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FLUID-STRUCTURE INTERACTION IN FREE VIBRATION ANALYSIS OF PIPELINES

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Summary. *We analyze the impact of the transported fluid on the natural frequencies of a pipeline. Solution of coupled acousto-mechanical problem for straight pipe is given by the Transfer Matrix Method. The equations for junction coupling are given. Several practical examples demonstrate the importance of taking into account the fluid-structure interaction. It is shown that treating fluid as an added mass at the axial vibrations can lead to significant errors.*

Key words: *pipeline, acousto-mechanical vibration,* natural frequency

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Introduction. The presence of the fluid in the pipe affect the dynamic behavior of the considered system. In its simplest form, one can take into account the fluid as an added mass, but this approach can lead to significant errors, especially in the calculation of dynamic events such as a water hammer. In a more detailed analysis is necessary to consider the interaction that occurs between the pipe and the fluid. The interaction between the internal fluid (liquid or gas) and the pipeline is performed by superimposing of the four mechanisms [1 – 3]:

- Fitting coupling;
- Poisson coupling;
- Friction coupling;
- Burdon coupling.

The interaction may be strong or bilateral, and unilateral or weak. One-way interaction is a classical scheme, at the beginning the speed and pressure in the fluid are calculated, and then these values as external loads are applied to the pipeline. Calculation scheme of bilateral interaction is more complicated, but allows you to get the correct dynamic response of the system.

The main effects of fluid-structure interaction are problem dependent [1]. When compared to predictions of conventional water hammer and uncoupled analyses, predictions including fluid-structure interaction may lead to: higher or lower extreme pressures and stresses, changes in the natural frequencies of the system, and more damping and dispersion in the pressure and stress histories. Fully coupled analysis is required in the calculation of compliant structures, which are subject to sudden loads. Generally speaking, coupled analysis gives more accurate results in the calculation pressures, stresses and displacements, and its own resonance frequency and damping forces in the supports.

The natural frequencies of the pipeline are usually obtained by taking into account the fluid as an added mass. The natural frequencies of the fluid, if necessary, get on the assumption of the rigid walls. This approach will give wrong results in systems where the natural frequencies of the pipeline and the fluid are close to each other. A coupled analysis of the interaction on the fittings (i.e. pipe bends) will lead to a separation of coincident natural frequencies [1, 4]. Thus, the coupling accounting prevents resonance, which is obtained by the uncoupled analysis.

Despite the practical importance and numerous studies [1, 5], the calculation scheme of coupled FSI is in the area of academic interest and almost has no use in practice. It is due to several factors:

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- The solution of separate problems by numerical methods. These solutions are aimed to show the presence of the coupling effect and do not offer a universal algorithm for solving the entire class of similar problems. Such solutions are cumbersome and are hardly programmed, their practical application is limited.
- Pipeline software packages (ASTRA, CAESAR) do not have the possibility of coupled analysis, fluid is accounted as an added mass;
- Universal FEM computer codes (ANSYS, ABAQUS), can solve the problems of coupled FSI only in three-dimensional form, that is expensive and impractical in the calculation of piping systems.

Show practicability of coupled acousto-mechanical calculations in the analysis of piping vibration is the main goal of this work. To solve this purpose it is necessary to address the following tasks:

- Summarize the work on the analysis of the FSI for complex systems;
- Demonstrate the practical application of the proposed analysis algorithm and show the importance of considering FSI.

At the beginning the brief derivation of the basic equations is given, in more detail it can be found elsewhere [2, 3]. Then, the search algorithm of the natural frequencies is presented, and in the second part a few practical examples are considered.

Mathematical equations for the straight pipe. Set of the Skalak equations for the straight pipe run:

$$-\frac{\partial u_f(x,t)}{\partial x}=\frac{P_f(x,t)}{K_f}\left(1+2\frac{RK_f}{hE}\right)+2\mu\frac{N_t(x,t)}{FE}; \tag{1a}$$

$$-\frac{\partial P_f(x,t)}{\partial x}=\rho_f\frac{\partial^2 u_f(x,t)}{\partial t^2}; \tag{1b}$$

$$-\frac{\partial v_t(x,t)}{\partial x}=\frac{N_t(x,t)}{FE}+\mu\frac{P_f(x,t)R}{hE}; \tag{1c}$$

$$-\frac{\partial N_t(x,t)}{\partial x}=F\rho_t\frac{\partial^2 v_t(x,t)}{\partial t^2}, \tag{1d}$$

where h is the pipe wall thickness; R is the inner radius, F – is the cross-sectional area of the pipe material, $F=\pi h(2R+h)$; $\rho_{f/t}$ – is the density of the fluid or pipe material; t is time, N_t is the axial force, v_t is the pipe longitudinal displacement, P_f is the fluid pressure pulsation u_f is fluid longitudinal displacement. K_f is bulk elasticity of fluid and E – is the Young modulus of pipe material.

In classical case when we assume that Poisson coefficient $\mu=0$, we get two independent system of equations. The first one will describe acoustical vibrations of fluid in the pipe, notice that expression $\left(1+2RK_f/hE\right)$ in the eq.(1a) is adjusted for pipe thinness. The second one describe axial vibration of a pipe.

We assume all the processes to be harmonic ones, i.e., proportional to $\sin\omega t$, where ω is some frequency of harmonic vibrations. The same notation will be used for the new introduced functions that depend on x only. Differentiation of (1a) and (1c) and substitution of (1b) and (1d) into them give the following system of initial differential equations:

$$\frac{d^2 v_t}{dx^2} = -v_t \cdot a - u_f \cdot \mu\alpha d \; ; \tag{2a}$$

$$\frac{d^2 u_f}{dx^2} = -2v_t \cdot \mu a - u_f \cdot d \; , \tag{2b}$$

where:

$$a = \omega^2 \frac{\rho_t}{E}, \; d = \frac{\rho_f}{K_f}\left(1 + \frac{2RK_f}{hE}\right)\omega^2 \; ; \tag{3a}$$

$$\alpha = \alpha_1/(1 + 2\alpha_1) < 1, \; \alpha_1 = \frac{RK_f}{hE} \, . \tag{3b}$$

A general solution of the system (2) is written in TMM form:

$$\begin{aligned} v_t(x) &= v_{t0}\left(\cos\lambda_1 x + \gamma F_2(x)\right) - u_{f0}\mu\gamma_y F_2(x) - \\ &- \tilde{N}_{t0}\left(\sin\lambda_1 x/\lambda_1 + \gamma F_1(x)\right) + \tilde{P}_{f0}\mu\gamma_y F_1(x); \end{aligned} \tag{4a}$$

$$\begin{aligned} u_f(x) &= v_{t0}\mu\gamma_x F_2(x) + u_{f0}\left(\cos\lambda_2 x - \gamma F_1(x)\right) - \\ &- \tilde{N}_{t0}\mu\gamma_x F_1(x) - \tilde{P}_{f0}\left(\sin\lambda_2 x/\lambda_2 - \gamma F_1(x)\right), \end{aligned} \tag{4b}$$

$$\begin{aligned} \tilde{N}_t(x) &= v_{t0}\left(\lambda_1 \sin\lambda_1 x + \gamma F_3(x)\right) - u_{f0}\gamma_y\mu F_3(x) + \\ &+ \tilde{N}_{t0}\left(\cos\lambda_1 x + \gamma F_2(x)\right) - \tilde{P}_{f0}\mu\gamma_y F_2(x); \end{aligned} \tag{4c}$$

$$\begin{aligned} \tilde{P}_f(x) &= v_{t0}\mu\gamma_x F_3(x) + u_{f0}\left(\lambda_2 \sin\lambda_2 x - \gamma F_3(x)\right) + \\ &+ \tilde{N}_{t0}\mu\gamma_x F_2(x) + \tilde{P}_{cp0}\left(\cos\lambda_2 x - \gamma F_2(x)\right). \end{aligned} \tag{4d}$$

where functions $\tilde{N}_t, \tilde{P}_f$ are connected with initial functions N_t, P_f with relations:

$$\tilde{P}_f = \frac{P_f}{K_f}\left(1 + \frac{2RK_f}{hE}\right) + 2\frac{\mu N_t}{FE} \, , \tag{5a}$$

$$\tilde{N}_t = \frac{N_t}{FE} + \mu\frac{P_f R}{hE} \, . \tag{5b}$$

The following functions are entered for convenience:

$$F_1(x) = \sin \lambda_1 x / \lambda_1 - \sin \lambda_2 x / \lambda_2 \tag{6a}$$

$$F_2(x) = \cos \lambda_1 x - \cos \lambda_2 x\,; \tag{6b}$$

$$F_3(x) = \lambda_1 \sin \lambda_1 x - \lambda_2 \sin \lambda_2 x\,, \tag{6c}$$

Also following notations are introduced:

$$\gamma_x = \frac{a}{(\lambda_1)^2 - (\lambda_2)^2}\,;\ \gamma_y = -\frac{\alpha d}{(\lambda_1)^2 - (\lambda_2)^2}\,;\ \gamma = -\frac{(\lambda_1)^2 - a}{(\lambda_1)^2 - (\lambda_2)^2} = \frac{(\lambda_2)^2 - d}{(\lambda_1)^2 - (\lambda_2)^2}\,. \tag{7a}$$

$$(\lambda_{1,2})^2 = \frac{(a+d) \pm \sqrt{(a-d)^2 + 8ad\mu^2\alpha}}{2} \tag{7b}$$

Solutions of eq. (4) have been obtained in the form suitable for programming in TMM. The advantage of such solutions over similar ones is that the set of equations does not degenerate for any fluid and pipe parameters and any Poisson's ratio μ. Furthermore, these expressions are easy to use in practice.

Fitting coupling. Fittings are specific points of the pipelines (tees, bends, valves) in which there is a strong interaction between the fluid and the pipeline. For the calculation of complex pipe systems we need to know the transmission conditions in these particular points.

Closed end(Valve). It can serve as the source of considerable effort, if the closed end of the tube is rigidly fixed pressure wave can be doubled. strong fluid-structure interaction may occur for a flexible pipe end:

$$P_f \pi R^2 + N_t = 0 \tag{8a}$$

$$u_f = v_t \tag{8b}$$

In the calculations also must be consider the presence of the masses at the end, for example a closed valve mass, which modifies (8a):

$$P_f \pi R^2 + N_t = \frac{\partial^2 v}{\partial t^2} M \tag{8a'}$$

Tee element. Also is the source of strong fluid-structure interaction. The equation for pressure and displacement at a junction point is a trivial one and based on the condition for pressure and volume continuity at the point:

$$P_f^1 = P_f^2 = P_f^3 = \ldots \tag{9a}$$

$$\sum_{\text{in}}^{i} F_i \left(u_f^i - v_t{}^i\right) = \sum_{\text{out}}^{j} F_j \left(u_f^j - v_t{}^j\right) \tag{9b}$$

The equation of continuity for mechanical forces at a junction of several elements is well known and based on the equilibrium condition for the junction. To allow for the pressure pulsation P_f , we should put down the total axial force $\vec{N}_n = \vec{N}_x + F \cdot P_f \cdot \vec{t}$ (where $\vec{t}$ is the tangent vector in the section and $F \cdot P_f$ represents a force in the bore due to the inner pressure) in place of the axial force $\vec{N}_x$ in the respective equations. Thus, the equilibrium equation for the junction is written, in view of P_f , as follows:

$$\sum_{\text{in}}^{i}\left((\vec{N}_x^i + F_i \cdot P_f^i \cdot \vec{t}_i) + \vec{Q}_y^i + \vec{Q}_z^i\right) = \sum_{\text{out}}^{j}\left((\vec{N}_x^j + F_j \cdot P_f^j \cdot \vec{t}_j) + \vec{Q}_y^j + \vec{Q}_z^j\right). \tag{10}$$

Mass and dimensions of the tee are neglected, as well as forces change in the fluid. It is believed that the right angle between the pipes is maintained in the process of dynamic interaction. Out of plane vibration, which isn't connected with in-plane vibration, has no FSI mechanism.

Pipe bend (elbow). Elbow – is the most common source of FSI. It can be modeled more efficiently with concept of «dimensionless rotary element» [6], thus curved pipes are presented by a finite set of linear and rotary elements.

In [6] the solution of a common problem for screw rotary element is given. For a planar rotary element the solution is greatly simplified:

$$Q_{y1} = Q_{y0} \cos\theta - N_0 \sin\theta - P_f \cdot F \sin\theta \tag{11a}$$

$$N_1 = N_0 \cos\theta + Q_{y0} \sin\theta - P_f \cdot F(1 - \cos\theta) \tag{11b}$$

In addition to the force transmission, the movement transmission is also present:

$$u_{f1} = u_{f0} - v_{t0}(1 - \cos\theta) + w_{t0} \sin\theta, \tag{11c}$$

$$w_{y1} = w_{y0} \cos\theta - v_0 \sin\theta \tag{11d}$$

$$v_1 = v_0 \cos\theta + w_{y0} \sin\theta \tag{11e}$$

The rest of the equation for the rotary element are similar to the straight pipe connection relations:

$$P_{f1}F_1 = P_{f0}F_0;\ M_1 = M_0;\ \theta_1 = \theta_0. \tag{11f}$$

Out of plane vibration, which isn't connected with in-plane vibration, has no FSI mechanism.

Frequency search algorithm. Acousto-mechanical vibrations are modeled by supplementing the mechanical-vibration TMM with two unknown parameters for each section, two coupling equations for each element, two parameters-continuity equations for each boundary between elements and for the edge conditions. The design model and the method of constructing the mathematical model remain unchanged. Since all of the 14 parameters of the stress-strain state of the fluid and pipe (12 mechanical parameters and two acoustic ones) are related to each other, we should model the coupled acousto-mechanical vibrations «as a whole» rather than split the vibration analysis into subtasks.

In some cases, a separate analysis of acousto-mechanical vibrations without any influence of the pipe displacements on the fluid should be carried out. This is easy to accomplish by using equations (4) as written without beam components. Such a «simplified» analysis makes it possible to find natural frequencies of acousto-mechanical vibrations of a fluid in a pipeline of a preset configuration, disregarding the pipe displacements during vibrations; this is important, for example, for TMM verification and for comparing the results to those published elsewhere.

There are many algorithms to search for natural frequencies, TMM formulation of the governing equations greatly simplifies this algorithm. We use a matrix formulation for a whole system, in contrast to the finite element method the size of this matrix is much smaller, and it is not very tenuous. Boundary conditions, or equations that consider the boundary conditions, such as (8) are also included in this matrix. After all the boundary conditions are substitute, determinant is set to zero, the natural frequencies are determined from this equation. Since we get a non-linear equation with respect to frequency, the most convenient is to use numerical methods for solving frequency (characteristic) equation.

Test examples. To verify the correctness of implementation of the mutual influence of acoustical and mechanical vibrations we considered a number of test cases. In the solutions obtained we checked whether all of the coupling equations, boundary conditions, and conjugation conditions were met.

Straight pipe. The use of the analysis is illustrated for the case of a fluid-filled pipe, closed at both ends and subjected to axial excitation [7, 8]. The pipe is suspended on wires and is excitated be axial impact of a 5 m long, 51 mm diameter steel rod. Pressures, strains and structural velocities are measured at several positions along the pipe. Transient cavitation does not occur because of the initial static pressure, the movement of the pipe on the wires is considered horizontal. It is an accurate experiment because of the avoidance of complications encountered in conventional reservoir-pipe-valve systems, such as unknown support conditions, unsteady valve behavior, non-constant reservoir pressure, disturbing pump vibrations, etc.

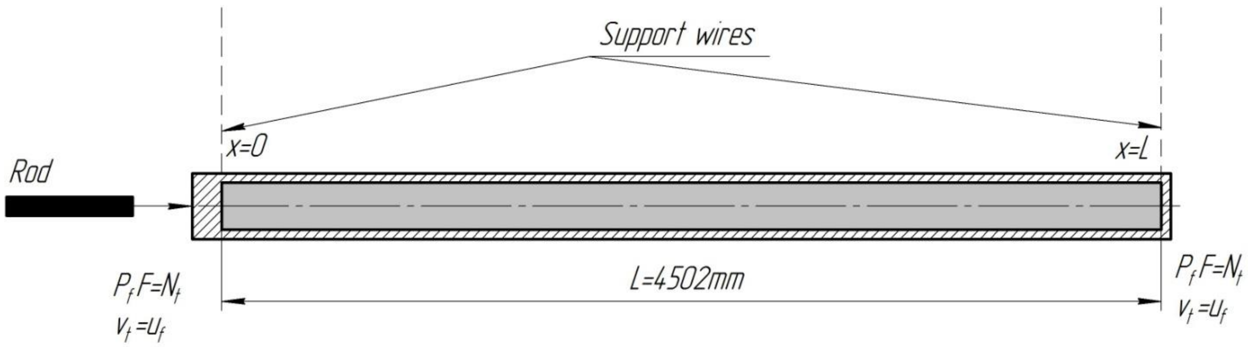

Figure 1. Discrete axial impact of free hanging pipe.

The pipe parameters: $h = 3.945mm$, $R = 26.01mm$, $\rho_m = 7985 kg/m^3$, $E = 1.68 \cdot 10^{11} Pa$, $\mu = 0.3$; the fluid parameters: $\rho_f = 999 kg/m^3, K_f = 2.14 \cdot 10^9 Pa$. On the left there is cap about 60mm thick, on the right cap about 5 mm.

Pure acoustic frequencies can be determined by the formula:

$$f_n = \frac{c \cdot n}{2 \cdot L}, \quad n = 1, 2, 3... \tag{12}$$

Here $c = \sqrt{K/\rho}$ – speed of sound in fluid. Speed of sound with wall correction:

$$c^* = \sqrt{\frac{K_f}{\rho}\left(1 + 2\frac{RK_f}{hE}\right)} \tag{13}$$

Mechanical frequency determined trivially, for beam with free ends. For a coupled analysis of such a system we will use eq (4) for the straight pipe directly. Both ends of the pipe are closed, so the boundary conditions (8) are used, more precisely is to take into account mass of end caps, i.e. instead of (8a) to use the expression (8a').

Table № 1.
Natural frequencies (in Hz) of axial vibration of water-filled pipe with free ends

No	Mechanical		Acoustical		Coupled		Experiment [6]
	ρ=7985	ρ*=11046	c=1464	c*=1354	μ=0	μ=0.3	
1	509	433	163	150	169	172	173
2	1019	866	325	301	288	286	289
3	1528	1299	488	451	453	453	459
4	2038	1733	650	602	493	493	485
5	2547	2166	813	752	629	633	636
6	3057	2599	975	902	743	741	750
7	3566	3032	1138	1053	906	907	918
8	4075	3465	1300	1203	980	980	968

Table 1 summarizes the results of measurements and calculations. Mechanical frequency are defined excluding water (ρ=7985 kg/m^3) and accounting for the water as the added mass (ρ*=11046 кг/м^3). Analysis of Table 1 shows that this approach leads to significant difference between the experimental and calculated values, but exactly this approach is incorporated in pipeline software (CAESAR, ASTRA). The acoustic frequency are calculated both with (c*=1354 m/s) and without (c=1464 m/s) accounting for pipe thinness. This approach also does not provide acceptable accuracy especially for higher frequencies. Coupled calculations are presented in two versions: with and without taking into account the Poisson coupling. In this example, the interaction on the fittings is dominant, Poisson coupling makes an insignificant contribution. Analyzing the results of table 1 is obvious the need to consider FSI when calculating the frequency of the axial vibrations.

In the next experimental study pipe was struck lateraly [7,8] (Figure 2).

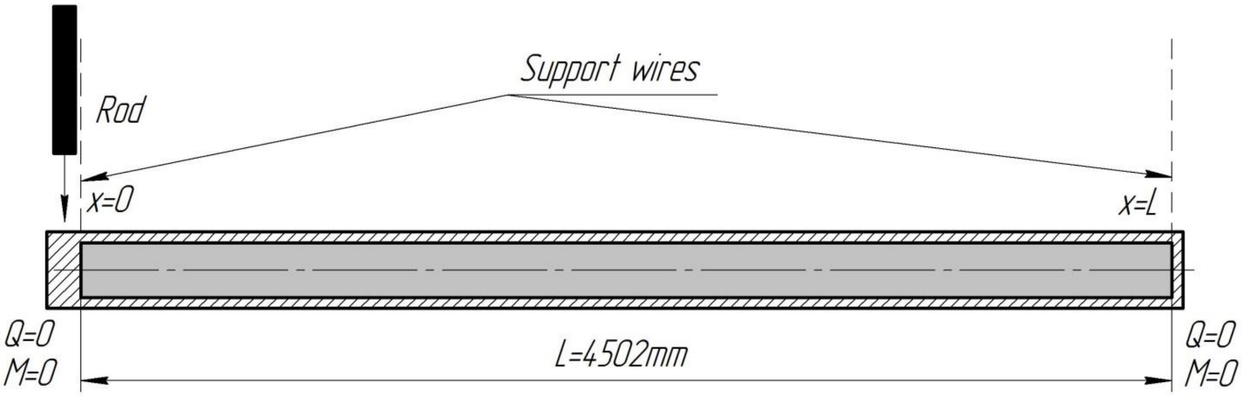

Figure 2. Discrete lateral impact of free hanging pipe.

Calculation results for air-filled pipe and water-filled pipe are summarized in table.2. In this results fluid is considered as an added mass.

Table № 2.
Natural frequencies (in Hz) of lateral vibration of pipe with free ends.

Air-filled pipe						Water-filled pipe					
No	Exper.	Calc.	No	Exper.	Calc.	No	Exper.	Calc.	No	Exper.	Calc.
1	15	16	7	373	385	1	13	14	7	320	328
2	41	44	8	478	491	2	36	37	8	411	418
3	81	86	9	595	607	3	70	73	9	510	518
4	135	141	10	723	735	4	116	120	10	619	627
5	202	210	11	859	872	5	173	179	11	737	744
6	281	291	12	1008	1019	6	241	248	12	864	870

As we can see, the fluid does not have a noticeable effect on the lateral vibration. Coupling in the transverse direction occurs due to the centripetal force and the Coriolis force, and is directly proportional to the flow rate, the dynamic buckling of pipe [9] occurs at considerable high fluid speeds. However, such speeds are much higher than the fluid speed in industrial pipes [9].

Pipeline system [4]. Let's consider system which is shown at Fig.3 [4]. The system has a total length of $L = 33.944m$, pipe outer radius $R = 0.15m$, wall thickness $h = 5mm$, bend compliance $K = 15.95$, fluid density $\rho_f = 1000 kg/m^3$, metal density $\rho_t = 7800 kg/m^3$, Young modulus $E = 2 \cdot 10^{11} Pa$, speed of sound in fluid $c = 1000 m/s$.

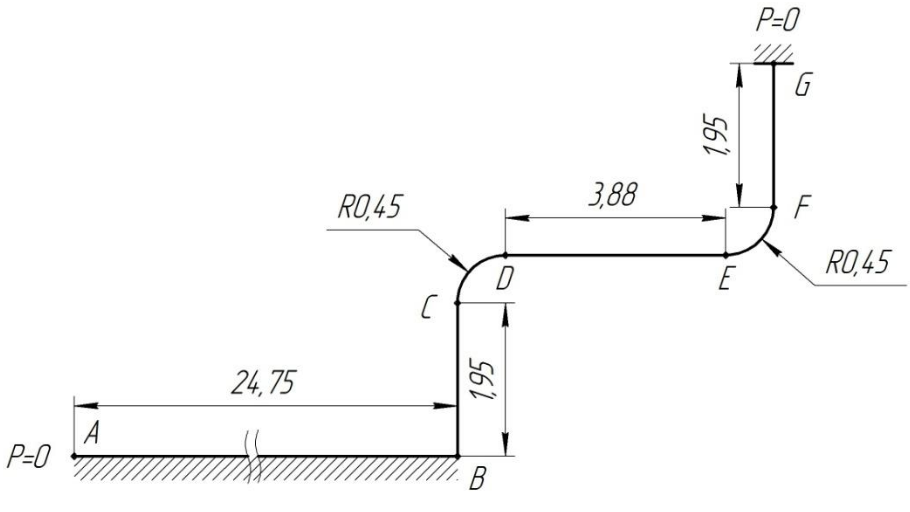

Figure 3. Model of a fluid-filled piping system [4].

Points B and C at the straight runs are mechanically fixed, the acoustical edge conditions at points A and G are as follows: $P_f = 0$. In [4] the mechanical and coupled acousto-mechanical vibrations were modeled only for the pipe portion from point B to point G, while the run between points A and B was taken to be purely acoustical. Geometrical and mechanical parameters of the piping system are designed so that the natural frequencies of the mechanical model and purely acoustical model would have the same value, in order to increase the coupling influence.

The acoustical natural frequencies are not discussed as they are easy determine by simple formulation (12), using numerical values we can find the acoustical frequencies:

$f_1 = 14.73 Hz$, $f_2 = 29.46 Hz$. Then if we consider fluid as an added mass for axial and lateral vibrations, we can find mechanical frequencies 14.96 Hz and 26.89 Hz, which are closed to acoustical ones. Afterwards we performed coupled calculations, where fluid was considered as an added mass only in lateral direction, in axial direction we used eq. (4). Strong coupling occurs at junctions, bends CD and EF, in accordance to eq. (11). Results of calculations are summarized in table 3. Calculations were performed for in-plane and out of plane motion.

Table № 3.
Comparison of natural frequencies for pipeline (Fig. 3).

	Frequency, Hz					
	In plane			Out of plane		
Frequency No	1	2	3	1	2	3
	Mechanical (Added mass)					
CIRCUS [4]	14.8	26	–	–	–	–
Code_ASTER [4]	14.91	27.05	–	14.50	21.01	–
Our calculation	14.96	26.89	78.33	15.44	21.10	–
	Acoustomechanical (Coupled)					
CIRCUS [4]	12.65	18.04	26.30	–	–	–
Code_ASTER [4]	12.72	18.09	26.66	14.29	20.90	14.29
Our calculation	12.93	18.24	26.86	15.44	21.09	48.80

Figure 4 shows amplitude-frequency characteristics for pressure at points D and F (Fig. 3), which are close to those obtained in [4]. It should be mentioned that because of the complex mutual influence of acoustical and mechanical vibrations the values of natural frequencies of coupled vibrations (the resonance spikes in the pressure curves) do not coincide with the values of acoustical and mechanical frequencies and are not mean ones in between.

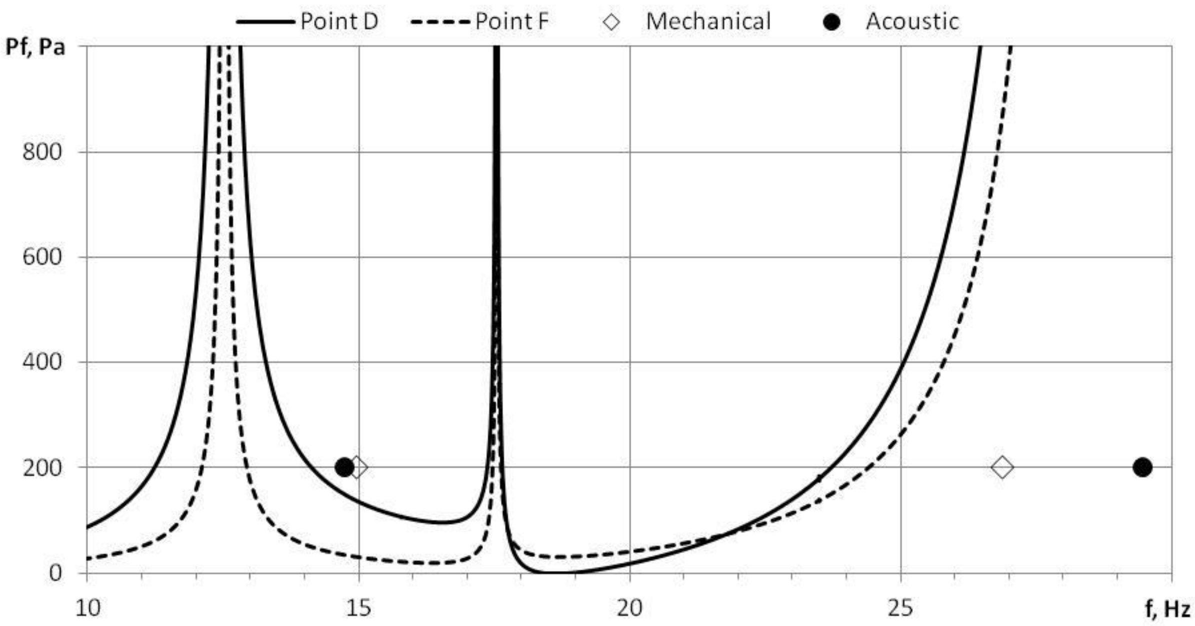

Figure 4. Amplitude-frequency characteristics for pressure at points D and F. Solid symbols – natural frequencies of acoustic vibrations, open symbols – natural frequencies of mechanical vibrations.

This piping system is a good illustration the difference between the coupled and the mechanical calculations. It is shown that due to the coupling, the natural frequencies of the piping system can't coincide, and that no super-resonance can occur.

Conclusions. In this paper we summarize the experience of the analysis of pipeline natural frequencies with Fluid-Structure Interaction. For the straight section of the pipeline acousto-mechanical vibrations equations are written in form of TMM, Poisson coupling is considered. The equation of junctions coupling at bends, tees, caps are formulated. Several practical examples show the importance of taking into account FSI, in particular it is established that:

- in the calculation of axial vibrations, treating fluid as an added mass can lead to significant errors, for a more realistic simulation it is necessary to use coupled analysis;
- in the calculation of lateral vibrations fluid can be considered as an added mass;
- due to mutual interactions, coupled vibrations frequencies do not coincide with the values of acoustical or mechanical frequencies and are not mean ones in between;
- strong liquid-pipe coupling occurs at the closed ends and elbows, weak coupling exists along the pipes due to axial-radial Poisson contraction/expansion.

Coupled analysis gives more accurate results in the calculation of pressures, stresses and displacements, and natural frequency and damping forces in the supports.

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УДК 539.4

ВРАХУВАННЯ ВЗАЄМОДІЇ СЕРЕДОВИЩЕ-ТРУБОПРОВІД ПРИ АНАЛІЗІ ВЛАСНИХ ЧАСТОТ ТРУБОПРОВІДНОЇ СИСТЕМИ

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Резюме. *Аналізується вплив середовища, що транспортується, на власні частоти трубопроводу. Рішення зв'язаної акусто-механічної задачі для прямої труби записані з використанням методу початкових параметрів. Наведені рівняння взаємодії на фітингах. На декількох практичних прикладах продемонстрована важливість врахування взаємодії середовище-трубопровід. Показано що врахування середовища як додаткової маси при поздовжніх коливаннях може привести до значних помилок.*

Ключові слова: *трубопровід, акусто-механічні коливання, власна частота.*

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