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THERMAL FAILURE OF THREE–DIMENSIONAL VISCOELASTIC CYLINDRICAL PANEL WITH INDEPENDENT TEMPERATURE CHARACTERISTICS UNDER THE FORCED RESONANT VIBRATIONS

Vasyl Кarnaukhov [1]; Volodymyr Кozlov [1]; Igor Umrykhin [1]; Biktor Sichko [2]

[1] *S.P. Tymoshenko Institute of Mechanics of the NAS of Ukraine, Kyiv, Ukraine*
[2] *V.O. Suhomlynskogo Mykolaev national university, Mykolaev, Ukraine*

Summary. *Thermal failure of three–dimensional viscoelastic cylindrical panel with independent temperature electromechanical characteristics under the forced resonant vibrations is investigated. The problem is reduced to the solution of the linear problems of thermo electrovisco elasticity and linear problems of thermal conductivity with known heat source. The solutions of these linear problems are obtained by a finite element method. The influence of the heat transfer coefficient on a critical potential difference is investigated.*
Key words: *three–dimensional layered cylindrical panel, forced resonant vibrations, dissipative heating, thermal failure, critical potential difference.*

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Problem setting. Cylindrical panels made of passive and piezoelectric materials are widely used in various fields of modern technology [1-4] as sound emitters and receivers that operate in a wide range of frequencies. The resonance mode is among their main operational modes.

Hysteretic losses inherent to inelastic piezoelectric materials lead to an increase in their temperature, also called dissipative heating temperature. Its level depends on many factors: the geometric parameters of the body, load amplitude, frequency, dissipative properties of the material, mechanical and thermal boundary conditions etc. The greatest increase in temperature occurs during oscillation at the resonant frequencies.

Large number of studies, such as provided in [5-19] for reviewing purposes, deal with resonant vibrations of piezoelectric active material cylinders. However, almost all of them do not take into account dissipative heating which can lead not only to quantitative but also to qualitative changes in oscillatory process. For example, when the piezoelectric active material temperature reaches Curie point a specific type of thermal destruction occurs when structure ceases to perform its functionality due to its loss of piezoelectricity. The importance of taking into account the temperature of dissipative heating as one of the main reasons for limiting the power of emitters was already mentioned in earlier works dealing with piezoelectric transducers [4].

Analysis of known research results. Vibrations of elastic and viscoelastic cylindrical bodies in three-dimensional setting were analyzed in a number of works of both domestic and foreign scientists [5-19]. The authors know of only one article [20], which investigates the vibrations and dissipative heating of three-dimensional viscoelastic cylindrical panels. However, the references list no works which study the thermal destruction of such panels at harmonic electric load due to the temperature reaching the material degradation point, or the Curie point, at which the piezoelectric effect is lost and the panel no longer fulfills its functional purpose.

Objectives. To investigate the thermal destruction of viscoelastic cylindrical three-dimensional panels at harmonic electric load depending on the heat transfer coefficient with the environment.

Formulation of the problem. We consider the three-dimensional problem of thermal destruction of viscoelastic cylindrical piezoelectric panel under the action of time harmonic potential difference. The potential difference is fed to an infinitely thin electrodes deposited on the cylindrical surface. Other surfaces have no electrodes. To simulate the electromechanical state of piezoelectric material at harmonic deformation we use the concept of integrated characteristics [21-22], according to which the defining equation of piezoelectric active materials have the same form as the constitutive equations for elastic material with the only difference that they are complex. It is believed that their real and imaginary components are independent of temperature. In this case the problem is divided into two independent problems: 1) the problem of electro mechanics and 2) the problem of thermal conductivity with a known source of heat. The first of these tasks is reduced to solution of system of differential equations in partial derivatives with complex coefficients. Next step after solving this problem is finding the average for the period electromechanical power, serving as a heat source in energy equation. Then the problem of thermal conductivity with a known source of heat is solved. Variation settings of each of these problems are given. Variation problems are solved by finite element method. As a thermal destruction criterion we select reaching Curie point by dissipative heating temperature at which piezoelectric material ceases to fulfill its functional purpose due to its loss of piezoelectric effect and transformation of piezoelectric active material into a piezoelectric passive one. To calculate the critical potential difference at which a thermal destruction of piezoelectric panel occurs it is necessary to repeatedly solve the above problems for different amplitudes of electric load and find a potential difference at which the maximum temperature reaches the Curie point. Critical load depends on many factors: the conditions of heat transfer, mechanical and electrical boundary conditions, geometric parameters of the panel, material properties and others. When the electric load level exceeds the critical point, it brings the problem of determining the critical time at which the maximum dissipative heating temperature equals Curie point. By determining the critical time for different levels of supercritical load we can build Weller type curve that links the critical load and critical time. This curve also depends on the factors mentioned above. In this paper, the main focus is on the calculation of the critical potential difference. We also present temperature and amplitude-frequency characteristics and a graph of the critical potential difference of heat transfer coefficient.

Let us assign cylindrical coordinate system *(r, z, θ)* to a cylindrical panel made of viscoelastic piezoelectric material. Electromechanical properties of this material are considered independent of temperature. The panel is under time harmonic electric load with a frequency close to its first resonant frequency of the panel. Electromechanical behavior of inelastic material is described by the concept of integrated characteristics [21-22]. In this case, the dynamic problem of vibration and dissipative heating of piezoelectric panels comes down to the interpretation of three-dimensional equations of motion and equations of electrostatics in a cylindrical coordinate system [1]:

$$\frac{\partial\sigma_{zz}}{\partial z}+\frac{1}{r}\frac{\partial(r\sigma_{zr})}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{z\theta}}{\partial\theta}+\rho\omega^2 w=0\,;$$

$$\frac{\partial\sigma_{z\theta}}{\partial z}+\frac{1}{r}\frac{\partial(r\sigma_{r\theta})}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}-\frac{\sigma_{r\theta}}{r}+\rho\omega^2 v=0\,;\,div\,\vec{D}=\frac{1}{r}\frac{\partial}{\partial r}(rD_r)+\frac{1}{r}\frac{\partial D_\theta}{\partial\theta}+\frac{\partial D_z}{\partial z}=0, \qquad (1)$$

where $\sigma_{ij}\ (i,j=r,\theta,z)$ – amplitude value of stress tensor components*; w, u, v* – components of displacement vector; ρ – material density; ω – oscillations frequency; $\vec{D}$ – Electric displacement field of radial D_r, circumferential D_θ and axial D_z components.

On the surface of the body Σ_p where surface forces are set, stress tensor satisfies the following boundary conditions $P_{\alpha n} = \sigma_{\alpha\beta} l_{\beta n}$. Here $l_\beta (\beta = 1,2,3)$ – the direction cosines of outer normal $\vec{n}$ to the surface of the body Σ_p; $P_{\alpha n}$ – the projection of surface forces on the axis of the cylindrical coordinate system. In another part of the body surface Σ_u components of the displacement vector can be specified.

When electric field on infinitely thin electrodes covering the cylindrical surface of the panel excites oscillation, we set the value of the electric potential

$$\varphi = \varphi_0 e^{i\omega t}. \tag{2}$$

In those parts of the body surface where electrodes are absent, we assume that the normal component of electric displacement field D_n is zero $(D_n = 0)$.

Strain tensor is associated with the displacement vector by Cauchy ratios [1.23]

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}; \varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + u\right); \varepsilon_{zz} = \frac{\partial w}{\partial z}; \varepsilon_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\left(\frac{\partial u}{\partial \theta} - v\right) + \frac{\partial v}{\partial r}\right];$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right); \quad \varepsilon_{z\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right). \tag{3}$$

The electric potential φ is associated with the electric field strength components by ratios

$$E_z = -\frac{\partial \varphi}{\partial z}; \quad E_r = -\frac{\partial \varphi}{\partial r}; \quad E_\theta = -\frac{1}{r}\frac{\partial \varphi}{\partial \theta} \tag{4}$$

For its closure the system (1) – (4) has to be supplemented by equations of state. For pre-polarized radially viscoelastic piezoelectric materials complex equations of state in the cylindrical coordinate system (r, z, θ) are of the form [1, 22]

$$\sigma_{rr} = c_{33}^E \varepsilon_{rr} + c_{13}^E \varepsilon_{\theta\theta} + c_{13}^E \varepsilon_{zz} - e_{33} E_r; \sigma_{\theta\theta} = c_{13}^E \varepsilon_{rr} + c_{11}^E \varepsilon_{\theta\theta} + c_{12}^E \varepsilon_{zz} - e_{13} E_r;$$
$$\sigma_{zz} = c_{13}^E \varepsilon_{rr} + c_{12}^E \varepsilon_{\theta\theta} + c_{11}^E \varepsilon_{zz} - e_{13} E_r; \sigma_{rz} = 2c_{44}^E \varepsilon_{rz} - e_{15} E_z; \sigma_{z\theta} = \left(c_{11}^E - c_{12}^E\right)\varepsilon_{z\theta}; \sigma_{r\theta} = 2c_{44}^E \varepsilon_{r\theta} - e_{15} E_\theta;$$
$$D_r = \mu_{33}^S E_r + e_{13}(\varepsilon_{zz} + \varepsilon_{\theta\theta}) + e_{33}\varepsilon_r; D_\theta = \mu_{11}^S E_\theta + 2e_{15}\varepsilon_{r\theta}; \quad D_z = \mu_{11}^S E_z + 2e_{15}\varepsilon_{rz}. \tag{5}$$

The temperature field of dissipative heating is found after solving thermal conductivity equation [21, 22]

$$\frac{\partial}{\partial z}\left(\lambda_q \frac{\partial T}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_q r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\lambda_q \frac{\partial T}{\partial \theta}\right) + D_M = c\rho \frac{\partial T}{\partial t} \tag{6}$$

at primary

$$T = T_0 \text{ at } t = t_0 \tag{7}$$

and boundary conditions of convective heat exchange with the environment, the temperature of which T_c:

$$\lambda_q \frac{\partial T}{\partial n} = -\alpha_T \left(T - T_c\right), \tag{8}$$

where λ_q – coefficient of thermal conductivity; α_T – heat exchange factor; *c* – specific mass ratio of the material heat capacity.

Dissipative function D_M, part of the energy equation (6), is determined by the formula [21, 22]

$$D_M = \frac{\omega}{2}\left(\sigma''_{ij}\varepsilon'_{ij} - \sigma'_{ij}\varepsilon''_{ij} + D'_i E''_i - D''_i E'_i\right);(i, j = r, \theta, z). \tag{9}$$

Analysis of numerical results. The system of equations (1) – (9) with appropriate boundary and initial conditions is a complex linear system of differential equations in partial derivatives with complex coefficients. For independent from temperature characteristics the system (1) – (9) splits into two independent linear problems: the problem of calculating the electromechanical state of viscoelastic piezoelectric material body and the problem of thermal conductivity with the known heat source [5, 7].

We solve these problems using linear finite element method (FEM). For this we give variation formulation of the problem of electro mechanics, which brings its solution to determining the stationary points of functional

$$\begin{aligned} Э = \frac{1}{2}\int\limits_V &\begin{bmatrix} c_{11}^E\varepsilon_{zz}^2 + 2c_{13}^E\varepsilon_{zz}\varepsilon_{rr} + 2c_{12}^E\varepsilon_{zz}\varepsilon_{\theta\theta} + c_{33}^E\varepsilon_{rr}^2 + 2c_{13}^E\varepsilon_{rr}\varepsilon_{\theta\theta} + c_{11}^E\varepsilon_{\theta\theta}^2 + \\ + 4c_{44}^E\varepsilon_{r\theta}^2 + 4c_{44}^E\varepsilon_{rz}^2 + 2\left(c_{11}^E - c_{12}^E\right)\varepsilon_{z\theta} - 2e_{13}\varepsilon_{zz}E_r - 2e_{33}\varepsilon_{rr}E_r - \\ - 2e_{13}\varepsilon_{\theta\theta}E_r - 4e_{15}\varepsilon_{r\theta}E_\theta - 4e_{15}\varepsilon_{rz}E_z - \mu_{33}E_r^2 - \mu_{11}E_z^2 - \mu_{11}E_\theta^2 - \\ - \rho\omega^2\left(u^2 + w^2 + v^2\right) \end{bmatrix} rdvdzd\theta - \\ &- \int\limits_{\Sigma_p}\left(p_{nz}w + p_{nr}u + p_{r\theta}v - \sigma^e\Psi\right)d\Sigma_p \end{aligned} \tag{10}$$

Solution of heat conduction problem with a known source of heat is reduced to solution of variation problem for the functional

$$\begin{aligned} I = \frac{1}{2}\int\limits_V &\left[\lambda_q\left(\frac{\partial T}{\partial z}\right)^2 + \lambda_q\left(\frac{\partial T}{\partial r}\right)^2 + \frac{\lambda_q}{r^2}\left(\frac{\partial T}{\partial\theta}\right)^2 + 2\rho c\frac{\partial T}{\partial t} - 2D_M T\right] rdrdzd\theta + \\ &+ \int\limits_S \alpha\left(\frac{T}{2} - T_c\right)TdS. \end{aligned} \tag{11}$$

For approximation of stress-strain state and the electric potential in the meridian plane of intersection (r, z) apply algebraic polynomial of the second degree. For approximation of movement and potential in the circumferential direction in each element we use trigonometric polynomials

$$H(\theta) = a_0 + a_1\cos\theta + b_1\sin\theta. \tag{12}$$

We divide the region occupied by the body N by M spatial elements using nodal points. In this case, let us assume that the displacement and electric potential within the element are approximated by expressions

$$w = \sum_{i=1}^{24} K_i w_i \quad u = \sum_{i=1}^{24} K_i u_i \quad v = \sum_{i=1}^{24} K_i v_i \quad \Psi = \sum_{i=1}^{24} K_i \Psi_i \,. \tag{13}$$

where w_i, u_i, v_i, Ψ_i – the nodal values of displacements and electric potential; K_i – approximation functions that are a combination of algebraic $L_j\left(j = 1,2,3,\ldots 8\right)$ and normalized trigonometric polynomials $H_j(\, j = 1,2,3\,)$:

$$\begin{aligned}
&L_1 = \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1), \quad L_2 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1),\\
&L_3 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta+1), \quad L_4 = \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1),\\
&L_5 = \frac{1}{2}\left(1-\xi^2\right)(1-\eta), \quad L_6 = \frac{1}{2}\left(1-\eta^2\right)(1+\xi),\\
&L_7 = \frac{1}{2}\left(1-\xi^2\right)(1+\eta), \quad L_8 = \frac{1}{2}\left(1-\eta^2\right)(1-\xi),
\end{aligned} \tag{14}$$

$$\begin{aligned}
H_1(\theta) &= \frac{\sin(\theta-\theta_2)-\sin(\theta-\theta_3)+\sin(\theta_2-\theta_3)}{\sin(\theta_1-\theta_2)-\sin(\theta_1-\theta_3)+\sin(\theta_2-\theta_3)} \,,\\
H_2(\theta) &= \frac{\sin(\theta-\theta_3)-\sin(\theta-\theta_1)+\sin(\theta_3-\theta_1)}{\sin(\theta_2-\theta_3)-\sin(\theta_2-\theta_1)+\sin(\theta_3-\theta_1)} \,,\\
H_3(\theta) &= \frac{\sin(\theta-\theta_1)-\sin(\theta-\theta_2)+\sin(\theta_1-\theta_2)}{\sin(\theta_3-\theta_1)-\sin(\theta_3-\theta_2)+\sin(\theta_1-\theta_2)} \,.
\end{aligned} \tag{15}$$

Thus defined form functions $K_1 = L_1 H_1\,;\; K_2 = L_2 H_1\,;\ldots;K_{24} = L_8 H_3$ are zero in all nodal points of the element except the node whose number matches the number of the corresponding form function. In addition, they satisfy the condition $\sum_{i=1}^{24} K_i = 1$. The relationship between cylindrical r,z and local ξ,η coordinates is established by using dependencies

$$r = \sum_{i=1}^{24} K_i r_i\,; \quad z = \sum_{i=1}^{24} K_i z_i \,. \tag{16}$$

Since it is impossible to get dependencies of the kind $\xi(r,z)$, $\eta(r,z)$, that are inverse to (16), partial derivatives in determining strains are to be calculated using ξ,η, and then solve thus obtained dependences regarding derivatives to cylindrical coordinates. As a result, the expression for the strain tensor components and vector components of the electric field through the nodal values of displacements and electric potential can be written as

$$\varepsilon_{rr} = \sum_{i=1}^{24} \Psi_i u_i; \quad \varepsilon_{zz} = \sum_{i=1}^{24} \Phi_i w_i; \quad \varepsilon_{\theta\theta} = \sum_{i=1}^{24}\left(\frac{1}{r}\chi_i v_i + \frac{1}{r}K_i u_i\right);$$

$$\varepsilon_{zr} = \frac{1}{2}\left(\sum_{i=1}^{24}\Phi_i u_i + \sum_{i=1}^{24}\Psi_i w_i\right); \quad \varepsilon_{z\theta} = \frac{1}{2}\left(\sum_{i=1}^{24}\Phi_i v_i + \sum_{i=1}^{24}\frac{1}{r}\chi_i w_i\right); \tag{17}$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\sum_{i=1}^{24}\frac{1}{r}\chi_i u_i + \sum_{i=1}^{24}\Psi_i v_i - \sum_{i=1}^{24}\frac{1}{r}K_i v_i\right), E_r = -\sum_{i=1}^{24}\varphi_i \psi_i, E_z = -\sum_{i=1}^{24}\Phi_i \varphi_i, E_\theta = -\sum_{i=1}^{24}\chi_i \frac{\varphi_i}{r},$$

$$\Phi_i = \frac{1}{|j|}\left(\frac{\partial K_i}{\partial \xi}\frac{\partial r}{\partial \eta} - \frac{\partial K_i}{\partial \eta}\frac{\partial r}{\partial \xi}\right), \Psi_i = \frac{1}{|j|}\left(\frac{\partial K_i}{\partial \eta}\frac{\partial z}{\partial \xi} - \frac{\partial K_i}{\partial \xi}\frac{\partial z}{\partial \eta}\right), \chi_i = \frac{\partial K_i}{\partial \theta}, \tag{18}$$

where

$$|j| = \frac{\partial z}{\partial \xi}\frac{\partial r}{\partial \eta} - \frac{\partial z}{\partial \eta}\frac{\partial r}{\partial \xi} \tag{19}$$

– Jacobi determinant (Jacobian).

We also approximate mechanical stress $\vec{P}$ by shape functions within each finite element:

$$P_{nz} = \sum_{i=1}^{24} K_i P_{inz}, \quad P_{nr} = \sum_{i=1}^{24} K_i P_{inr}, \quad P_{n\theta} = \sum_{i=1}^{24} K_i P_{in\theta}, \tag{20}$$

Substituting the expressions for the strains and electric field vector components in the functional (10), within the conditions of its stability we obtain the system of linear algebraic equations for nodal displacement vector component values and electric potential $(w_i,\ u_i,\ v_i, \varphi_i)$ for a specific finite element:

$$\frac{\partial Э_m}{\partial w_1} = a_{11}^w w_1 + b_{11}^w u_1 + c_{11}^w v_1 + d_{11}^w \varphi_1 + \ldots + a_{124}^w w_{24} + b_{124}^w u_{24} + c_{124}^w v_{24} + d_{124}^w \varphi_{24} = P_1^{(z,w)};$$

$$\frac{\partial Э_m}{\partial u_1} = a_{11}^u w_1 + b_{11}^u u_1 + c_{11}^u v_1 + d_{11}^u \varphi_1 + \ldots + a_{124}^u w_{24} + b_{124}^u u_{24} + c_{124}^u v_{24} + d_{124}^u \varphi_{24} = P_1^{(r,u)};$$

$$\frac{\partial Э_m}{\partial v_1} = a_{11}^v w_1 + b_{11}^v u_1 + c_{11}^v v_1 + d_{11}^v \varphi_1 + \ldots + a_{124}^v w_{24} + b_{124}^v u_{24} + c_{124}^v v_{24} + d_{124}^v \varphi_{24} = P_1^{(\theta,v)};$$

$$\frac{\partial Э_m}{\partial \varphi_1} = a_{11}^\varphi w_1 + b_{11}^\varphi u_1 + c_{11}^\varphi v_1 + d_{11}^\varphi \varphi_1 + \ldots + a_{124}^\varphi w_{24} + b_{124}^\varphi u_{24} + c_{124}^\varphi v_{24} + d_{124}^\varphi \varphi_{24} = 0;$$

$$\frac{\partial Э_m}{\partial w_{24}} = a_{241}^w w_1 + b_{241}^w u_1 + c_{241}^w v_1 + d_{241}^w \varphi_1 + \ldots + a_{2424}^w w_{24} + b_{2424}^w u_{24} + c_{2424}^w v_{24} + d_{2424}^w \varphi_{24} = P_{24}^{(z,w)};$$

$$\frac{\partial Э_m}{\partial u_{24}} = a_{241}^u w_1 + b_{241}^u u_1 + c_{241}^u v_1 + d_{241}^u \varphi_1 + \ldots + a_{2424}^u w_{24} + b_{2424}^u u_{24} + c_{2424}^u v_{24} + d_{2424}^u \varphi_{24} = P_{24}^{(r,u)};$$

$$\frac{\partial Э_m}{\partial v_{24}} = a_{241}^v w_1 + b_{241}^v u_1 + c_{241}^v v_1 + d_{241}^v \varphi_1 + \ldots + a_{2424}^v w_{24} + b_{2424}^v u_{24} + c_{2424}^v v_{24} + d_{2424}^v \varphi_{24} = P_{24}^{(\theta,v)};$$

$$\frac{\partial Э_m}{\partial \varphi_{24}} = a^{\varphi}_{241} w_1 + b^{\varphi}_{241} u_1 + c^{\varphi}_{241} v_1 + d^{\varphi}_{241} \varphi_1 + \ldots + a^{\varphi}_{2424} w_{24} + b^{\varphi}_{2424} u_{24} + c^{\varphi}_{2424} v_{24} + d^{\varphi}_{2424} \varphi_{24} = 0 . \quad (21)$$

Expressions for the coefficients $a_{ij},\ldots,d_{ij}$ of these equations are determined by the physical and mechanical characteristics of the material and geometric parameters of the considered body.

Adding up expression (21) on all finite elements for the general global node numbering we get a system of equations units, where volume integration is replaced by the amount of integrals taken according to the volume of separate finite elements, and surface integration is replaced by the amount of the surface elements integrals, where the boundary conditions in stresses are set.

The resulting system of algebraic equations we solve in complex domain using Gauss method. This allows us to accurately obtain the solution of large dimension systems without violating the symmetry and strip-like appearance of their structure.

According to the nodal values of displacements and electric potential we determine the strain tensor components and electric field vector components at any point of the finite element. In this case, the estimation accuracy of the strain and stress induction and electric field due to the necessary process of differentiation is lower than estimation accuracy of the displacement and electric potential. It will also be different in different parts of the element. The most accurate values we obtain in Gauss integration points that correspond to the minimum order of integration [24-26] (in this case $n = 8$).

In solving the variation problem (11) for the heat equation time derivative $\frac{\partial T}{\partial t}$ does not vary and is replaced by the expression

$$\frac{\partial T}{\partial t} = \frac{T(t+\Delta t) - T(t)}{\Delta t} \quad (22)$$

This allows us to implement implicit scheme of solving heat conductivity problems.

Thus calculations of the deformations, the electric field, as well as mechanical stresses and electrical induction are conducted in eight points of integration. It is known [24-26], that the position of integration points is determined by irrational numbers. This causes some discomfort when analyzing the results. For many practical calculations it is important that required values of stress and electrical induction are calculated at nodal points of a finite element grid, as well as at the boundaries of the body. The most simple definition of strain, stress and other variables in each finite element is obtained by extrapolation of the values calculated at the points of integration, at any point, including the border. For the case considered the most accurate calculation results are obtained by extrapolating the stress and induction of electric field by bilinear polynomials.

Research results. Consider a three-dimensional problem of vibrations and dissipative heating of a cylindrical piezoelectric panel with thickness H. Infinitely thin electrodes, which get potential difference $\varphi = \varphi_0 cos\omega t.$ are applied on the cylindrical surface. Hinged ends of the panel are free from mechanical load. In solving the problem we consider a quarter of the panel with symmetry conditions $w = 0$ at $z = 0$, $v = 0$ at $\theta = 0.$ through the load symmetry and boundary conditions. Calculations are performed for piezoelectric ceramic panels $PZT - TC - 65$ with radial polarization. The panel load together with its geometric, physical and mechanical properties are characterized by the following parameters:

$\varphi_0 = 220\,V; r_1 = 0{,}09\ m; r_2 = 0{,}11\ m; H = r_2 - r_1 = 0{,}02\ m; h_1 = 0{,}005\ m; h_2 = 0{,}015\ m; L = 0{,}1\ m; \theta_0 = \pi/3;$
$E_a = 7{,}3 \cdot 10^{10}\ N/m^2; \nu_a = 0{,}34; \rho_a = 0{,}75 \cdot 10^4\ Kg/m^3.$

Thermal conductivity and density of the material have the following meanings: $\lambda = 1{,}25\,W/m \cdot {}^0C$, $\rho = 0{,}75 \cdot 10^4\,Kg/m^3$. When calculating amplitude and frequency characteristics of temperature and heat transfer coefficient between the environment and the panel material is constant $(\alpha_T = 5\dfrac{W}{m^2 \cdot {}^0C})$.

Complete specifications for the specified material are presented in [27]. Their dependence on temperature is approximated by second degree polynomials:

$$
\begin{aligned}
&S'_{11} = [0{,}171 \cdot 10^2 + 0{,}48335 \cdot 10^{-2}T - 0{,}48511 \cdot 10^{-4}T^2] \cdot 10^{-12}\,м^2/N;\\
&S'_{12} = -[0{,}568 \cdot 10^1 + 0{,}48333 \cdot 10^{-2}T - 0{,}19444 \cdot 10^{-4}T^2] \cdot 10^{-12}\,м^2/N;\\
&S'_{13} = -[0{,}91 \cdot 10^1 + 0{,}97231 \cdot 10^{-2}T - 0{,}38544 \cdot 10^{-4}T^2] \cdot 10^{-12}\,м^2/N;\\
&S'_{33} = [0{,}184 \cdot 10^2 - 0{,}43333 \cdot 10^{-1}T + 0{,}11111 \cdot 10^{-3}T^2] \cdot 10^{-12}\,м^2/N;\\
&S'_{55} = [0{,}460 \cdot 10^2 - 0{,}29167 \cdot 10^{-1}T - 0{,}6944 \cdot 10^{-4}T^2] \cdot 10^{-12}\,м^2/N;\\
&d'_{31} = -[189{,}7 + 0{,}4545T - 0{,}1515 \cdot 10^{-2}T^2] \cdot 10^{-12}\,м^2/N;\\
&d'_{33} = [357 + 0{,}17T - 0{,}41 \cdot 10^{-3}T^2] \cdot 10^{-12}\,м^2/N;\\
&d'_{15} = [609 - 0{,}385T + 0{,}45 \cdot 10^{-3}T^2] \cdot 10^{-12}\,м^2/N;\\
&\mu'_{11} = [0{,}20541 \cdot 10^5 + 0{,}4163 \cdot 10^2T - 0{,}576 \cdot 10^{-1}T^2] \cdot 10^{-12};\\
&\mu'_{33} = [0{,}14803 \cdot 10^5 + 0{,}76783 \cdot 10^2T - 0{,}145 \cdot 10^{-1}T^2] \cdot 10^{-12}.
\end{aligned}
\tag{23}
$$

Imaginary compliance components equal to

$$
\begin{aligned}
&S''_{11} = -\frac{0{,}2}{17{,}1}S'_{11}; S''_{12} = -\frac{0{,}1}{5{,}8}S'_{12}; S''_{13} = -\frac{0{,}2}{9{,}1}S'_{13}; S''_{33} = -\frac{0{,}4}{18{,}4}S'_{33}; S''_{55} = -\frac{5{,}6}{468}S'_{55}.\\
&d''_{31} = \frac{4{,}8}{189{,}7}d'_{31}; d''_{33} = -\frac{14{,}7}{357}d'_{33}; d''_{15} = -\frac{253{,}6}{609}d'_{15}; \mu''_{11} = -\frac{11270}{20541}\mu'_{11};\\
&\mu''_{33} = -\frac{342}{14805}\mu'_{33}.
\end{aligned}
\tag{24}
$$

Complex characteristics c_{ij} piezoelectric moduli e_{ij}, dielectric permittivity μ^S_{ij} are determined by the formulas:

$$
\begin{aligned}
&c_{11}=\frac{S_{11}^E S_{11}^E - S_{13}^E S_{13}^E}{(S_{11}^E - S_{12}^E)\Delta},\ c_{12}=\frac{S_{13}^E S_{13}^E - S_{12}^E S_{13}^E}{(S_{11}^E - S_{12}^E)\Delta},\ c_{13}=-\frac{S_{13}^E}{\Delta},\\
&c_{33}=\frac{S_{11}^E - S_{12}^E}{\Delta},\ c_{55}=\frac{1}{S_{55}^E},\ c_{66}=\frac{1}{2}(c_{11}-c_{12}),\\
&e_{33}=\frac{d_{33}(S_{11}^E+S_{12}^E)-2d_{31}S_{13}^E}{\Delta}, e_{15}=\frac{d_{15}}{S_{55}^E}, e_{31}=\frac{d_{31}S_{33}^E - d_{33}S_{13}^E}{\Delta},\\
&\mu_{11}=\mu_{33}^T-\frac{d_{33}^2(S_{11}^E+S_{12}^E)+2d_{31}^2S_{33}^E-4d_{33}d_{31}S_{13}^E}{\Delta}, \Delta=(S_{11}^E+S_{12}^E)S_{33}^E-2S_{13}^E S_{13}^E.
\end{aligned}
\tag{25}
$$

Fig. 1, 2 shows the frequency dependence of the displacement vector radial component and fixed temperature at the point of the middle surface, which lies at the intersection $z=0;\ R=0{,}1м;\ \theta=0.$

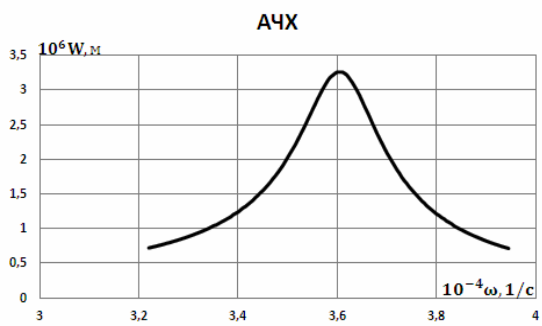

Figure 1. Amplitude-frequency response

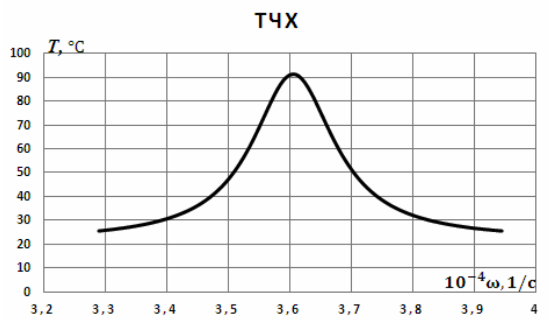

Figure 2. Temperature-frequency response

As can be seen from these figures, amplitude and temperature of dissipative heating reach maximum values at resonant frequencies. Therefore from now on we calculate critical values of electrical loads at the resonant frequency.

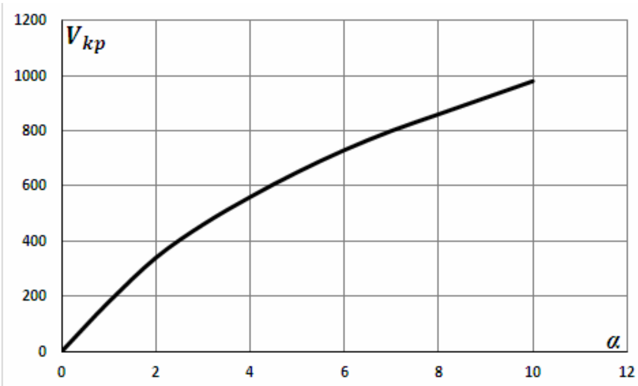

Figure 3. Dependence of critical potential difference on a heat-transfer coefficient

The curve of the critical potential differences dependence on heat transfer coefficient α is shown in Fig. 3. As can be seen, with the increase of this factor the critical potential difference increases monotonically. Based on physical considerations it is clear that in case of thermal insulation of the panel critical potential difference is zero, since piezoelectric panel thermal destruction occurs at any electrical load no matter how infinitely small it is.

Conclusions. The paper presents a three-dimensional mathematical model and research methods of forced resonant vibrations and dissipative heating of cylindrical piezoelectric panel with hinged leaning ends; material characteristics are considered to be independent of temperature. The main results are as follows:

1) based on the concept of integrated features we set a three-dimensional linear boundary problem of forced resonant vibrations and dissipative heating of viscoelastic cylindrical piezoelectric panel with characteristics independent of temperature at harmonic electric load. Solution of this problem is reduced to solving linear problem of electric viscoelasticity with complex electromechanical coefficients dependent on spatial coordinates, and heat conduction problem with a known source of heat;

2) solutions to these linear problems of electro mechanics and thermal conductivity were obtained by finite elements method;

3) using solutions of these problems we offered a problem setting concerning thermal destruction of viscoelastic cylindrical piezoelectric panel; where the destruction criterion occurs at temperature reaching Curie point at dissipative heating of piezoelectric panel at which it loses its functionality due to its loss of piezoelectric effect;

4) based on the analysis of numerical results the influence of heat transfer coefficient on the critical potential difference has been researched.

References

1. Grinchenko V.T., Ulitko A.F., Shul'ga N.A. The mechanics of coupled fields in elements of designs. At 6-T V.5. Electroelasticity. Naukova Dumka, (1998), 290 p.
2. Shul'ga N.A., Karlash V.I. Resonant electromechanical vibrations of piezoelectric plates [in Russian]. Naukova Dumka, (2008), 290 p.
3. Hamid R. Hamidzadeh, Reza Jazar N. Vibrations of Thick Cylindrical Structurals, New York – Dordrecht – Heidelberg-London: Springer, 2010, 201 p.
4. Berlinkur D. Curran, D., Jaffe G. Piezomagnetic piezoelectric materials and their use in converters, Physical Acoustics, Ed. U.Mezona. Volume 1, Part A., M.: World, 1966, pp. 204 – 326.
5. Chen C., Shen Y., Liang X. Three dimentional analysis of piezoelectric circular cylindrical shell of finite length, Acta mechanica, 1999, 134, pp. 235 – 249.
6. Ebenezer D.D., Ramesh R. Exact analysis of axially polarized piezoelectric ceramic cylinders with certain uniform boundary conditions, Current science, 2003, 85, pp. 1173 – 1179.
7. Ebenezer D.D., Ravichandran K., Ramesh R., Padmaranabhan C. Forced response of solid axially polarized piezoelectric ceramic finite cylinders with internal losses, JASA, 2005, 117, pp. 3645 – 56.
8. Daneshmehr A.R., Akbari S., Nateghi A. Dynamical thermal analysis of laminated cylinder with piezoelectric layer based on three-dimentional elasticity solution , Appl. Mech. And Materials, 2012, 186, pp. 16 – 25.
9. Desai P., Kant T. Accurate stresses in laminated piezoelectric finite length cylinders subjested to electro-thermo-mechanical loadings, Current science, 2012, 102, pp. 50 – 60.
10. Oveisi A., Gudarzi M., Hasheminejad S.M. An investigation on free/forced vibration of a piezoelectric circular cylindrical panel located on an elastic foundation, Intern. J. of Mech, 2013, 7, pp. 45 – 56.
11. Grigorenko A.J., Efimova T.L., Loza I.A. Osesymerychnoyi solving the problem of free oscillations of piezoceramic hollow cylinder of finite length method of spline collocation, Mat - physical methods and fur. Field, 2008, 51, no. 3, pp. 1 – 9.
12. Grigorenko A.J., Efimova T.L., Loza I.A. Application of spline approximation and discrete orthogonalization method for solving the problems of free oscillations of piezoceramic hollow tsilndrov, Theor. J. Mechanics, 2008, 44, pp. 61 – 65.
13. Grigorenko A.J., Efimova T.L., Loza I.A. An approach to the study of oscillations pezokeramicheskeih hollow cylinders of finite length, Reports of National Academy of Sciences of Ukraine, 2009, no. 6 pp. 61 – 66.
14. Grigorenko A.J., Efimova T.L., Loza I.A. Study of free oscillations of a hollow piezoceramic cylinder of finite length with axial polarization, J. Appl. Mechanics, 2010, 46, no. 6, pp. 17 – 26.
15. Grigorenko A.J., Loza I.A. Axisymmetrical vibrations of hollow cylinders of functionally gradient materials, polarized axially, News. Dnipropetrovskogo University, 2011, Release 15, vol. 2, no. 5, pp. 48 – 53.
16. Grigorenko A.J., Loza I.A. On free axisymmetric vibrations of a hollow piezoceramic cylinder of finite length with radial polarization, J. Appl. Mechanics, 2010, 46, no. 10, pp. 17 – 26.

17. Grigorenko A.J., Loza I.A. Axisymmetrical vibrations hollow tsilndrov of functionally gradient materials, polarized axially, News Dnipropetrovsky University, 2011, Release 15, vol. 2, no. 5, pp. 48 – 53.
18. Loza I.A. The solution of the problem of axisymmetric vibrations of a hollow piezoceramic cylinder of finite length, Reports of National Academy of Sciences of Ukraine, 2010, no. 6, pp. 52 – 58.
19. Loza I.A. Free oscillations of piezoceramic hollow cylinder with radial polarization, Mate Methods and phy - meh. Field, 2010, 55, pp. 135 – 140.
20. Karnaukhov V.G., Kozlov V.I., Umrykhin I.N., Zavgorodnii A.V. Forced resonance vibrations and dissipative heating of viscoelastic bodies of rotation of a piezoelectric material, Applied mechanics, 2015, vol. 51, no. 6, pp. 12 – 22.
21. Karnaukhov V.G., Kirichok I.F. The mechanics of coupled fields in elements of designs. At 6-T T.4. Elektrotermovyazko-elasticity. Naukova Dumka, 1988, 320 p.
22. Karnaukhov V.G., Myhilenko V.V. Nonlinear one-frequency vibrations and dissipative heating of nonelastic piezoelectric bodies [in Russian]. ZTSU, Zhytomir, 2005, 428 p.
23. Kovalenko A.D. Fundamentals of thermoelasticity, B: Naukova Dumka, 1970, 305 p.
24. Bathe K., Wilson E. Numerical methods of analysis. Finite element technique, M: Stroyizdat, 1982, 447 p.
25. Washizu.K. Variational methods in the theory of elasticity and plasticity, Dergamon press, Oxford, New York, Toronto, Sydney, Davos, Frankfurt,1982, 545 p.
26. Zienkiewicz O.C. The finite element method in engineering science, MCGraw-HILL.London, 1971, 542 p.
27. Sabat R.G., Mukherjee B., Ren W., Yung G. Temperature dependence of the complete material coefficients matrix of soft and hard doped piezoelectric lead zirconate titanate ceramics, J. Appl. Phys, 2007, 101 p, 06411–1–7.

Список використаної літератури

1. Гринченко, В.Т. Механика связанных полей в элементах конструкций. В 6т.–Т.5. Электроупругость [Текст] / В.Т. Гринченко, А.Ф. Улитко, Н.А. Шульга. – К.: Наук. думка, 1989. – 290 с.
2. Шульга, М.О. Резонансі електромеханічні коливання п'єзоелектричних пластин [Текст] / М.О. Шульга, В.Л. Карлаш. – К.: Наук. думка, 2008. – 290 с.
3. Hamid R. Hamidzadeh, Reza N.Jazar. Vibrations of Thick Cylindrical Structurals. – New York – Dordrecht – Heidelberg–London: Springer. – 2010. – 201 p.
4. Берлинкур, Д. Пьезоєлектрические и пьезомагнитные материалы и их применение в преобразователях [Текст] / Д. Берлинкур, Д. Керран, Г. Жаффе // Физическая акустика. Под ред. У. Мезона. Т.1, часть А. – М.: Мир, 1966. – С. 204 – 326.
5. Chen C., Shen Y., Liang X. Three dimentional analysis of piezoelectric circular cylindrical shell of finite length, Acta mechanica. 1999, 134, pp. 235 – 249.
6. Ebenezer D.D., Ramesh R. Exact analysis of axially polarized piezoelectric ceramic cylinders with certain uniform boundary conditions//Current science. 2003, 85, pp. 1173 – 1179.
7. Ebenezer D.D., Ravichandran K., Ramesh R., Padmaranabhan C. Forced response of solid axially polarized piezoelectric ceramic finite cylinders with internal losses, JASA. 2005, 117, pp. 3645 – 56.
8. Daneshmehr A.R., Akbari S., Nateghi A. Dynamical thermal analysis of laminated cylinder with piezoelectric layer based on three-dimentional elasticity solution, Appl. Mech. And Materials. 2012, 186, pp. 16 – 25.
9. Desai P., Kant T. Accurate stresses in laminated piezoelectric finite length cylinders subjested to electro-thermo-mechanical loadings, Current science. 2012, 102, pp. 50 – 60.
10. Oveisi A., Gudarzi M., Hasheminejad S.M. An investigation on free/forced vibration of a piezoelectric circular cylindrical panel located on an elastic foundation//Intern. J. of Mech. 2013, 7, pp. 45 – 56.
11. Григоренко, О.Я. Розв'язання осесиметричної задачі про вільні коливання п'єзокерамічних порожнистих циліндрів скінченої довжини методом сплайн-колокацій [Текст] / О.Я. Григоренко, Т.Л. Єфімова, І.А. Лоза. //Мат. методи та фіз.-мех. поля. – 2008. – 51, №3. – С. 1 – 9.
12. Григоренко, А.Я. Применение сплайн-аппроксимации и метода дискретной ортогонализации для решения задач о свободных колебаниях полых пьезокерамических цилндров [Текст] / А.Я. Григоренко, Т.Л. Ефимова, И.А. Лоза. // Теор. и прикл. механика. – 2008. – 44. – С. 61 – 65.
13. Григоренко, А.Я. Об одном подходе к исследованию колебаний полых пьезокерамическеих цилиндров конечной длины [Текст] / А.Я. Григоренко, Т.Л. Ефимова, И.А. Лоза. // Доповіді НАН України. – 2009, № 6. – С.61 – 66.
14. Григоренко, А.Я. Исследование свободных колебаний полых пьезокерамических цилиндров конечной длины с осевой поляризацией [Текст] / А.Я. Григоренко, Т.Л. Ефимова, И.А. Лоза. // Прикл. механика. – 2010, – 46, № 6. – С. 17 – 26.
15. Григоренко, А.Я. Неосесимметричные колебания полых цилиндров из функционально градиентных материалов, поляризованных в осевом направлении [Текст] / А.Я. Григоренко, И.А. Лоза. // Вісник Дніпропетровського університету. – 2011. – Вип. 15. Т. 2, № 5. – С. 48 – 53.

16. Григоренко, А.Я. О свободных неосесимметричных колебаниях полых пьезокерамических цилиндров конечной длины с радиальной поляризацией [Текст] / А.Я. Григоренко, И.А. Лоза. //Прикл. механика. – 2010, – 46, № 10. – С. 17 – 26.
17. Григоренко, А.Я. Неосесимметричные колебания полых цилиндров из функционально градиентных материалов, поляризованных в осевом направлении [Текст] / А.Я. Григоренко, И.А. Лоза. // Вісник Дніпропетровського університету. – 2011. – Вип. 15. Т. 2, № 5. – С. 48 – 53.
18. Лоза, И.А. Решение задачи о неосесимметричных колебаниях полых пьезокерамических цилиндров конечной длины [Текст] / И.А. Лоза. // Доповіді НАН України. – 2010, № 6. – С. 52 – 58.
19. Лоза, И.А. Свободные колебания пьезокерамических полых цилиндров с радиальной поляризацией [Текст] / И.А. Лоза. // Мат. методи та фіз.-мех. поля. – 2010. – 55. – С. 135 – 140.
20. Вынужденные резонансные колебания и диссипативный разогрев тел вращения из вязкоупругого пьезоэлектрического материала [Текст] / В.Г. Карнаухов, В.И. Козлов, И.Н. Умрихин, А.В. Завгородний. // Прикладная механика. – 2015. – Т. 51, № 6. – С. 12 –22.
21. Карнаухов, В.Г. Механика связанных полей в элементах конструкций. В 6т.–Т.4. Электротермовязко-упругость [Текст] / В.Г. Карнаухов, И.Ф. Киричок. – К.: Наук. думка, 1988. – 320 с.
22. Карнаухов, В.Г. Нелинейная термомеханика пьезоэлектрических неупругих тел при моногармоническом нагружении [Текст] / В.Г. Карнаухов, В.В. Михайленко. – Житомир: ЖТТУ, 2005. – 428с .
23. Коваленко, А.Д. Основы термоупругости [Текст] / А.Д. Коваленко. – К.: Наук.думка, 1970. – 305 с.
24. Бате, К.Ю. Численные методы анализа. Метод конечных элементов [Текст] / К.Ю. Бате. – М.:Стройиздат, 1982. – 447 с.
25. Васидзу, К. Вариационные методы в теории упругости и пластичности [Текст] / К. Васидзу. – М.: Мир, 1987. – 542 с.
26. Зенкевич, О. Конечные элементы и аппроксимация [Текст] / О. Зенкевич, К. Морган. – М: Мир, 1986. – 318 с.
27. Sabat R.G., Mukherjee B., Ren W., Yung G. Temperature dependence of the complete material coefficients matrix of soft and hard doped piezoelectric lead zirconate titanate ceramics, J. Appl. Phys, 2007, 101 p. 06411–1–7.

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ТЕПЛОВЕ РУЙНУВАННЯ ТРИВИМІРНОЇ В'ЯЗКОПРУЖНОЇ ЦИЛІНДРИЧНОЇ ПАНЕЛІ З НЕЗАЛЕЖНИМИ ВІД ТЕМПЕРАТУРИ ХАРАКТЕРИСТИКАМИ ПРИ ВИМУШЕНИХ РЕЗОНАНСНИХ КОЛИВАННЯХ

Василь Карнаухов[1], Володимир Козлов[1], Ігор Умрихін[1], Віктор Січко[2]

[1]Інститут механіки ім. С.П. Тимошенка НАН України, Київ, Україна
[2]Миколаївський національний університет ім. В.О. Сухомлинського, Миколаїв, Україна

Резюме. *Досліджено теплове руйнування тривимірної циліндричної п'єзопанелі з незалежними від температури електромеханічними характеристиками при вимушених резонансних коливаннях. Задачу зведено до розв'язання двох лінійних крайових задач: задачі електромеханіки й задачі теплопровідності з відомим джерелом тепла. Ці задачі розв'язано методом скінченних елементів. Досліджено вплив коефіцієнта теплообміну на критичну різницю потенціалів.*

Ключові слова: *тривимірна циліндрична панель, вимушені резонансні коливання, дисипативний розігрів, теплове руйнування, критична різниця потенціалів.*

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