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SEQUENCES OF SEMIGROUPS OF NONLINEAR OPERATORS AND THEIR APPLICATIONS TO STUDY THE CAUCHY PROBLEM FOR PARABOLIC EQUATIONS

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Summary. *We consider the operator function of exponential type, studied the link between these functions (semigroup) and Cauchy problem for differential parabolic equation. We establish conditions under which the semigroup is associated with Cauchy problem; we investigate semigroups sequences and their convergence to function of exponential type which is semigroup. We consider maximal dissipative operators and maximum semigroups. We study the problem of existence of the solution of nonlinear partial differential equations of parabolic type with measurable coefficients, nonlinear term which satisfies the forms – bouded conditions.*

Key words: *quasi-linear differential equations, dissipative operators, the method of forms, semigroup, maximal operators, sequence of semigroups.*

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Introduction. Consider the Cauchy problem for a parabolic equation in the form $\frac{d}{dt}u(t) \in \mathrm{A}\left(u(t)\right), \quad u(t) \in L^p(R^l, d^l x),$ for almost all $t \in [0, t_0],$ with the initial condition $u(0) = u_0$, $u_0 \in D(A)$, where the operator $\mathrm{A} : L^p(R^l, d^l x) \to L^p(R^l, d^l x)$ is determined by the operator $A^p : W_1^p \to W_{-1}^p$, which operates as follows: $h_\lambda^p(u, v) = \langle A^p(u), v \rangle$. The form $h_\lambda^p : W_1^p \times W_1^q \to R$ is based on the left part of elliptic equation $\lambda u - \sum_{i,j=1,\ldots,l} \frac{\partial}{\partial x_i}\left(a_{ij}(x,u)\frac{\partial}{\partial x_j}u\right) + b(x,u,\nabla u) = f$, where $u(x)$ is the unknown function, $\lambda > 0$ – a real number, and $f(x)$ – given function [8, 9]. Here $b(x,u,\nabla u)$ – a function of three variables: the dimension vector l, scalar, dimension vector l. Dimensional matrix $a_{ij}(x,u)$ of dimension $l \times l$ satisfies ellipticity condition: $\nu(u)\sum_{i=1}^{l}\xi_i^2 \le \sum_{ij=1,\ldots,l} a_{ij}(x,u)\xi_i\xi_j \le \mu(u)\sum_{i=1}^{l}\xi_i^2 \quad \forall \xi \in R^l$ for almost all $x \in R^l$ [4-5, 8, 9].

Let us construct form $h_\lambda^p : W_1^p \times W_1^q \to R$: $h_\lambda^p(u,v) \equiv \lambda\langle u,v\rangle + \langle \nabla v \circ a \circ \nabla u\rangle + \langle b(x,u,\nabla u), v\rangle$, which is assumed to be specified for all elements $u \in W_1^p(R^l, d^l x), v \in W_1^q(R^l, d^l x)$.

The function $b(x,y,z)$ is a measurable function of its arguments and $b \in L_{loc}^1(R^l)$; function almost everywhere satisfies $\left|b(x,u,\nabla u)\right| \le \mu_1(x)|\nabla u| + \mu_2(x)|u| + \mu_3(x)$.

We introduce the class of functions $PK_\beta(A) = \left\{ f \in L_{loc}^1(R^l, d^l x) : \left|\langle h\, f\, h\rangle\right| \le \beta\langle \nabla h \circ a \circ \nabla h\rangle + c(\beta)\| h\|_2^2 \right\},$ where $\beta > 0,\ \ c(\beta) \in R^1$. Features

$\mu_1^2 \in PK_\beta(A)$, $\mu_2 \in PK_\beta(A)$. Growth of the function $b(x,y,z)$ almost everywhere satisfies the condition: $\left|b(x,u,\nabla u)-b(x,v,\nabla v)\right| \le \mu_4(x)\left|\nabla(u-v)\right|+\mu_5(x)\left|u-v\right|$ where $\mu_4^2 \in PK_\beta(A)$, $\mu_5 \in PK_\beta(A)$ [8, 9].

Preliminary information.

Definition 1. *A set of one-parameter nonlinear operators* $\mathrm{T}_t, t \ge \mathbf{0},$ *is called a continuous one-parameter semigroup if the following conditions are met: for any fixed* $t \ge \mathbf{0}$ *operator* T_t *is a continuous nonlinear operator which operates from* $L^p\left(R^l, d^l x\right)$ in $L^p\left(R^l, d^l x\right)$*; for any fixed* $f \in L^p\left(R^l, d^l x\right)$ *set of elements* $\mathrm{T}_t f$ *is strongly continuous on* t*; there exists a property of group* $\mathrm{T}_{s+t} = \mathrm{T}_s \mathrm{T}_t$*, for* $t, s \ge \mathbf{0}$ *at* $\mathrm{T}_{\mathbf{0}} = \mathrm{I}$*, where* I *– the identical motion.*

Definition 2. *Let function* $u_n(t) \in D(\mathrm{A}) \subset C_{L^p(R^l,d^l x)}[\mathbf{0},t],\ n \in N,$ *satisfy (in the classical sense) equation* $\dfrac{d}{dt}u_n(t) = \mathrm{A}_n u_n(t),\ n \in N,$ *where* $\mathrm{A}_n,\ n \in N,$ *– motion of* $u - vn^{-\mathbf{1}} \to v$ *at* $u \in D(\mathrm{A})$, $v \in \mathrm{A}u$*. If the sequence* $\{u_n(t), n \in N\}$ *matches evenly with* $u(t) \in D(\mathrm{A})$, $t \in [\mathbf{0}, t_{\mathbf{0}}]$ *in the strong topology, and there is a subsequence* $\left\{\dfrac{d}{dt}u_{n_k}(t),\ n_k \in N\right\}$ *that matches a* σ*-weak topology to the element* $\dfrac{d}{dt}u(t) \in L_{L^p(R^l,d^l x)}[\mathbf{0},t]$*, the element* $u(t) \in D(\mathrm{A})$ *is called a solution of generalized parabolic equation.*

We know that if $u(t)$ is an absolutely continuous function than $u(t)$ is differentiable for almost all t because $L^p(R^l, d^l x)$ is a reflexive Banach space, and can be written by the integral of its derivative, which exists for almost all t.

Definition 3. *Own solution of the Cauchy problem for a parabolic equation is a function* $u(t)$*, if* $u(t) \in D(A)$ *and this function is absolutely continuous for almost all* t *and satisfies for almost all* t *this generalized parabolic equation.*

Denote with $C_{L^p(R^l,d^l x)}[\mathbf{0},t]$ the space of all $L^p(R^l, d^l x)$ – significant highly continuous functions on the interval of real axis $[\mathbf{0},t]$, i.e. if $u(\cdot) \in C_{L^p(R^l,d^l x)}[\mathbf{0},t]$, then $u:[\mathbf{0},t] \to L^p(R^l, d^l x)$; and through $L = L_{L^p(R^l,d^l x)}[\mathbf{0},t]$ – space of all $L^p(R^l, d^l x)$ – significant highly integrable functions on the interval $[\mathbf{0},t]$, that is, if $u(\cdot) \in L_{L^p(R^l,d^l x)}[\mathbf{0},t]$ then $u:[\mathbf{0},t] \to L^p(R^l, d^l x)$ and $\| u \|_L = \int_{\mathbf{0}}^{t} \| u(s) \| ds < \infty$.

We assume that the operator A is valid from $L^p(R^l, d^l x)$ to $L^p(R^l, d^l x)$ and is one that generates mapping from $C_{L^p(R^l,d^l x)}[\mathbf{0},t]$ to $L_{L^p(R^l,d^l x)}[\mathbf{0},t]$ which can be determined by the rule $C_{L^p(R^l,d^l x)}[\mathbf{0},t] \ni u \to \{v \in L_{L^p(R^l,d^l x)}[\mathbf{0},t] : v(s) \in \mathrm{A}u(s)$ almost everywhere $\{s\}\}$, this mapping is also denoted by the letter A.

Nonlinear semigroups and local generators.

Definition 4. *Semigroup T_t is called the maximum compression semigroup if there is no compression semigroup with a broader definition domain to which it may be extended.*

Remark. Any compression semigroup can be extended to a maximum compression semigroup.

Remark. Maximum dissipative operator does not necessarily generate a maximum compression semi-group.

Lemma 1. *Let* $\{B_\alpha : \alpha \in \Gamma\}$ *and* $\{B'_\alpha : \alpha \in \Gamma\}$ *– two systems of areas in* $L^p(R^l, d^l x)$: $B_\alpha = \left\{ f \in L^p(R^l, d^l x) : \| f - f_\alpha \| \le r_\alpha \right\}$, $B'_\alpha = \left\{ f \in L^p(R^l, d^l x) : \| f - f'_\alpha \| \le r'_\alpha \right\}$. *If* $\| f_\alpha - f_\beta \| \ge \| f'_\alpha - f'_\beta \|$ and $r_\alpha \le r'_\alpha \quad \forall \alpha, \beta \in \Gamma$, *then because of* $\bigcap_{\alpha\in\Gamma} B_\alpha \ne \phi$ *stems condition* $\bigcap_{\alpha\in\Gamma} B'_\alpha \ne \phi$.

Lemma 2*. Let* Ω *– convex closed shell* $D(T_t)$. *For any fixed natural number* k *there is mapping* $U_k : \Omega \to \Omega$ *that* $\| U_k f - U_k q \| \le \| f - q \| \quad \forall f, q \in \Omega$ *and* $U_k f = T_{2-k} f \quad \forall f \in D(T_t)$.

If f'_0 and f''_0 of Ω *satisfy correlation* $\| f''_0 -_{2-k} f \| \le \| f'_0 - f \| \quad \forall f \in D(T_t)$, *there is an* U_k *expansion of* T_{2-k} *that* $U_k f'_0 = f''_0$.

Proof. Let us assume that set $\Omega - D(T_t)$ is ordered like $\{f_\alpha\}$. Using transfinite induction, we construct the map U_k. Suppose that U_k – a reflection of compression that is defined for $\forall f_\beta : \beta < \alpha \forall q \in D(T_t) : U_k q = T_{2-k} q$. Let $B(f - \varphi_1; \varphi_2) = \{ f_1 : \| f_1 - \varphi_2 \| \le \| f - \varphi_1 \| \}$.

Thus, for systems of areas $\{B(f_\alpha - f_\beta; f_\beta), B(f_\alpha - q; q) : \beta < \alpha, q \in D(T_t)\}$ and $\{B(f_\alpha - f_\beta; U_k f_\beta), B(f_\alpha - q; U_k q) : \beta < \alpha, q \in D(T_t)\}$, then $\bigcap_{\beta<\alpha} B(f_\alpha - f_\beta : f_\beta) \bigcap_{y\notin D(T_t)} B(f_\alpha - q : q) \ne \phi$, as f_α belongs to this intersection, it follows: $\bigcap_{\beta<\alpha} B(f_\alpha - f_\beta : U_k f_\beta) \bigcap_{y\notin D(T_t)} B(f_\alpha - q; U_k q) \le B^0_\alpha \ne \phi.$

Denote the projection $L^p(R^l, d^l x) \to \Omega$ with p, thus $pf = q$ at $\| q - f \| = \inf_{q_1\in\Omega} \| q_1 - f \|$.

As inequality for the norms is true: $\| p\phi_1 - f \| \le \| \phi - f \| \, \forall f \in \Omega, \quad \phi_1 \in L^p\left(R^l, d^l x\right)$ therefore, there is identity $\varphi_3 \in B^0_\alpha$, from where we get $pf_3 \in B^0_\alpha$.

Select an item $f^0_\alpha \in B^0_\alpha \bigcap \Omega$ and let $U_k f_\alpha = f^0_\alpha$, then U_k compression is on $D(T_t) \bigcup \{f_\beta : \beta \le \alpha\}$.

Using transfinite induction we have received the necessary map U_k. The Lemma statement is proved.

Consider the semi group $T^\alpha = \left\{ T^\alpha_t : t = \dfrac{j}{2^k}, \; j = 0,1,2,... \right\}$, where $T^\alpha_{2-k} = U^\alpha_k$ and $T^\alpha_{t+s} = T^\alpha_t T^\alpha_s$ when $t = \dfrac{j}{2^k}$, $s = \dfrac{j}{2^k}$, and the set of maps - $\{U^\alpha_k : \alpha \in \Gamma\}$ from Lemma 2.

Denote with τ_k the set $\{T^\alpha : \alpha\}$ and define canonical map for $n \ge k$ $J_{n,k} : \tau_n \to \tau_k$ as

$J_{n,k}T^{\alpha}=T^{\beta}$ at $T^{\alpha}_{2-k}=T^{\beta}_{2-k}$, where $T^{\alpha}\in\tau_k$, $T^{\beta}\in\tau_n$. Noting that $J_{m,n}:J_{n,k}:\tau_m\to\tau_n\to\tau_k$, $J_{m,n}J_{n,k}T^{\alpha}=J_{m,n}J_{n,k}T^{\beta}=T^{\gamma}$, $T^{\alpha}_{2-k}=T^{\beta}_{2-k}=T^{\gamma}_{2-k}$, $T^{\alpha}\in\tau_k$, $T^{\beta}\in\tau_n$, $T^{\gamma}\in\tau_m$, obtain approval $J_{m,n}J_{n,k}=J_{m,k}, D(J_{n,k})=\tau_n$. Therefore, we can put $A^{\alpha}_k=(T^{\alpha}_{2-k}-I)$ for $T^{\alpha}\in\tau_k$.

Theorem 1. Scope definition maximum compression semigroup $\{T_t\}$ is a closed convex set that is not contained in any closed hyperplane.

Proof. On the opposite, let $D(T_t)$ be its own subset of convex closed shell Ω.

Define dissipative operator A tightly defined in Ω as $Aq=\{2^n(q-f):\exists_n, q_n(f)=q\}$, where $q_n(f)=\lim\limits_{\Psi_\infty}(\mathrm{I}-\mathbf{2}^{-n}A^{\alpha}_k)^{-\mathbf{1}}f$ and Ψ_∞ – filter defined earlier.

The set A is the union of sets A^n where A^n – operator defined earlier.

Having used the equality $D\left((\mathrm{I}-\mathbf{2}^{-n}A^n)^{-\mathbf{1}}\right)=\Omega$, we can conclude that $D\left((\mathrm{I}-\mathbf{2}^{-n}A^n)^{-\mathbf{1}}\right)\supset\Omega, n\in N\cup\{\mathbf{0}\}$.

Consider the Cauchy problem:

$$\begin{cases}\dfrac{d}{dt}u(t)\in Au(t),\\ u(t)\in\Omega, u(\mathbf{0})=q_o\in(\Omega\setminus D(T_t))\cap D(A)\end{cases}$$

it can be approached by the sequence of Cauchy problems appearing as:

$$\begin{cases}\dfrac{d}{dt}u(t)=A_nu_n(t),\\ u_n(t)\in\Omega, u_n(\mathbf{0})=q_o-\mathbf{2}^{-n}q^{\mathbf{0}}_{\mathbf{0}}, q^{\mathbf{0}}_{\mathbf{0}}=\lim\limits_{\varphi_\infty}(\mathrm{I}-A^{\alpha}_k)^{-\mathbf{1}}f_{\mathbf{0}}-f_{\mathbf{0}}\end{cases},$$

where $f_{\mathbf{0}}$ is part of $q_{\mathbf{0}}-Aq_{\mathbf{0}}$, and A_n is a mapping of: $f-\mathbf{2}^{-n}f_{\mathbf{1}}\to f_{\mathbf{1}}$, $f\in D(A)$, $f_{\mathbf{1}}\in Af$.

Let $P:L^p(R^l,d^lx)\to\Omega$ be the projection, thus $Pf=q\in\Omega\subset L^p(R^l,d^lx)$ and $\|q-f\|=\inf\limits_{q^{\mathbf{1}}\in\Omega}\|q^{\mathbf{1}}-f\|$. Define the sequence $u^m_n(t)$ by placing the induction $u^{\mathbf{0}}_n(t)=q_{\mathbf{0}}-\mathbf{2}^{-n}q^{\mathbf{0}}_{\mathbf{0}}$,

$$u^{m+1}_n(t)=P(q_0+\int\limits_0^t A_nu^m_n(s)ds).$$

Then $u^m_n(t)\in\Omega\subset L^p(R^l,d^lx)$, and

$$\left\|u^{m+\mathbf{1}}_n(t)-u^m_n(t)\right\|=\left\|P\left(q_{\mathbf{0}}+\int\limits_{\mathbf{0}}^t A_nu^m_n(s)ds\right)-P\left(q_{\mathbf{0}}+\int\limits_{\mathbf{0}}^t A_nu^m_n(s)ds\right)\right\|\le$$

$$\le\int\limits_{\mathbf{0}}^t\|A_nu^m_n(s)-A_nu^{m-\mathbf{1}}_n(s)\|ds.$$

As a result of Lipchitz condition A_n we have $\sum\|u^{m+\mathbf{1}}_n(t)-u^m_n(t)\|<\infty$, so there $u_n(t)=\lim\limits_{m\to\infty}u^m_n(t)$.

Resulting from theorem given earlier, $u_n(t)$ satisfies the equation in the approximate Cauchy problem and $\{u_n(t)\}^{\infty}_{n=\mathbf{0}}$ tends to function $u_n(t)$ evenly at t on $[\mathbf{0},t_{\mathbf{0}}]$ and function $u_n(t)$ satisfies the equation in the initial Cauchy problem.

Show that $\tilde{T}_t(f)=\begin{cases} T_t f, & npu \quad f\in D(T_t), \\ u(t+s), & npu \quad f=u(s),\ s\geq \mathbf{0}, \end{cases}$ is really a compression semigroup.

Because of dissipativity A of $\| U_k f - U_k q \| \leq \| f-q \| \quad \forall f,q\in\Omega$ and $U_k f = T_{2-k} f$ $\forall f\in D(T_t)$ we get $\| u(s+t)-u(s_1+t)\| \leq \| u(s)-u(s_1)\|$ at $s,s_1,t\geq \mathbf{0}$ thus $\| u(s+t)-T_t f \| \leq \| u(s)-f \|$ at $f\notin D(T_t)$, $s,t>\mathbf{0}$.

Let $s,t,s+t\in[\mathbf{0},r]$, $r\in R_+$. Define descriptive semigroups: $T^{\alpha,n}_{2-k}=\mathbf{2}^{-k}(\mathrm{I}-\mathbf{2}^n A^{\alpha}_k)^{-\mathbf{1}}+\mathrm{I}$, $T^{\alpha,n}_{\mathbf{0}}=\mathrm{I}$, $T^{\alpha,n}_{t+s}=T^{\alpha,n}_t T^{\alpha,n}_s$ at $t=J\mathbf{2}^{-k}, s=J_{\mathbf{1}}\mathbf{2}^{-k}$.

Because $(\mathrm{I}-\mathbf{2}^{-n}A^{\alpha}_k)^{-\mathbf{1}}$ and $\mathbf{2}^{-k}A^{\alpha}_k=T^{\alpha}_{2-k}-\mathrm{I}$ is the compression, $\{T^{\alpha,n}_t : t=J\mathbf{2}^{-k}, J=\mathbf{0},\mathbf{1},\ldots\}$ is the compression semigroup.

As $A^{\alpha,n}_k=\mathbf{2}^{-k}(T^{\alpha,n}_{2-k}-\mathrm{I})=A^{\alpha}_k(\mathrm{I}-\mathbf{2}^{-n}A^{\alpha}_k)^{-\mathbf{1}}$ is dissipative, we have $\| A^{\alpha,n}_k T^{\alpha,n}_t f \| \leq \| A^{\alpha,n}_k f \|$, $f\in\Omega\subset L^p(R^l,d^l x)$.

We show that $\| T^{\alpha,n}_t u^{\alpha}_n - T^{\alpha,m}_t u^{\alpha}_m \| \leq \varepsilon$ for $u^{\alpha}_n=q^{\alpha}_k-\mathbf{2}^{-n}q^{\alpha_1}_k$, $q^{\alpha}_k=(\mathrm{I}-A^{\alpha}_k)^{-\mathbf{1}}f_{\mathbf{0}}$, $q^{\alpha_1}_k=q^{\alpha}_k-f_{\mathbf{0}}=A^{\alpha}_k q^{\alpha}_k$, at $n,m\geq n_{\mathbf{0}}$, $(\alpha,k)\in\varphi_{\mathbf{0}}\in\Psi_{\infty}$, $\mathbf{0}\leq t\leq J\mathbf{2}^{-k}\leq r$. Really $\| T^{\alpha,n}_{j2^{-k}}u^{\alpha}_n - T^{\alpha,m}_{j2^{-k}}u^{\alpha}_m \|^p - \| u^{\alpha}_n-u^{\alpha}_m \|^p=$,

$$=\sum_{i=\mathbf{1}}^{j-\mathbf{1}}\left(\| T^{\alpha,n}_{(i+\mathbf{1})2^{-k}}u^{\alpha}_n - T^{\alpha,m}_{(i+\mathbf{1})2^{-k}}u^{\alpha}_m \|^p - \| T^{\alpha,n}_{(i+\mathbf{1})2^{-k}}u^{\alpha}_n - T^{\alpha,m}_{(i+\mathbf{1})2^{-k}}u^{\alpha}_m \|^p\right)=$$

$$=\sum_{i=\mathbf{0}}^{j-\mathbf{1}}\mathbf{2}^{\mathbf{1}-k}\langle A^{\alpha,n}_k T^{\alpha,n}_{j2^{-k}}u^{\alpha}_n - A^{\alpha,m}_k T^{\alpha,m}_{j2^{-k}}u^{\alpha}_m, \left(T^{\alpha,n}_{j2^{-k}}u^{\alpha}_n - T^{\alpha,m}_{j2^{-k}}u^{\alpha}_m\right)\left|T^{\alpha,n}_{j2^{-k}}u^{\alpha}_n - T^{\alpha,m}_{j2^{-k}}u^{\alpha}_m\right|^{p-\mathbf{2}}\rangle+$$

$$+\sum_{i=\mathbf{0}}^{j-\mathbf{1}}\mathbf{2}^{\mathbf{1}-k}\| A^{\alpha,n}_k T^{\alpha,n}_{j2^{-k}}u^{\alpha}_n - A^{\alpha,m}_k T^{\alpha,m}_{j2^{-k}}u^{\alpha}_m \|^p$$
.

Since $\langle A^{\alpha,n}T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n - A^{\alpha,m}_k T^{\alpha,m}_{i2^{-k}}u^{\alpha}_m, \left((\mathrm{I}-\mathbf{2}^{-n}A^{\alpha}_k)^{-\mathbf{1}}T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n - (\mathrm{I}-\mathbf{2}^{-m}A^{\alpha}_k)^{-\mathbf{1}}T^{\alpha,m}_{i2^{-k}}u^{\alpha}_m\right) \left|(\mathrm{I}-\mathbf{2}^{-n}A^{\alpha}_k)^{-\mathbf{1}}T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n - (\mathrm{I}-\mathbf{2}^{-m}A^{\alpha}_k)^{-\mathbf{1}}T^{\alpha,m}_{i2^{-k}}u^{\alpha}_m\right|^{p-\mathbf{2}}\rangle\leq\mathbf{0}$

and $\| T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n - (\mathrm{I}-\mathbf{2}^{-n}A^{\alpha}_k)^{-\mathbf{1}}T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n \| \leq \mathbf{2}^{-n}\| A^{\alpha,n}T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n \|$ then from $\| A^{\alpha,n}_k T^{\alpha,n}_t f \| \leq \| A^{\alpha,n}_k f \|$, $f\in\Omega\subset L^p(R^l,d^l x)$, follows that $\| T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n - T^{\alpha,m}_{i2^{-k}}u^{\alpha}_m \|^p \leq \mathbf{4}r\| q^{\alpha_1}_k \|^p\ (\mathbf{2}^{-n}+\mathbf{2}^{-m}+\mathbf{2}^{-k})$ with $j\mathbf{2}^{-k}\leq\mathbf{2}$.

Denote $t_k=j_k\mathbf{2}^{-k}$, $j_k=[t\mathbf{2}^k], \mathbf{0}\leq t\leq r$, where n – arbitrary fixed number with N, and $\rho_j=\| T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n - u_n(j\mathbf{2}^{-k})\|$.

Because $T^{\alpha,n}_{(i+\mathbf{1})2^{-k}}u^{\alpha}_n = T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n + \int_{\mathbf{0}}^{2^{-k}} A^{\alpha,n}T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n ds$, $u_n\left((j+\mathbf{1})\mathbf{2}^{-k}\right)=u_n(j\mathbf{2}^{-k})+\int_{\mathbf{0}}^{2^{-k}} A_n u_n(j\mathbf{2}^{-k}+s)ds$

and $\lim_{\Psi_{\infty}} A^{\alpha,n}_k u_n(t)=A_n u_n(t)$, for $(\alpha,k)\in\varphi$, we get

$$\rho_{j+\mathbf{1}}\leq\| u_n(j\mathbf{2}^{-k})-T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n + \int_{\mathbf{0}}^{2^{-k}}\|\left(A_n-A^{\alpha,n}\right)u_n(j\mathbf{2}^{-k}+s)ds + \int_{\mathbf{0}}^{2^{-k}}\| A^{\alpha,n}u_n(j\mathbf{2}^{-k}+s)-A^{\alpha,n}T^{\alpha,n}_{i2^{-k}}u^{\alpha}_n \| ds\leq$$

$\leq \rho_j + 2^{-k}\varepsilon + 2^k 2^n(\rho_j + 2^{-k} \| q_0^0 \|) \leq \rho_j(1+2^{n-k}) + 2^{1-k}\varepsilon \quad \exists \varphi \in \Psi_\infty$ is performed for inequality $2^n \| q_0^0 \| < 2^k \varepsilon$.

Then, using induction, we get: $\rho_j \leq \rho_0 e^{r2^n} + 2re^{r2^n}\varepsilon \leq (\| q_k^\alpha - q_0 \| 2^{-n} \| q_k^{\alpha_1} - q_0^0 \|)e^{r2^n} + 2re^{r2^n}\varepsilon$ when $0 \leq j2^{-k} \leq r$, therefore we have $\lim_{\Psi_\infty} T_t^{\alpha,n} u_n^\alpha = u_n(t)$.

As $\lim_{n\to\infty} A_k^\alpha(\mathrm{I} - 2^{-n} A_k^\alpha)^{-1} q = A_k^\alpha q$ when $q \in D(A_k^\alpha) = \Omega \subset L^p(R^l, d^l x)$, for fixed α, we have $\lim_{n\to\infty} \| T_{i2^{-k}}^{\alpha,n} u_n^\alpha - T_{i2^{-k}}^\alpha u_n^\alpha \| = 0$, so $\{u_n^\alpha\}_{n=0}^\infty$ – is a relatively compact set in $\Omega \subset L^p(R^l, d^l x)$.

When $s = i2^{-k}, t = j2^{-k}, s+t = (i+j)2^{-k} \in [0, r]$ we get

$$\| u(s+t) - T_t f \| \leq \| u(s+t) - u_n(s+t) \| + \| u_n(s+t) - T_{t+s}^{\alpha,n} u_n^\alpha \| +$$
$$+ \| T_{t+s}^{\alpha,n} u_n^\alpha - T_{t+s}^\alpha u_n^\alpha \| + \| T_{t+s}^\alpha u_n^\alpha - T_t f \| \leq \| T_s^\alpha u_n^\alpha - f \| + 3\varepsilon \leq \| u(s) - f \| + 6\varepsilon$$

since $\| u(s) - T_s^\alpha u_n^\alpha \| \leq \| u(s) - u_n(s) \| + \| u_n(s) - T_s^{\alpha,n} u_n^\alpha \| + \| T_s^{\alpha,n} u_n^\alpha - T_s^\alpha u_n^\alpha \| \leq 3\varepsilon.$

Because of the uniform continuity $T_t f$ and $u(t)$ indeed there is an inequality $\| u(s+t) - T_t f \| \leq \| u(s) - f \|$ with $f \in D(T_t)$, $s, t, s+t \in [0, r]$. In other words $\{\tilde{T}_t\}$ is a contraction semigroup, but this leads to conflict with maximality of semigroup $\{T_t\}$, so we get a contradiction. We show that the region $D(T_t)$ is not contained in some closed hyperplane in $L^p(R^l, d^l x)$.

Suppose $D(T_t) \subset \{ f \in L^p(R^l, d^l x) : \langle f, e \rangle = M \}$ for some $e \in L^p(R^l, d^l x)$ and $\| e \| = 1$.

Let $S_t(f+e) = T_t f + e$ at $f \in D(T_t)$, then $\{S_t\}$ is also a compression semigroup and the definitional domain of S_t is the set $e \bigcup D(T_t)$, which has an empty intersection with $D(T_t)$, so

$$\tilde{T}_t f = \begin{cases} T_t f & npu \quad f \in D(T_t) \\ S_t f & npu \quad f \in e \bigcup D(T_t) \end{cases}$$

is an extension of $\{T_t\}$, but $\{\tilde{T}_t\}$ is a compression semigroup, ie there is a conflict with maximality $\{T_t\}$. Theorem 1 is proved.

Theorem 2. *The closure of the set* $D(A)$, *where* A *is the maximum dissipative operator is a set which is convex in* $L^p(R^l, d^l x)$.

Proof. We will use contradiction method. Let $q, \Psi \in D(A)$ and $f = \mu q - (1-\mu)\Psi$ when $0 < \mu < 1$. Suppose that $f \notin [D(A)]$, then put $q_\lambda = q - \lambda q_1$ with $q_1 \in Aq$. Using statement 8, we get $\left\| (\mathrm{I} - \lambda A)^{-1} f - q \right\| = \left\| (\mathrm{I} - \lambda A)^{-1} f - (\mathrm{I} - \lambda A)^{-1} q_\lambda \right\| \leq \left\| f - q_\lambda \right\|$, $\lim_{\lambda \downarrow 0} \| f - q_\lambda \| = \| f - q \|$, just as $\overline{\lim_{\lambda \downarrow 0}} \left\| (\mathrm{I} - \lambda A)^{-1} f - \Psi \right\| \leq \| f - \Psi \|$, since $\| q - \Psi \| \leq \overline{\lim_{\lambda \downarrow 0}} \left(\left\| q - (\mathrm{I} - \lambda A)^{-1} f \right\| + \left\| (\mathrm{I} - \lambda A)^{-1} f - \Psi \right\| \right) =$
$= \| q - f \| + \| f - \Psi \| = \| q - \Psi \|$, we have that $\lim_{\lambda \downarrow 0} (\mathrm{I} - \lambda A)^{-1} f = \sigma q - (1-\sigma)\Psi, \sigma \in [0, 1]$ and $\lim_{\lambda \downarrow 0} (\mathrm{I} - \lambda A)^{-1} f - q = \| f - q \|$, thus we get that $\lim_{\lambda \downarrow 0} (\mathrm{I} - \lambda A)^{-1} f = f$, but this contradicts the

assumption that $f \notin [D(A)]$. Theorem 2 is proved.

Theorem 3. 1) The maximum compression semigroup $\{T_t\}$ has tightly defined generator and it has been generated by a maximum dissipative operator. 2) If the maximum dissipative operator is single valued, then the semigroup generated by this operator is a maximal contraction semigroup.

Proof. Proposition 1) is a consequence of previous theories and assertions. Indeed local generator $\{T_t\}$ A_0 is consistently expressed in $D(T_t)$, maximum dissipative extension A operator A_0 generates a compression semi-group $\{S_t\}$, then semigroup $\{S_t\}$ is an extension of semigroup $\{T_t\}$, but out of maximality $\{T_t\}$ we get $\{S_t\}$ = $\{T_t\}$. 1) has been proven.

Proposition 2) prove by contradiction. Let the operator A generates semigroups $\{T_t\}$ and $\{S_t\}$ – the maximum extension $\{T_t\}$. Assume the opposite $D(S_t) \supset D(T_t)$ and $D(S_t) \neq D(T_t)$, use 1) generator $\{S_t\}$ let $\tilde{A}$ be consistently defined in $D(S_t)$.

Because of closure $D(T_t)$ there is an element $f \in D(\tilde{A})$ and $f \notin D(T_t) \subset L^p(R^l, d^l x)$ using the maximum dissipativity A get that exist $q_\lambda = (\mathrm{I} - \lambda A)^{-1} f, \forall \lambda > \mathbf{0}$ and $\lim\limits_{\lambda \downarrow \mathbf{0}} q_\lambda = q \in [D(A)] \subset L^p(R^l, d^l x)$. Since $q_\lambda \in D(A)$, $T_t q_\lambda$ weakly differentiable and on t, and $\omega - \lim\limits_{h \downarrow \mathbf{0}} A_h q_\lambda = A q_\lambda$ and $A q_\lambda = \dfrac{q_\lambda - f}{\lambda}$ thus $\lim\limits_{h \downarrow \mathbf{0}} \dfrac{\lambda}{h} \left\langle T_h q_\lambda - q_\lambda, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle = \| q_\lambda - f \|^p$,

$$\left\langle S_h f - f, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle = \left\langle S_h f - T_h q, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle +$$

$$+ \left\langle T_h q_\lambda - q, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle + \| q_\lambda - f \|^p \geq \left\langle T_h q_\lambda - q_\lambda, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle,$$

since $\| T_h q_\lambda - S_h f \| = \| S_h q_\lambda - S_h f \| \leq \| q_\lambda - f \|$.

That is, we have $\lim\limits_{h \downarrow \mathbf{0}} \dfrac{\left\langle S_h f - f, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle}{h} \geq \dfrac{\|q_\lambda - f\|^p}{\lambda}$, directing λ to zero, we obtain a contradiction

$$\lim_{h \downarrow \mathbf{0}} \left\langle \frac{S_h f - f}{h}, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle =$$

$= \left\langle \tilde{A} f, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle \to \left\langle \tilde{A} f, (q_\lambda - f)\left|q_\lambda - f\right|^{p-2} \right\rangle$ and $\dfrac{\| q_\lambda - f \|^p}{\lambda} \xrightarrow[\lambda \downarrow \mathbf{0}]{} \infty$. Theorem is proved.

Sequences of nonlinear semigroups in spaces $L^p(R^l, d^l x)$. established that: Cauchy problem $\dfrac{d}{dt} u(t) \in \mathrm{A} u(t), \quad u(t) \in L^p(R^l, d^l x),$, $t \in [0, t_0], \; u(0) = u_0$, with each $u_0 \in D(A)$ has only a weak solution if the operator $\mathrm{A} : L^p(R^l, d^l x) \to L^p(R^l, d^l x)$ is maximal dissipative operator; Let A_0 be consistently defined generator of compression semigroup T_t, while its maximum dissipative expansion A generates the same compression semi-group T_t; In addition, it was found that the generator of nonlinear compression semigroup is consistently defined in $L^p(R^l, d^l x)$.

Theorem 4. Let $\{T_t^n : t \geq \mathbf{0}, n \in N\}$ be a sequence of nonlinear semigroups, satisfying condition: $\left\|T_t^n f - T_t^n g\right\| \leq e^{\omega t} \| f - g\|$ and the sequence of operators $\{\mathrm{A}_n : n \in N\}$ is a generator of nonlinear semigroups $\{T_t^n : t \geq \mathbf{0}, n \in N\}$ sequence and there is a sequence of numbers $\mu^n \in \left(\mathbf{0}, \frac{\mathbf{1}}{\omega}\right)$, so that $R\left(\mathrm{I} - \mu^n \mathrm{A}_n\right) = L^p\left(R^l, d^l x\right)$. Denote the border of elements from $L^p\left(R^l, d^l x\right)$ $\lim\limits_{n\to\infty} \mathrm{A}_n f$ with $\mathrm{A}f$ as a border in $L^p\left(R^l, d^l x\right)$-norm.

Then the closure of A in $L^p\left(R^l, d^l x\right)$ norm, which we denote with $[\mathrm{A}]$ generates nonlinear semigroup $\{T_t : t \geq \mathbf{0}\}$, which can be defined as $L^p\left(R^l, d^l x\right)$-uniform border: $T_t f = \lim\limits_{n\to\infty} T_t^n f$ on any finite interval $t \in [\mathbf{0}, t_\mathbf{0}]$.

In addition, the semigroup $\{T_t : t \geq \mathbf{0}\}$ is the only one in the class of semigroups which satisfies the following conditions:

1) for any element $f \in D(\mathrm{A})$ function $T_t f$ is strongly absolutely continuous on any finite interval;

2) for any element $f \in D(\mathrm{A})$ for all $t \geq \mathbf{0}$ $T_t f \in D(\mathrm{A})$ and $D^+ T_t f = \mathrm{A}_0 T_t f$ and $\mathrm{A}_0 T_t f$ *is continuous by the norm* $L^p(R^l, d^l x)$ *at* $t \geq \mathbf{0}$ by t.

3) for any element $f \in D(\mathrm{A})$, there is a strong continuous derivative $\frac{d}{dt} T_t f = \mathrm{A}^0 T_t f$ except perhaps countable number of points.

Proof. The proof methods used are similar to those that were used above. For convenience and to avoid confusion code sequence is set in brackets, i.e. semigroup T_t^n generator A_n will continue to be marked as $T_t^{(n)}$ and $\mathrm{A}^{(n)}$ respectively.

Let the elements $f, g \in D(\mathrm{A}) \subseteq L^p(R^l, d^l x)$

$$\left\langle \mathrm{A}^{(n)} f - \mathrm{A}^{(n)} g, (f - g)\left|f - g\right|^{p-2} \right\rangle =$$

$$= \lim_{\alpha \to \mathbf{0}} \left\langle \frac{T_\alpha^{(n)} f - f}{\alpha} - \frac{T_\alpha^{(n)} g - g}{\alpha}, (f - g)\left|f - g\right|^{p-2} \right\rangle \leq \omega \| f - g\|^p,$$

therefore the operator $\mathrm{A}^{(n)} - \omega \mathrm{I}$ is a dissipative operator. Since $R(\mathrm{I} - \mu_n \mathrm{A}^{(n)}) = L^p\left(R^l, d^l x\right)$, then for $\eta_n = \mu_n \frac{\mathbf{1}}{\mathbf{1} - \mu_n \omega}$ and every n the equality $R\left(\mathrm{I} - \eta_n\left(\mathrm{A}^{(n)} - \omega \mathrm{I}\right)\right) = L^p\left(R^l, d^l x\right)$ is true, so $\mathrm{A}^{(n)} - \omega \mathrm{I}$ is the maximum dissipative operator. Then fix sequence index, one that is in parentheses, and use previous results, which is always possible when $m > \omega$ then there is $\left(\mathrm{I} - \frac{\mathrm{A}^{(n)}}{m}\right)^{-\mathbf{1}}$ for this operator the evaluation is true for $f, g \in L^p\left(R^l, d^l x\right)$ $\left\|\left(\mathrm{I} - \frac{\mathrm{A}^{(n)}}{m}\right)^{-\mathbf{1}} f - \left(\mathrm{I} - \frac{\mathrm{A}^{(n)}}{m}\right)^{-\mathbf{1}} g\right\| \leq \left(\mathbf{1} - \frac{\omega}{m}\right)^{-\mathbf{1}} \| f - g\|$.

Let us put by definition $A_m^{(n)} = m\left(\left(I - \frac{A^{(n)}}{m}\right)^{-1} - I\right)$. So nonlinear operators $A_m^{(n)}$ are generators of nonlinear semigroups $\{T_{mt}^{(n)} : t \geq 0, n \in N\}$ and for these semigroups for $f, g \in L^p\left(R^l, d^l x\right)$ assessment $\left\|T_{mt}^{(n)} f - T_{mt}^{(n)} g\right\| \leq e^{\frac{\omega t}{1-m^{-1}\omega}} \|f - g\|$ is true. For elements $f \in [D\left(A^{(n)}\right)]$ a border $T_t^{(n)} f = \lim_{m\to\infty} T_{mt}^{(n)} f, \quad t \geq 0$ exists.

Define the Cauchy difference for semigroups $\{T_{mt}^{(n)} : t \geq 0, n \in N\}$ sequences using index m : $T_{mt}^{(n)} f - T_{kt}^{(n)} f, \quad t \geq 0$, and mark it $P_{mkt}^{(n)}$, that is $P_{mkt}^{(n)} f = T_{mt}^{(n)} f - T_{kt}^{(n)} f, \quad t \geq 0$, and because the way $P_{mkt}^{(n)}$ acts is important only for large k, m, it can be assumed that $k, m \geq p\omega$, thus the "tail" of $P_{mkt}^{(n)}$ sequence is investigated.

Using estimates obtained and given the already introduced symbols, we get for $\alpha \geq 0$: $\left\|P_{mk\alpha}^{(n)} f\right\| \leq 2pe^{p\alpha\omega} \left\|A^{(n)} f\right\| \alpha$,

$$\left\|P_{mkt}^{(n)} f - \left(I - \frac{A^{(n)}}{m}\right)^{-1} T_{m\alpha}^{(n)} f + \left(I - \frac{A^{(n)}}{k}\right)^{-1} T_{k\alpha}^{(n)} f\right\| \leq$$

$$\leq \frac{\left\|A_m^{(n)} T_{m\alpha}^{(n)} f\right\|}{m} + \frac{\left\|A_k^{(n)} T_{k\alpha}^{(n)} f\right\|}{k} \leq$$

$$\leq \left(\frac{1}{m-\omega} + \frac{1}{k-\omega}\right) e^{p\alpha\omega} \left\|A^{(n)} f\right\|,$$

then if we denote

$$K_{mk}^{(n)}(\beta) = const(p) e^{p\omega\beta} \inf\{\|g\| : g \in A^{(n)}\} \times$$

$$\times \int_0^\beta \left\| \left(T_{m\eta}^{(n)} f - T_{k\eta}^{(n)} f\right) \left|T_{m\eta}^{(n)} f - T_{k\eta}^{(n)} f\right|^{p-2} - \right.$$

$$\left. - \left(\left(I - \frac{A_m^{(n)}}{m}\right)^{-1} T_{m\eta}^{(n)} f - \left(I - \frac{A_k^{(n)}}{k}\right)^{-1} T_{k\eta}^{(n)} f\right) \left|\left(I - \frac{A_m^{(n)}}{m}\right)^{-1} T_{m\eta}^{(n)} f - \left(I - \frac{A_k^{(n)}}{k}\right)^{-1} T_{k\eta}^{(n)} f\right|^{p-2} \right\| d\eta +$$

$$+ \left(c_1(p)\right)^p \omega \left(\frac{1}{m-\omega} + \frac{1}{k-\omega}\right)^p e^{2p\beta\omega} \beta \left\|A^{(n)} f\right\|^p,$$

we get similar assessment $L^p\left(R^l, d^l x\right)$ of the norm $P_{mk\alpha}^{(n)}$ in the segment $\alpha \in [0, \beta]$: $\left\|P_{mk\alpha}^{(n)}\right\| \leq \sqrt[p]{K_{mk}^{(n)}(\beta)} e^{p\alpha\omega}$.

Then we prove that the convergence in the limit $T_t^{(n)} f = \lim_{m\to\infty} T_{mt}^{(n)} f, \quad t \geq 0$, of any number $\alpha \in [0, \beta]$ is uniform regarding n.

Proof. Fix an arbitrary number $\alpha \in [0, \beta]$ and element $f \in D(A)$. Since $\lim_{n\to\infty} A^{(n)} f = Af$, there is a natural number n_0 and the number $M > 0$ that at $n > n_0$,

$f \in D\left(\mathrm{A}^{(n)}\right)$, and $\left\|\mathrm{A}^{(n)} f\right\| \leq M$, that for large indexes of norm sequence generators are uniformly bounded on each element.

Obviously, the set:

$$\mathrm{B}=\{\left(T_{m\eta}^{(n)} f-T_{k\eta}^{(n)} f\right)\left|T_{m\eta}^{(n)} f-T_{k\eta}^{(n)} f\right|^{p-2},\left(\mathrm{I}-\frac{\mathrm{A}_m^{(n)}}{m}\right)^{-1} T_{m\eta}^{(n)} f-\left(\mathrm{I}-\frac{\mathrm{A}_k^{(n)}}{k}\right)^{-1} T_{k\eta}^{(n)} f:$$

$$\eta \in[0, \beta], n>n_0, m>p\omega, k>p\omega\}$$

is limited.

Using the previous inequalities we find out that for every number $\varepsilon>0$ there is such a number $\delta(\varepsilon)>0$, that for elements $f, g \in \mathrm{B}$ and $\|f-g\|<\delta(\varepsilon)$ inequality $\left\|f\,|f|^{p-2}-g\,|g|^{p-2}\right\|<\dfrac{\varepsilon^p}{2 p^2 M \beta e^{p\omega\beta}} const$ is true.

We choose among numbers $k, m>k_0>\omega p$ large enough, so they fit the inequality $2 \dfrac{p^2 M e^{p\omega\beta}}{\left(k_0-\omega\right) c_1(p)} \leq \min \left(\delta, \dfrac{\varepsilon}{\sqrt[p]{p^2 \omega \beta}}\right)$, and get at $\alpha \in[0, \beta]$ and $\left\|P_{mkt}^{(n)} f-\left(\mathrm{I}-\dfrac{\mathrm{A}^{(n)}}{m}\right)^{-1} T_{m\alpha}^{(n)} f+\left(\mathrm{I}-\dfrac{\mathrm{A}^{(n)}}{k}\right)^{-1} T_{k\alpha}^{(n)} f\right\| \leq 2 \dfrac{M e^{p\omega\beta}}{\left(k_0-\omega\right)} \leq \delta$ thus at $n>n_0$ we have:

1.

$$\left\|\begin{array}{l}P_{mkt}^{(n)} f\left|P_{mkt}^{(n)} f\right|^{p-2}- \\ -\left(\left(\mathrm{I}-\dfrac{\mathrm{A}^{(n)}}{m}\right)^{-1} T_{m\alpha}^{(n)} f+\left(\mathrm{I}-\dfrac{\mathrm{A}^{(n)}}{k}\right)^{-1} T_{k\alpha}^{(n)} f\right)\left|\left(\mathrm{I}-\dfrac{\mathrm{A}^{(n)}}{m}\right)^{-1} T_{m\alpha}^{(n)} f+\left(\mathrm{I}-\dfrac{\mathrm{A}^{(n)}}{k}\right)^{-1} T_{k\alpha}^{(n)} f\right|^{p-2}\end{array}\right\| \leq$$

$$\leq \frac{\varepsilon^p}{2 p^2 M \beta e^{p\omega\beta}} const$$

Let us show that for $n>n_0$ inequality $\mathrm{K}_{mk}^{(n)}(\beta) \leq \varepsilon^p$. is true.

Indeed we have estimates

$$const(p) e^{p\omega\beta} \inf \{\|g\|: g \in \mathrm{A}^{(n)}\} \times \times\int_0^\beta\left\|\left(T_{m\eta}^{(n)} f-T_{k\eta}^{(n)} f\right)\left|T_{m\eta}^{(n)} f-T_{k\eta}^{(n)} f\right|^{p-2}-\right.$$

$$\left.-\left(\left(\mathrm{I}-\frac{\mathrm{A}_m^{(n)}}{m}\right)^{-1} T_{m\eta}^{(n)} f-\left(\mathrm{I}-\frac{\mathrm{A}_k^{(n)}}{k}\right)^{-1} T_{k\eta}^{(n)} f\right)\left|\left(\left(\mathrm{I}-\frac{\mathrm{A}_m^{(n)}}{m}\right)^{-1} T_{m\eta}^{(n)} f-\left(\mathrm{I}-\frac{\mathrm{A}_k^{(n)}}{k}\right)^{-1} T_{k\eta}^{(n)} f\right)\right|^{p-2}\right\| d\eta \leq$$

$$\leq \frac{\varepsilon^p p^2 M \beta e^{p\omega\beta}}{2 p^2 M \beta e^{p\omega\beta}};$$

$$c_1(p) \omega\left(\frac{1}{m-\omega}+\frac{1}{k-\omega}\right)^p e^{2 p\beta\omega} \beta\left\|\mathrm{A}^{(n)} f\right\|^p \leq\left(c_1(p)\right)^p\left(\frac{p^2 M e^{p\omega\beta}}{\left(k_0-\omega\right) c_1(p)}\right)^p \beta \leq \frac{\varepsilon^p}{2}.$$

So, for $k, m>k_0$ a valid assessment is $\sup\limits_{\eta \in[0, \beta],\ n>n_0}\left\|T_{m\eta}^{(n)} f-T_{k\eta}^{(n)} f\right\| \leq e^{p\omega\beta} \varepsilon$. Since $\left[R\left(\mathrm{I}-\mu_0 \mathrm{A}\right)\right]=L^p\left(R^l, d^l x\right)$ and operator $\mathrm{A}-\mathrm{I}\omega$ is dissipative as the border of operators

$\mathrm{A}^{(n)} - \mathrm{I}\omega$ at $n \to \infty$, we get the assertion of uniform convergence of semigroups $T_t^{(n)} f = \lim_{m\to\infty} T_{mt}^{(n)} f, \quad t \geq \mathbf{0}$. To complete the proof we use Lemma 4 [6, 7].

Lemma 4. Let $m > \omega$ then the operator $\left(\mathrm{I} - \frac{\mathrm{A}}{m}\right)^{-\mathbf{1}}$ has a unique extension B_m which is determined in the entire area $L^p(R^l, d^l x)$ and for which in the entire $L^p(R^l, d^l x)$ assessment $\|\mathrm{B}_m f - \mathrm{B}_m g\| \leq \frac{\|f-g\|}{\mathbf{1} - m^{-1}\omega}$ is correct. In addition $\mathrm{B}_m = \left(\mathrm{I} - \frac{[\mathrm{A}]}{m}\right)^{-\mathbf{1}}$ and the operator $([\mathrm{A}] - \mathrm{I}\omega)^{-\mathbf{1}}$ is a maximal dissipative operator.

Theorem 4. (on the generalized Cauchy problem in $L^p(R^l, d^l x)$). Generalized Cauchy problem $\frac{d}{dt}u(t) \in \mathrm{A}u(t), \quad u(t) \in L^p(R^l, d^l x), \quad t \in [0, t_0],$ $u(0) = u_0$, where $\mathrm{A} : L^p(R^l, d^l x) \to L^p(R^l, d^l x)$, each has at every $u_0 \in D(A)$ only a single weak solution.

Conclusions. We have constructed operator functions of exponential type, investigated the link between these operator functions and generalized initial Cauchy problem for equations of parabolic type. The existence of a solution of the generalized Cauchy problem for equations of parabolic type has been proven. The results can be generalized to classes of differential operators of more general type operating in certain functional spaces.

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ПОСЛІДОВНОСТІ НАПІВГРУП НЕЛІНІЙНИХ ОПЕРАТОРІВ ТА ЇХ ЗАСТОСУВАННЯ ДЛЯ ДОСЛІДЖЕННЯ ЗАДАЧІ КОШІ ДЛЯ РІВНЯННЯ ПАРАБОЛІЧНОГО ТИПУ

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Резюме. *Розглянуто операторні функції експоненціального типу, досліджено зв'язок між такими функціями (напівгрупами) та задачами Коші для диференціального параболічного рівняння. Встановлено умови, за яких напівгрупа буде асоційованою з задачею Коші, досліджено послідовності напівгруп та їх збіжність до певної напівгрупи. Розглянуто максимальні дисипативні оператори та максимальні напівтрупи, а також задачу про існування розв'язку нелінійних диференціальних рівнянь у частинних похідних параболічного типу з вимірними коефіцієнтами, нелінійний доданок яких задовольняє умови форм – обмеженості коефіцієнтів.*

Ключові слова*: квазілінійні диференціальні рівняння, дисипативні оператори, метод форм, напівгрупа, максимальні оператори, послідовності напівгруп.*

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