюме. *Розв'язок широкого кола задач математичної фізики може бути отримано у вигляді рядів Фур'є. При розгляді задач, що пов'язані з дослідженням локалізованих дій, отримують швидкозмінні розв'язки, у зв'язку з чим ряди повільно збігаються. Для розв'язування складних задач метод рядів Фур'є використовують сумісно з іншими підходами. Зокрема, при додатковому застосуванні методу граничних елементів коефіцієнти рядів знаходяться шляхом розв'язування одно- або двовимірних інтегральних рівнянь, що відповідно вимагає значного обсягу обчислень. При цьому коефіцієнти рядів знаходять з певними похибками, що відповідно може призвести до втрати точності розрахунків. У таких випадках актуальною є проблема покращення збіжності рядів з контрольованою точністю розрахунків. Нижче запропоновано метод покращення збіжності рядів Фур'є для функцій, які можуть бути з достатньо високою точністю апроксимовані методом найменших квадратів кусково-неперервними поліномами першого степеня на всьому проміжку задавання ряду.*

Ключові слова: *ряди Фур'є, покращення збіжності рядів, кусково-неперервні поліноми, конформне відображення, числове обернення перетворення Лапласа.*

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