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ROBUST STABILITY OF LINEAR CONTROL SYSTEM WITH MATRIX UNCERTAINTY

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Resume. *The work is devoted to working out of new methods for analysis of robust stability and robust stabilization of linear dynamic systems. Sufficient stability conditions of the zero solution are formulated for a linear system with uncertain coefficient matrices and a measured output feedback. In addition, a common quadratic Lyapunov function and ellipsoidal set of stabilizing matrixes of amplification factors of a output feedback are given for the whole set of system. Application of the results is reduced to a solution of systems of linear matrix inequalities.*

Keywords*: control system, output feedback, robust stability, matrix uncertainty, ellipsoid.*

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Problem setting. In the applied problems of analysis and synthesis of real objects differential and difference systems with uncertain parameters and functional structure are used (see, eg, [1] – [4]). This focuses on the objectives of robust stability and robust stabilization.

As multiple robust stability of dynamic systems we mean parametric or functional set characterizing uncertainty of the given structure of the system and its controlling elements. In particular, in the uncertain linear models matrix of coefficients and reverse connection may belong to some given sets in the corresponding spaces (polytopes, ellipsoids, matrix spacing, etc.).

The task of stabilizing of the control system is to build a static or dynamic control to ensure the asymptotic stability of the equilibrium of the closed system with arbitrary values of uncertain elements. Typically, this problem is reduced to solving systems of linear matrix inequalities (LMN).

Analysis of recent research and publications. To describe the uncertainties and conditions of robust stability of systems matrix intervals and polytopes are used [1, 5, 6]. In the works [3, 7] in terms of linear matrix inequalities sufficient conditions for the stability of linear control systems with uncertain coefficient matrix and reverse connection have been obtained. One can find the view of problems and known methods of robust stability analysis and stabilization of control systems in [8, 9].

The aim of the research is to develop new methods of analysis of robust stability and robust stabilization of linear dynamic systems with limited at a rate matrix uncertainties and static reverse connection in dimensional output.

Robust stabilization of control systems. We will consider the continuous linear dynamical system of control:

$$\dot{x} = (A + \Delta A(t))x + (B + \Delta B(t))u, \quad u = Ky, \quad y = Cx + Du, \tag{1}$$

where $x \in \mathrm{P}^n$, $u \in \mathrm{P}^m$ і $y \in \mathrm{P}^l$ – vectors in the appropriate state, control and monitoring of the facility A, B, C і D – stable matrix of the corresponding sizes $n \times n$, $n \times m$, $l \times n$ і $l \times m$, and

$$\Delta A(t) = F_A \Delta_A(t) H_A, \; \Delta B(t) = F_B \Delta_B(t) H_B, \tag{2}$$

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where F_A, F_B, H_A, H_B – stable matrix of appropriate size and matrix uncertainties $\Delta_A(t)$ i $\Delta_B(t)$ satisfy the constraints

$$\|\Delta_A(t)\| \le 1,\ \|\Delta_B(t)\| \le 1 \text{ or } \|\Delta_A(t)\|_F \le 1,\ \|\Delta_B(t)\|_F \le 1, \quad t \ge 0. \tag{3}$$

Hereinafter, $\|\cdot\|$ – Euclidean vector norm and spectral matrix norm, $\|\cdot\|_F$ – matrix Frobenius norm, I – single matrix of appropriate size. To simplify the records of the matrices dependency on t we will omit.

It should be noted that when $\Delta_A(t) = 0$, $\Delta_B(t) = 0$ the system changes into the system without uncertainties under review [10].

We will formulate known criteria of positive and inalienable uncertainties of block matrices.

Lemma 1. [11] *There is an equivalence:*

$$\begin{bmatrix} U & Z \\ Z^T & V \end{bmatrix} > 0 \Leftrightarrow V > 0, \quad U - ZV^{-1}Z^T > 0. \tag{4}$$

If the power V is nondegenerated, then

$$\begin{bmatrix} U & Z \\ Z^T & V \end{bmatrix} \ge 0 \Leftrightarrow V > 0, \quad U - ZV^{-1}Z^T \ge 0. \tag{5}$$

Lemma 2. [12] *If the system of matrix inequalities is implemented*

$$\begin{bmatrix} R - P^{-1} & D^T \\ D & -Q^{-1} \end{bmatrix} < 0, \quad \begin{bmatrix} W & U^T & V^T \\ U & R - P^{-1} & D^T \\ V & D & -Q^{-1} \end{bmatrix} \le 0\ (<0), \tag{6}$$

where $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T \ge 0$, $W = W^T \ge 0$, U, V *i* D *– matrices of appropriate sizes. Then for any matrix the inequality is implemented*

$$W + U^T \Delta(K) V + V^T \Delta^T(K) U + V^T \Delta^T(K) R \Delta(K) V \le 0\ (<0). \tag{7}$$

Lemma 3. [13] *If* L *is symmetric matrix, the matrix* $M_1, \dots, M_r$ *i* $N_1, \dots, N_r$ *have appropriate dimensions. Then, if the numbers are* $\varepsilon_1, \dots, \varepsilon_r > 0$ *matrix inequality is performed*

$$L + \sum_{i=1}^{r} \left(\varepsilon_i M_i M_i^T + \frac{1}{\varepsilon_i} N_i^T N_i \right) \le 0,$$

then the inequality is true

$$L + \sum_{i=1}^{r} \left(M_i \Delta_i N_i + (M_i \Delta_i N_i)^T \right) \le 0,$$

for all $\|\Delta_i\| \le 1$ *or* $\|\Delta_i\|_F \le 1$, $i = 1, \dots, r$.

We will note that Lemmas 2 and 3 are generalizations of the famous statement of the adequacy criterion called Petersen's lemma on matrix uncertainty [14].

The set of management matrices K providing stability in the closed system we will build as an ellipsoid

$$\mathrm{E} = \left\{ K \in \mathrm{P}^{m \times l} : K^T P^{-1} K \le Q \right\}, \tag{8}$$

where $P = P^T > 0$ i $Q = Q^T > 0$ – some positively defined matrices.

We introduce on the set of matrices $\mathrm{K} = \{K : \det(I_m - KD) \ne 0\}$ nonlinear operator

$$\Delta : \mathrm{P}^{m \times l} \to \mathrm{P}^{m \times l}, \ \ \Delta(K) = (I_m - KD)^{-1} K \equiv K (I_l - DK)^{-1}.$$

For the operator Δ performed the property is performed [12]: if $K_1 \in \mathrm{K}$ i $K_2 \in \mathrm{K}$,

$$K_3 = (I_m - K_1 D)^{-1} K_2 \in \mathrm{K}, \ \ \Delta(K_1 + K_2) \equiv \Delta(K_1) + \Delta(K_3)\left[I_l + D\Delta(K_1)\right]. \tag{9}$$

With (1) and (8) follows the inequality

$$w_0(x,u) = \left[x^T, u^T\right] \begin{bmatrix} C^T QC & C^T QD \\ D^T QC & D^T QD - P^{-1} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \ge 0 . \tag{10}$$

Supposed

$$D^T QD < P^{-1}. \tag{11}$$

This is equivalent to the first block inequality of Lemma 2 with $R = 0$. Then out of $x = 0$ follows that $u = 0$ and $x \equiv 0$ are state of the equilibrium system whose stability we are examining. The closed system has the following structure

$$\dot{x} = M(t)x, \ \ M(t) = A + \Delta A + (B + \Delta B)\Delta(K)C. \tag{12}$$

In conditions (8) and (9) we have according to Lyapunov's theorem for discrete systems $\rho(KD) < 1$, and therefore (12) $I_m - KD$ is a non-degenerated matrix.

We will describe the conditions of robust stabilization of the class systems (1).

Theorem 1. *Let for the matrix* A *of the system (1) some* $\varepsilon_1, \varepsilon_2 > 0$ *are carried matrix inequalities (11) and*

$$\begin{bmatrix} A^T X + XA + \varepsilon_1 H_A^T H_A & XB & C^T & XF_A & XF_B \\ B^T X & -P^{-1} + \varepsilon_2 H_B^T H_B & D^T & 0 & 0 \\ C & D & -Q^{-1} & 0 & 0 \\ F_A^T X & 0 & 0 & -\varepsilon_1 I & 0 \\ F_B^T X & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} \le 0 , \tag{13}$$

where $X = X^T > 0$. *Then* $u = Ky$ *with arbitrary control matrix* $K \in \mathrm{E}$ *stabilizes the system (1). Moreover, if (13) holds strict matrix inequality, the given set of controls ensures asymptotic stability of the closed system (12) and a common Lyapunov's function* $v(x) = x^T Xx$.

Bringing. We construct Lyapunov's function for the closed system (12) as $v(x) = x^T Xx$. Then by Lyapunov's Theorem system (12) is stable (asymptotically stable) if for some positively defined matrix $X = X^T > 0$ matrix inequality is implemented

$$(A+\Delta A+(B+\Delta B)\Delta(K)C)^T X + X(A+\Delta A+(B+\Delta B)\Delta(K)C) \le 0 \;\; (<0). \qquad (14)$$

We rewrite the last inequality in the form

$$(A+\Delta A)^T X + X(A+\Delta A) + C^T \Delta^T(K)(B+\Delta B)^T X + X(B+\Delta B)\Delta(K)C \le 0$$

and use lemma 2 putting 2

$$U = (B+\Delta B)^T X\,,\;\; V = C\,,\;\; W = (A+\Delta A)^T X + X(A+\Delta A)\,,\;\; R = 0\,.$$

Then the second block inequality in (6) has the form

$$\begin{bmatrix} (A+\Delta A)^T X + X(A+\Delta A) & X(B+\Delta B) & C^T \\ (B+\Delta B)^T X & -P^{-1} & D^T \\ C & D & -Q^{-1} \end{bmatrix} \le 0\,. \qquad (15)$$

Using the structure of matrix uncertainties $\Delta_A(t)$, $\Delta_B(t)$, we decompose the last inequality

$$\begin{bmatrix} A^T X + XA & XB & C^T \\ B^T X & -P^{-1} & D^T \\ C & D & -Q^{-1} \end{bmatrix} + \begin{bmatrix} H_A^T \\ 0 \\ 0 \end{bmatrix} \Delta_A^T \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \Delta_A \begin{bmatrix} H_A & 0 & 0 \end{bmatrix} +$$
$$+ \begin{bmatrix} 0 \\ H_B^T \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \Delta_B \begin{bmatrix} 0 & H_B & 0 \end{bmatrix} \le 0\,,$$

which is done for lemma 3 if there are $\varepsilon_1, \varepsilon_2 > 0$ such as

$$\begin{bmatrix} A^T X + XA & XB & C^T \\ B^T X & -P^{-1} & D^T \\ C & D & -Q^{-1} \end{bmatrix} + \varepsilon_1 \begin{bmatrix} H_A^T H_A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_1} \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} +$$
$$+ \varepsilon_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & H_B^T H_B & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_2} \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} \le 0\,.$$

According to lemma 1 the obtained matrix inequality is equivalent to inequality (13).

The theorem is proved.

You can give another proof of Theorem 1 using the theorem of S- procedure for quadratic forms with one restriction [15]. It argues that inequality $w(x,u) \le 0 \;\; (<0)$ with limit $w_0(x,u) \ge 0$ is equivalent to the ratio

$$w(x,u)+\tau w_0(x,u)\le 0\ (<0),\quad x^T x+u^T u\ne 0\,, \tag{16}$$

where $\tau>0$ – a certain number. We can give $\tau=1$. Then according to (10) and the derivative of the system

$$w(x,u)=\left[x^T,u^T\right]\begin{bmatrix}(A+\Delta A)^T X+X(A+\Delta A) & X(B+\Delta B)\\ (B+\Delta B)^T X & 0\end{bmatrix}\begin{bmatrix}x\\ u\end{bmatrix}\le 0\ (<0)\,,$$

We rewrite (16) as

$$\left[x^T,u^T\right]\begin{bmatrix}\Omega & X(B+\Delta B)+C^T QD\\ (B+\Delta B)^T X+D^T QC & D^T QD-P^{-1}\end{bmatrix}\begin{bmatrix}x\\ u\end{bmatrix}\le 0\ (<0)\,,$$

where $\Omega=(A+\Delta A)^T X+X(A+\Delta A)+C^T QC$. Using lemma 1, we obtain the inequality (15).

In theorem 1 the system (1) without controlling ($u=0$) should be stable. If the zero state of the system (1) without control is unstable, then we will look for a plurality of stabilizing controls from the ellipsoid

$$\mathrm{E}_0=\left\{K\in \mathrm{P}^{\,m\times l}:(K-K_0)^T P^{-1}(K-K_0)\le Q\right\},$$

which is equivalent to the matrix choice

$$K=K_0+\tilde{K}\,,\quad \tilde{K}\in \mathrm{E}\,. \tag{17}$$

Firstly we have to obtain matrix K_0 which stabilizes the system

$$\dot{x}=M_0 x\,,\quad M_0=A+\Delta A+(B+\Delta B)\Delta(K_0)C\,.$$

Matrix K_0 can be obtained with methods described in [5].

We construct conditions of robust stabilization of class (1) with the matrix control (17). According to (1), (8) and (17) the inequality should be performed

$$\left[x^T,\,u^T\right]\begin{bmatrix}C^T QC-C^T K_0^T P^{-1}K_0 C & C^T QD+C^T K_0^T P^{-1}G\\ D^T QC+G^T P^{-1}K_0 C & \Delta\end{bmatrix}\begin{bmatrix}x\\ u\end{bmatrix}\ge 0\ ,$$

where $\Delta=D^T QD-G^T P^{-1}G$, $G=I_m-K_0 D$. We suppose that

$$\Delta<0\,. \tag{18}$$

Then if $x=0$ follows $u=0$ and $x\equiv 0$ is a state of the system equilibrium.

If (18) matrix G must be non-degenerated. Therefore, the defined values of the operator are $\Delta(K_0)=(I_m-K_0 D)^{-1}K_0$. If it is defined as the value $\Delta(K)$ i $\Delta(\hat{K})$, where $\hat{K}=G^{-1}\tilde{K}$. Truly in conditions (17) i (18) we obtain

$$D^T\tilde{K}^T P^{-1}\tilde{K}D\le D^T QD<G^T P^{-1}G\,,\quad F^T P^{-1}F<P^{-1}\,,$$

where $F=\tilde{K}DG^{-1}$ and $P>0$. So $\rho(F)<1$ and matrix I_m-F is non-degenerated and non-degenerated are matrices $I_m-KD=(I_m-F)G$ and $I_m-\hat{K}D=G^{-1}(I_m-KD)$.

Thus, the closed system (1), (17) under constraints (18) is provided in the form (12).

Theorem 2. *Suppose that for a positive defined matrix* $X=X^T>0$ *and some* $\varepsilon_1,\varepsilon_2>0$ *matrix inequalities share applied (18) and*

$$\begin{bmatrix} Z & XB+\varepsilon_2 C^T\Delta^T(K_0)H_B^T H_B & C_*^T & XF_A & XF_B \\ B^T X+\varepsilon_2 H_B^T H_B\Delta(K_0)C & -G^T P^{-1}G+\varepsilon_2 H_B^T H_B & D^T & 0 & 0 \\ C_* & D & -Q^{-1} & 0 & 0 \\ F_A^T X & 0 & 0 & -\varepsilon_1 I & 0 \\ F_B^T X & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} \le 0, \qquad (19)$$

where $Z=(A+B\Delta(K_0)C)^T X+X(A+B\Delta(K_0)C)+\varepsilon_1 H_A^T H_A+\varepsilon_2 C^T\Delta^T(K_0)H_B^T H_B\Delta(K_0)C$. *Then control* $u=Ky$ *with random matrix (17) stabilizes the system (1). Moreover, if (19) holds strict matrix inequality, the given set of controls ensures asymptotic stability of the closed system (12) and a common Lyapunov function* $v(x)=x^T Xx$.

Bringing. We construct Lyapunov function for the closed system (12) in the form of $v(x)=x^T Xx$. Resistance (asymptotic stability) of the zero-ensure equilibrium provide matrix inequality $X=X^T>0$ and not positive (negative) matrix inequality of the derivative of the system $\dot{v}(x)=w(x,u)$, ie taking into account (17) performance of matrix inequalities would be sufficient.

$$(A+\Delta A+(B+\Delta B)\Delta(K_0+\tilde{K})C)^T X+X(A+\Delta A+(B+\Delta B)\Delta(K_0+\tilde{K})C)\le 0\ (<0). \qquad (20)$$

Applying the property (9) operator $\Delta(K)=(I_m-KD)^{-1}K$, rewrite inequality (20) in the form

$$(A+\Delta A)^T X+X(A+\Delta A)+C^T\left(\Delta^T(K_0)+(I+\Delta^T(K_0)D^T)\Delta^T(\hat{K})\right)(B+\Delta B)^T X+$$
$$+X(B+\Delta B)\left(\Delta(K_0)+\Delta(\hat{K})(I+D\Delta(K_0))\right)C\le 0$$

Last inequality we rewrite as

$$M_*^T X+XM_*+C_*^T\Delta^T(\hat{K})(B+\Delta B)^T X+X(B+\Delta B)\Delta(\hat{K})C_*\le 0,$$

where $M_*=A+\Delta A+(B+\Delta B)\Delta(K_0)C$, $C_*=C+D\Delta(K_0)C$, $\hat{K}=G^{-1}\tilde{K}$. With
$\tilde{K}\in \mathrm{E}\Leftrightarrow \hat{K}\in\hat{\mathrm{E}}=\left\{K:K^T\hat{P}K\le Q\right\}$,

where $\hat{P}=G^T P^{-1}G$.

We use lemma 2 putting

$$W=M_*^T X+XM_*,\ U=(B+\Delta B)^T X,\ V=C_*,\ R=0.$$

Then the second block inequality in (6) has the form

$$\begin{bmatrix} M_*^T X + XM_* & X(B+\Delta B) & C_*^T \\ (B+\Delta B)^T X & -G^T P^{-1} G & D^T \\ C_* & D & -Q^{-1} \end{bmatrix} \le 0 .$$

Using the structure of the matrix uncertainties $\Delta_A(t)$, $\Delta_B(t)$, we decompose the last inequality

$$\begin{bmatrix} A^T X + XA + C^T \Delta^T (K_0) B^T X + XB\Delta(K_0)C & XB & C_*^T \\ B^T X & -G^T P^{-1} G & D^T \\ C_* & D & -Q^{-1} \end{bmatrix} + \begin{bmatrix} H_A^T \\ 0 \\ 0 \end{bmatrix} \Delta_A^T \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \Delta_A \begin{bmatrix} H_A & 0 & 0 \end{bmatrix} + \begin{bmatrix} C^T \Delta^T (K_0) H_B^T \\ 0 \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \Delta_B \begin{bmatrix} H_B \Delta(K_0)C & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ H_B^T \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} + \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \Delta_B \begin{bmatrix} 0 & H_B & 0 \end{bmatrix} \le 0 ,$$

which according to lemma 3 is done if there are $\varepsilon_1, \varepsilon_2 > 0$ such as

$$\begin{bmatrix} A^T X + XA + C^T \Delta^T (K_0) B^T X + XB\Delta(K_0)C & XB & C_*^T \\ B^T X & -G^T P^{-1} G & D^T \\ C_* & D & -Q^{-1} \end{bmatrix} +$$

$$+ \varepsilon_1 \begin{bmatrix} H_A^T H_A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_1} \begin{bmatrix} XF_A \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_A^T X & 0 & 0 \end{bmatrix} +$$

$$+ \varepsilon_2 \begin{bmatrix} C^T \Delta^T (K_0) H_B^T H_B \Delta(K_0) C & C^T \Delta^T (K_0) H_B^T H_B & 0 \\ H_B^T H_B \Delta(K_0) C & H_B^T H_B & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_2} \begin{bmatrix} XF_B \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} F_B^T X & 0 & 0 \end{bmatrix} \le 0 .$$

We apply Lemma 1 and get the conditions (18) and (19), in which the inequality (20) is for any matrix $\tilde{K} \in \mathrm{E}$. These conditions provide asymptotic stability of zero state closed system (12).

The theorem is proved.

The results of theorems 1 – 2 can be generalized in case when

$$\Delta A(t) = \sum_{i=1}^{r} F_A^{(i)} \Delta^{(i)}(t) H_A^{(i)} , \quad \Delta B(t) = \sum_{i=1}^{r} F_B^{(i)} \Delta^{(i)}(t) H_B^{(i)} .$$

Conclusions. In this work, new methods of analysis of robust stability of equilibrium states of control with reverse output connection been obtained. The values of matrix coefficients are set by restrictions on normal matrix uncertainties and dimensional vector output includes components of the system as well as control. Feasibility of the obtained methods is reduced to solving algebraic LMN. A distinctive feature of obtained LMN from known ones is the possibility of building an ellipsoid matrix stabilizing coefficients to stimulate reverse connection and common quadratic Lyapunov function.

The results are obtained based on the known generalizations statement on adequacy of Petersen's lemma about matrix uncertainties. Unfortunately, the conditions of theorem 1-2 are generally theoretical. Their practical use in problems of output robust stabilization based on quadratic Lyapunov functions with uncertain matrices requires special methods of matrix K_0 (see, e.g., [5, 8]). This is one of the topical tasks of the following studies.

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РОБАСТНА СТІЙКІСТЬ ЛІНІЙНИХ КЕРОВАНИХ СИСТЕМ З МАТРИЧНИМИ НЕВИЗНАЧЕНОСТЯМИ

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Резюме. *Присвячено розробленню нових методів аналізу робастної стійкості та робастної стабілізації лінійних динамічних систем. Для лінійних керованих систем з невизначеними матричними коефіцієнтами та зворотного зв'язку по вимірюваному виходу формулюються достатні умови стійкості нульового стану рівноваги. При цьому визначаються спільна квадратична функція Ляпунова та еліпсоїдальна множина стабілізуючих матриць коефіцієнтів підсилення зворотного зв'язку для всієї сім'ї систем. Практична реалізація отриманих методів зводиться до розв'язуванням систем лінійних матричних нерівностей.*

Ключові слова: *система керування, зворотний зв'язок, робастна стійкість, матрична невизначеність, еліпсоїд.*

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