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INTERRELATION AND KINETICS OF MATERSALS FATIGUE DAMAGE UNDER "SOFT" AND "RIGID" LOADING MODES

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Summary*. In the article the calculating estimation of fatigue damage and plasto-elastic stress-strain state (SSS) kinetics of steels under the "rigid" cyclic loading modes using the ultimate exhaustion of cyclic plasticity (UECP) model is presented. To do this the previously developed method of fatigue damage summation, which allows to calculate the kinetics of fatigue damage under irregular cyclic loading, was used. The difference in the accumulation of fatigue damage and the kinetics of elastic-plastic SSS for two "rigid" cyclic loading modes (total and inelastic strains control loadings) are shown on the example of two materials: cyclically hardening steel 45 and cyclically softening steel 1X2M. Besides, calculations with help of the UECP model show, that for the materials with cyclic inelastic strain instability during cyclic loading fatigue damage summation rate is significantly different as compared with the same obtained by the linear summation hypothesis. The results presented can be used for calculating lifetime estimation of structural elements operating under "rigid" loading modes with a higher accuracy as compared with the use of both stabilized values of inelastic strains and well-known fatigue damage summation hypotheses. This is due to the fact, that the proposed calculative UECP model describes the difference in fatigue damage accumulation under variable cyclic load amplitudes for the materials with different kinetics of inelastic strains. In its turn, it makes possible to substantiate theoretically the necessity of taking into account the inelastic strain kinetics peculiarities under the "rigid" loading modes and to describe the difference and nonlinear nature of fatigue damage accumulation under these loading modes for the specified groups of materials.*

Key words: *fatigue, "rigid" loading, damage, exhaustion of plasticity, plastico-elastic stress strain state, inelastic strains, calculating model.*

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Introduction. Real structural elements and equipment during their operation are subjected to the operation loading changing in time as to its intensity depending on the equipment operation mode and can cause the fatigue cracks initiation and growth, which result in the operation failure. The operation loading of different configuration structural elements creates in their different local areas the modes of the cyclic loading, which correspond to the material strain under the constant strain amplitudes or constant stress amplitudes although it is considered, that the difference in the material damage under such two loading modes is significant only for the low cyclic fatigue. Available inelastic strains under high-cyclic fatigue [1] results in the difference of the fatigue curves under these modes. As the cyclic loading under the changeable inelastic strain needs the current change of the loading degree in eccordance with the change of the inelastic strain kinetics, to calculate precisely the fatigue damage of the structural material and the residual operation life of the structural elements under different cyclic loading modes the sophisticated calculation methods must be available.

Usually to represent the conditions of the material operation in the structural element it is necessary to have the testing results under the "soft" and "rigid" modes of the regular and irregular loadings [1–5]. But, when experimental data under one loading mode is available, the methods of their calculation under the other mode are of interest too. That is why the calculation methods, taking advantage of the linear and non-linear hypothesis of the damage accumulation in particular, have been proposed.

But such dependences can not represent fully the plastic or inelastic strain and the material damage kinetics. As it was shown earlier the structural metal materials under the high-

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cyclic fatigue at different stages of cyclic loading can demonstrate cyclic hardening, softening, stability of the inelastic strain kinetics, which depend on the material nature and the loading level. As the inelastic strain kinetics shows its current damage, it is different in different materials, which must affect the damage summation under the “rigid” modes of the cyclic loading. That is why the development of theoretically true and substantiated calculation of the fatigue damage of materials under the operational “rigid” loading is the problem of paramount importance and of great need.

The authors have developed the model of ultimate exhaustion of the cyclic plasticity (UECP) earlier, which was used to calculate the inelastic strain kinetics and lifetime till the ultimate state of different structural material groups in the area of the high cyclic fatigue is approached. The model uses experimental data obtained in the conditions of the symmetric tensile-compression of the smooth specimens under the “soft” mode of loading [8–11]. Taking it into account the equation of the UECP model is used in the method of the fatigue damage summation to predict the lifetime under irregular modes of the cyclic loading. Taking advantage of this method the concrete equations system for summation of the fatigue damages has been obtained for the “rigid” modes of the cyclic loading, which does not need additional parameters and formal conditions of summation, which was not assumed in the model basis. The UECP model itself is based on the calculative determination of the cycle-by-cycle change of the material cyclic yield strength and the damage kinetics till they reach certain value, which is assumed to be the ultimate state of the material. The material damage being treated as the continuous hardening / softening of some material volumes, which results in the exhaustion of the material plasticity resource and approaching the ultimate state–fatigue crack initiation.

Objective and task of the paper is to take advantage of the developed earlier model of the ultimate exhaustion of the cyclic plasticity [8–11] for calculation of the materials lifetime under the “rigid” loading. Fatigue damage and ultimate exhaustion of plasticity is determined step-by-step at every loading semi-cycle. The calculations of the materials lifetime under the “rigid” loading are performed according to the criterion of the ultimate exhaustion of the material plasticity.

Description of the calculation model. The main aspects of the UECP model were proposed and described by the authors earlier [8–11]. In the UECP model it is not necessary to introduce the fatigue damage summation hypothesis, because the material damages are treated as the continuous hardening / softening till the exhaustion of the material plasticity resource which results in its ultimate state – damage. To describe the process of hardening / softening of the material, which is expressed as the change of the cyclic yield strength, let us introduce the function of the plasticity exhaustion. It is the dependence, which describes cycle-by-cycle change of the cyclic yield strength from the initial value $\sigma_{T,0}$ till the critical $\sigma_{T,cr}$, at which the ultimate state is approached. The current value of the cyclic yield strength in the *(*i+1*)* semicycle expressed in terms of exhaustion plasticity function derivative is written as follows:

$$\sigma_{T,i+1} = \sigma_{T,i} + \frac{df\left(\varepsilon_{ine}\right)}{d\varepsilon_{ine}} \cdot \left(\pm 1 - \frac{df\left(\varepsilon_{ine}\right)}{d\varepsilon_{ine}} \right)^{-1} \Bigg|_{\varepsilon_{ine,i}} \cdot \varepsilon_{ine,i}\,, \tag{1}$$

where $\sigma_{T,i+1}$, $\sigma_{T,i}$ –cyclic yield strength in the *(*i+1*)* and *i* semicycles of loading correspondingly, here $i \in 0 \ldots 2N$, N – number of cycles to fatigue failure under the given cyclic stress amplitude, σ_a; $\Delta\sigma_{T,i}$ – increment of the cyclic yield strength in cycling process from the $(i)^{th}$ – semicycle till the $(i+1)^{th}$; $f(\varepsilon_{ine})$ – plasticity exhaustion function; E – modulus of elasticity; ε_{ine}, $\varepsilon_{ine,I}$ – inelastic strain amplitude and its value in the $(i)^{th}$ cycle of loading.

In the (1) the upper sign (plus) is used for the hardening material, the lower (minus) – for the softening one [8]. The dependence (1) is the equation for finding the increment of the ultimate cyclic yield strength in every semicycle of loading in the recurrent form. Having integrated the differencial equation arising from (1) we obtain the equation of the fatigue curve, derived for the ultimate state of the material under the cyclic loading [10]:

$$N = \frac{1}{2} \cdot \int_{\sigma_{T,0}}^{\sigma_{T,cr}} \left(\frac{df\left(\varepsilon_{ine}\right)}{d\varepsilon_{ine}} \right)^{-1} \frac{d\sigma_T}{\varepsilon_{ine}}, \tag{2}$$

where σ_T, $\sigma_{T,0}$, $\sigma_{T,cr}$ – current, initial and critical values of the cyclic yield strength.

Specification of the model. To specify the equations (1) and (2) it is necessary to introduce a form of the plasticity exhaustion function and the equation of the cyclic stress-strain diagram, which take into account the fatigue damages accumulation resulted from the plasticity exhaustion (hereinafter referred to as complete one). With this purpose let us assume, that the plasticity exhaustion function kernel obeys the exponential function the value of which depends on the inelastic strain:

$$f\left(\varepsilon_{ine}\right) = b\left(\varepsilon_{ine}\right)^a, \tag{3}$$

where *a, b* – coefficients of the plasticity exhaustion function, which deal with the non-linearity and the rate of the cyclic yield strength change correspondingly.

For the analytical description of the complete cyclic stress-strain diagram (CCSSD) we use the Osgood-Rumberg equation with the modified part, which deals with the change of inelastic strains with damages D_ε, caused by the cyclic loading [9]:

$$\begin{aligned} \varepsilon_a\left(\sigma_a, D_\varepsilon\right) &= \varepsilon_e\left(\sigma_a\right) + \varepsilon_{ine}\left(\sigma_a, D_\varepsilon\right) = \\ &= \varepsilon_e\left(\sigma_a\right) + \varepsilon_{ine,s}\left(\sigma_a\right) \cdot f\left(\sigma_a, D_\varepsilon\right) = \\ &= \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K}\right)^{1/m} \cdot \left(L_0\left(\sigma_a\right) + D_\varepsilon \cdot \left(L_K\left(\sigma_a\right) - L_0\left(\sigma_a\right)\right)\right), \end{aligned} \tag{4}$$

where $\varepsilon_a(\sigma_a, D_\varepsilon)$, $\varepsilon_e(\sigma_a)$, $\varepsilon_{ine}(\sigma_a, D_\varepsilon)$ – functions, which describe the amplitudes of the total, elastic and inelastic components of the complete cyclic stress-strain diagram correspondingly; $\varepsilon_{ine,s}(\sigma_a)$ –inelastic strains function of σ_a at the saturation stage; $f(\sigma_a, D_\varepsilon)$ –function taking into account the change of the inelastic strain value owing to the fatigue degradation of material determed by the damage D_ε; *K, m* – coefficients of the diagram equation (4) at the inelastic strain saturation stage; $L_0(\sigma_a)$, $L_{cr}(\sigma_a)$ – unit functions of the relative inelastic strain, which are found as follows:

$$\begin{aligned} L_0\left(\sigma_a\right) &= L_{0,0} + \frac{L_{0,K} - L_{0,0}}{\sigma_K - \sigma_{-1}} \cdot \left(\sigma_a - \sigma_{-1}\right), \\ L_K\left(\sigma_a\right) &= L_{K,0} + \frac{L_{K,K} - L_{K,0}}{\sigma_K - \sigma_{-1}} \cdot \left(\sigma_a - \sigma_{-1}\right) \end{aligned} \tag{5}$$

where σ_K – critical fatigue stress which corresponds to N_K on fatigue curve; σ_{-1} – endurance limit under the symmetric cycle; $L_{0,0}$, $L_{0,K}$, $L_{K,0}$, $L_{K,K}$ – the inelastic strain values at the initial and final states divided by their average values at σ_{-1} and σ_K, correspondingly.

According to the model the criterion of the material ultimate state is the condition $\sigma_T=\sigma_{T,cr}$. At the beginning of loading, when $\sigma_T=\sigma_{T,0}$, then the damage is $D_\varepsilon=0$. When the ultimate state is reached then $D_\varepsilon=1$. D_ε may be found as:

$$D_{\varepsilon}=\frac{\sigma_{T}-\sigma_{T,0}}{\sigma_{T,cr}-\sigma_{T,0}}. \tag{6}$$

Having substituted the expressions (3) – (6) in the (1), the final expression for the determination of the cyclic yield strength in every semicycle of loading can be obtained. Taken into account the found dependence the expression (2) after simplifications takes the form:

$$N\left(\sigma_{a}\right)=\frac{1}{2}\frac{\sigma_{T,cr}-\sigma_{T,0}}{L_{K}\left(\sigma_{a}\right)-L_{0}\left(\sigma_{a}\right)}\left(\pm\frac{\left(L_{K}\left(\sigma_{a}\right)\right)^{1-a}-\left(L_{0}\left(\sigma_{a}\right)\right)^{1-a}}{a\cdot b\cdot\left(1-a\right)\cdot\left(\sigma_{a/K}\right)^{a/m}}\right). \tag{7}$$

The equation (7) specifies the fatigue curve in accordance with described criterion of the material ultimate state under the cyclic loading. It does not contain the integral relations that makes it easier the further calculations for finding the UECP model parameters.

The system of equations for finding the plasticity exhaustion diagram parameters *a* and *b* is derived using the equation (7) and the determined condition in relationship of the fatigue damages D_{ε} at the normalized loading cycles $D_N=n/N$ equals 0,5 (connection follows from the condition of normalization while analyzing the modified part of the complete cyclic stress-strain diagram equation (4)). Thus, if the experimental fatigue curve is known as $N(\sigma_a)=f(\sigma_a)$, then, using the obtained equation of the fatigue curve (7) according to the model, which looks like $N(\sigma_a)=F(a,b,\sigma_a)$ and the normalization condition (the function value $f(\sigma_a,\ D_{\varepsilon})=1$ at $D_N=0,5)$), we may obtain the system of non-linear equations for finding the parameters *a* and *b*, derived for the stress σ_a, which corresponds to the high-cyclic fatigue curve as to its value:

$$\begin{cases} f\left(\sigma_{a}\right)=F\left(a,b,\sigma_{a}\right); \\ D_{\varepsilon}\Big|_{D_{N}=0,5}=\dfrac{1-L_{0}\left(\sigma_{a}\right)}{L_{K}\left(\sigma_{a}\right)-L_{0}\left(\sigma_{a}\right)}. \end{cases} \tag{8}$$

Main relations of the model for the fatigue damages summation. To determine the lifetime under the variable cyclic loading modes, the previously obtained UECP model-based dependence of fatigue damage versus the current value of the cyclic yield strength has been considered [8, 9]. As it was stated, the calculation damage of this type accompanies the process of the material cyclic yield strength changes from its initial value till the critical one in the normalized values. In the general case of the variable loading, for finding of the total lifetime it is necessary to use the system which is composed of the found earlier equations to calculate both the inelastic strain and the cyclic yield strength in every semicycle [10, 11] taking into account the conditions of the ultimate state as follows:

$$D_{\varepsilon,i}\le\left[D_{cr}\right]=1. \tag{9}$$

where $D_{\varepsilon,i}$ – the material damage in the $(i)^{th}$ semicycle of the loading; D_{cr} – critical value of the fatigue damage.

To use the equations for the calculation of the inelastic strain $\varepsilon_{ine,i}$ and the ultimate cyclic yeild $\sigma_{T,i}$ in every cycle of loading under the variable amplitudes of the cyclic loading it is necessary to introduce the law of the stress change depending on the number of the loading cycles *(i)* as follows $\sigma_{a,i}=f(i)$. Taking into account $\sigma_{a,i}$ we will obtain:

$$\begin{cases} \varepsilon_{ine,i} = \left(\dfrac{\sigma_{a.i}}{K}\right)^{1/m} \cdot \left(L_0\left(\sigma_{a,i}\right) + \dfrac{\sigma_{T,i} - \sigma_{T,0}}{\sigma_{T,cr} - \sigma_{T,0}} \cdot \left(L_K\left(\sigma_{a.i}\right) - L_0\left(\sigma_{a,i}\right)\right)\right); \\ \sigma_{T,i+1} = \sigma_{T,i} \pm E \cdot \dfrac{a \cdot b \cdot \left(\varepsilon_{ine,i}\right)^{a-1}}{\left(1 \mp a \cdot b \cdot \left(\varepsilon_{ine,i}\right)^{a-1}\right) \cdot E - a \cdot b \cdot \left(\varepsilon_{ine,i}\right)^{a-1}} \cdot \varepsilon_{ine,i}, \end{cases}, \qquad (10)$$

where i – the index, which corresponds to the loading semi-cycles; $\sigma_{a,i}$ – the function of the stress amplitude change in every loading semi-cycle.

The lifetime for this case of loading is determined as the half of the loading semi-cycles number untill the critical state is approached according to the *(9)* – $N=i/2$. The system is composed of the equations (9) and (10), which forms the model of the fatigue damages summation for the irregular modes of the cyclic loadings. It is used for determination of the lifetime untill the ultimate state is approached and for the semi-cycle plotting of the plasticity exhaustion diagram, inelastic cyclic strains and damage kinetics. The summation coefficient S for the general case of the irregular loading is found as follows:

$$S = \frac{1}{2}\sum_{j=1}^{i} S_j = \frac{1}{2}\sum_{j=1}^{i}\frac{1}{N\left(\sigma_{a,j}\right)} \text{ чи } S = \frac{1}{2}\int_0^i \frac{dn}{N\left(\sigma_a(n)\right)}. \qquad (11)$$

For the case, when the cyclic loading is under the constantly changeable stress amplitude, for example, as it is under the "rigid" loading mode, the system of equations (9) – (10) can be written in another way. For this the (2) is used not for the material ultimate state, but for the intermediate one, changing in it the integration boundaries. After the analytical transformations we will obtain the system (12), which is similar to (10).

$$N = \sum_{i=1}^{p} n_i \text{ , where}$$

$$\begin{aligned} n_i = {} & \frac{1}{2}\frac{\sigma_{T,cr} - \sigma_{T,0}}{L_K\left(\sigma_{a,i}\right) - L_0\left(\sigma_{a,i}\right)} \cdot \left(\frac{\sigma_{a.i}}{K}\right)^{-1/m} \left[\frac{\pm\left(\sigma_{T,cr} - \sigma_{T,0}\right)^{a\left(\sigma_{a,i}\right)-1}}{a\left(\sigma_{a,i}\right)\cdot b\left(\sigma_{a,i}\right)\cdot\left(1 - a\left(\sigma_{a,i}\right)\right)} \cdot \left(\frac{\sigma_{a.i}}{K}\right)^{\frac{1-a\left(\sigma_{a,i}\right)}{m}} \cdot \right. \\ & \cdot \left\{ \left[L_0\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,cr} - \sigma_{T,i+1}\right) + L_K\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,i+1} - \sigma_{T,0}\right)\right]^{1-a\left(\sigma_{a,i}\right)} - \right. \\ & \left. - \left[L_0\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,cr} - \sigma_{T,i}\right) + L_K\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,i} - \sigma_{T,0}\right)\right]^{1-a\left(\sigma_{a,i}\right)} \right\} - \\ & \left. - \ln \frac{L_0\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,cr} - \sigma_{T,i+1}\right) + L_K\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,i+1} - \sigma_{T,0}\right)}{L_0\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,cr} - \sigma_{T,i}\right) + L_K\left(\sigma_{a,i}\right)\cdot\left(\sigma_{T,i} - \sigma_{T,0}\right)} \right] \end{aligned}, \qquad (12)$$

where i – the index, which corresponds to the loading stage; p – the number of all stages of the step-by-step or block loading; $\sigma_{a,j}$ – function of the stress amplitude change in the $(j)^{th}$ – stage of loading; n_j – the $(j)^{th}$ stage duration.

In the system (12) there are two expressions, one of which is the recurrent equation and two unknown values: $\sigma_{T,i}$ $(j=1, \ldots, p-1)$ and N. The system (12) is used taking into account the initial conditions, that is, in the initial moment of time the value of the ultimate yield strength equals $\sigma_{T,0}$, and before the fracture – $\sigma_{T,cr}$. The second equation of the system (12) shows the relation between the current value of the cyclic ultimate yield strength $\sigma_{T,i}$ and the number of the loading cycles n_i till the approaching of the given ultimate plasticity exhaustion $\sigma_{T,i+1}$ under the σ_a amplitude loading. Total lifetime for the case of the multi-cycle loading is found according to the first equation of the system (12) when all $\sigma_{T,i}$, have been found according to the second equation.

Determination of the lifetime under the “rigid” loading. The UECP model will be used further for the determination of the fatigue lifetime under the variable amplitudes of the cyclic loading, notably the “rigid” loading under the total and inelastic strain amplitude control by examples of the hardening steel 45 (IV) and softening steel 1X2M [12]. The main characteristics of the mechanical properties and thermal treatment of the materials investigated are presented in Table 1. Parameters of the materials studied under “soft” and “rigid” loading modes are presented in tabl. 2. The fatigue curves parameters were obtained for the bi-logarithmic approximation equation. Besides, in Table 2 the index “-1” indicates the endurance limit (boundary transition from the high-cyclic fatigue domain to the hyper-cyclic one), and “K” index indicates critical fatigue stress (boundary transition from the high-cyclic domain to the low-cyclic one). For all fatigue curves the testing base N_0 was $2{\cdot}10^6$ cycles. Endurance limit under the “soft” and “rigid” loadings was determined for the same lifetime as well. In the Table 3, the coefficients of stabilized stress (σ_a) – cyclic strain ($\varepsilon_{ine,s}(\sigma_a)$) diagrams described by the equation (4) are presented. In the Table 3, the values of the inelastic strain, which correspond to two limiting stresses on the high-cyclic fatigue curve (namely, the critical fatigue stress and endurance limit) are presented.

Table 1

Mechanical properties of materials investigated

Material	Heat treatment of material	σ_m, MPa	σ_u, MPa	E, MPa	δ, %	ψ, %
Steel 45(IV)	Normalizing 840…860^{0}C	316	580	$2{,}09{\cdot}10^5$	25,6	47,4
Steel 1X2M	As-received condition	332	529	$2{,}17{\cdot}10^5$	30,6	76,6

where σ_m – yield strength; σ_u – ultimate strength; E – the 1-st order modulus of elasticity; δ – ultimate elongation; ψ – ultimate contraction.

Table 2

Characteristics of the investigated materials under “soft” mode and “rigid” modes of cyclic loading

Material	Loading mode	σ_{-1}, $\varepsilon_{a,-1}$, $\varepsilon_{ine,-1}$	σ_K, $\varepsilon_{a,K}$, $\varepsilon_{ine,K}$	A	$-B$	$-R$	m
Steel 45(IV)	«soft»	264,6	323,4	50,837	18,386	0,946	13
	«rigid» on ε_a	$1{,}44{\cdot}10^{-3}$	$2{,}17{\cdot}10^{-3}$	-19,47	9,074	0,934	7
	«rigid» on ε_{ine}	$0{,}145{\cdot}10^{-3}$	$1{,}80{\cdot}10^{-3}$	0,687	1,462	0,946	16
Steel 1X2M	«soft»	270,5	333,6	49,059	17,584	0,913	9
	«rigid» on ε_a	$1{,}30{\cdot}10^{-3}$	$2{,}20{\cdot}10^{-3}$	-13,94	7,013	0,869	14
	«rigid» on ε_{ine}	$0{,}0293{\cdot}10^{-3}$	$1{,}52{\cdot}10^{-3}$	2,066	0,935	0,800	26

where A, B – coefficients for the bi-logarithmic approximation equation; R – correlation coefficient; m – specimen number.

Table 3

Parameters of the experimental stabilized diagrams of cyclic strain

Material	K	n	$\varepsilon_{ine}(\sigma_{-1})\cdot 10^4$	$\varepsilon_{ine}(\sigma_K)\cdot 10^3$
Steel 45(IV)	574,151	0,082292	0,816018	0,934825
Steel 1X2M	564,484	0,073052	0,423248	0,746608

The task of the transition to the "rigid" mode of loading has been solved as the option of the step-by-step "soft" cyclic loading: σ_a dependent on the current value D_ε at the given "rigid" mode was introduced into the main model equations.

Two types of the "rigid" mode loading are identified: under the total strain amplitude control *(ε_a=const)* and under the inelastic strain range control *(ε_{ine}=const)*.

The loading trajectories which correspond to the "soft" and two types of the "rigid" modes for the case, when the loading parameters (stress amplitude, the amplitude total or the inelastic strain rate) coinside for the half lifetime under these loading modes, are presented on CCSSD. It is presented schematically on Fig. 1 for the cyclically hardening material.

In Fig. 2 the projections of the obtained loading trajectories corresponding to the "soft" and two types of "rigid" loading modes are compared schematically in the relative lifetimes.

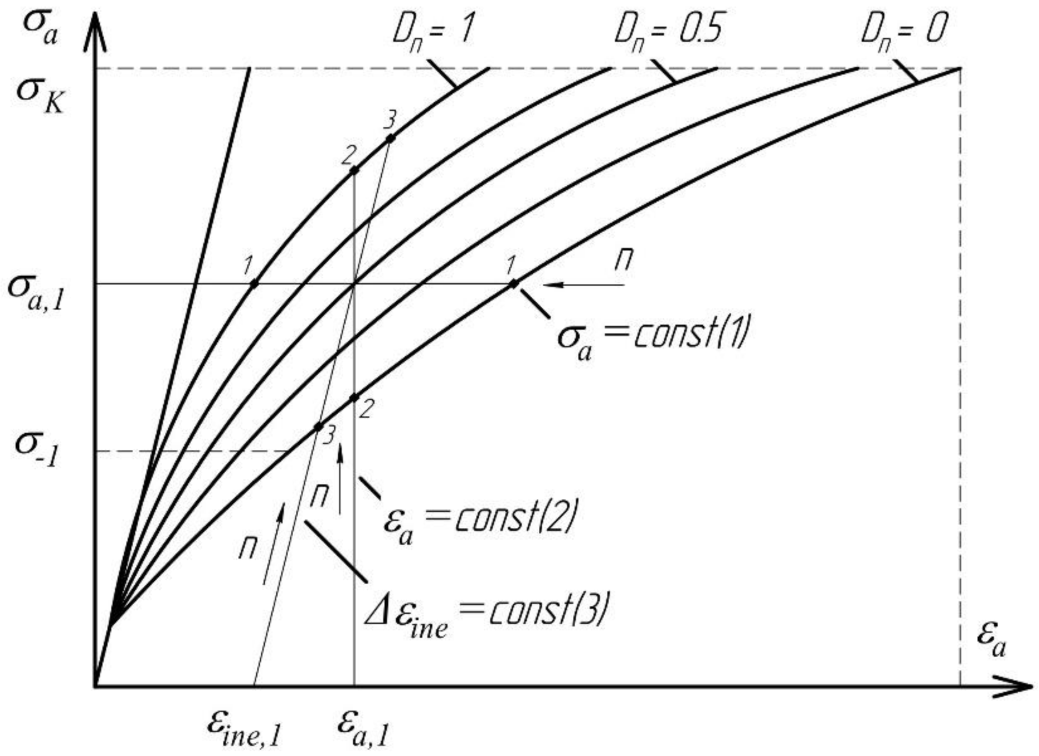


Figure 1. Loading trajectories by way of example of hardening steel, which correspond to "soft" loading (1) and "rigid" loading modes under total strain amplitude control (2) and inelastic strain range control (3). The arrow shows the direction of running cycles growth

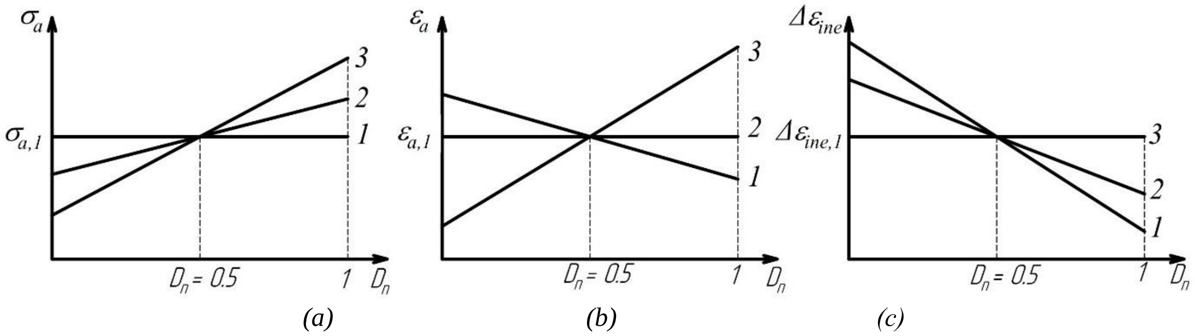


Figure 2. Comparison of kinetics of stress amplitude (*a*), total strain amplitude (*b*) and inelastic strain range (*c*) under different loading modes: 1 – “soft” loading mode; 2 – “rigid” loading mode under total strain amplitude control; 3 – “rigid” loading mode under inelastic strain range control

As it is seen from Fig. 2, the mode $\varepsilon_{ine}=const$ as compared with the $\varepsilon_a=const$ is more aggressive judging by stress changes, as the greater difference between the values of the stress amplitude at the beginning and before the fracture is noticed. The “soft” mode is more agressive judging by change of the inelastic strain. It is seen from the Fig. 2, that the described “rigid” loading modes can be considered as stepwise ones with variable stress amplitude loading at every semi-cycle. For the mode loading $\varepsilon_a=const$ both the stress amplitude and the inelastic strain component are changed under the loading, that is, $\sigma_a=f(\varepsilon_a, D_\varepsilon)$ and $\varepsilon_{ine}=f(\varepsilon_a, D_\varepsilon)$. Under the mode $\varepsilon_{ine}=const$ the stress amplitude and the total strain amplitude are changed during the loading, that is, $\sigma_a=f(\varepsilon_{ine}, D_\varepsilon)$ and $\varepsilon_a=f(\varepsilon_{ine}, D_\varepsilon)$. During the loading under the constant value of one of the loading parameters, two other parameters change continuously. Then, for the “rigid” loading modes we can state, that:

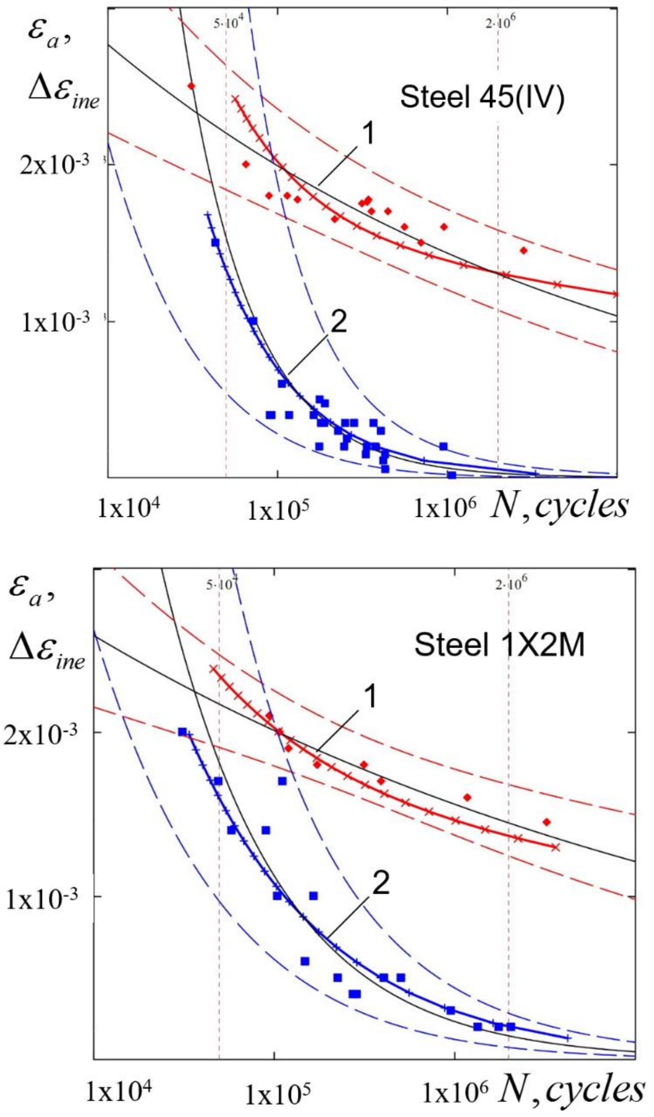
$$\begin{cases}\varepsilon_a = const;\\ \left\{\sigma_a, \varepsilon_{a,ine}\right\} = f\left(\varepsilon_a, D_\varepsilon\right);\end{cases} \text{and} \begin{cases}\Delta\varepsilon_{ine} = const;\\ \left\{\sigma_a, \varepsilon_a\right\} = f\left(\varepsilon_{ine}, D_\varepsilon\right).\end{cases} \tag{13}$$

The determination of the loading parameters change (the stress-strain state kinetics) for the “rigid” modes is presented due to the UECP model equations as the system (3) – (4).

$$\begin{cases}\varepsilon_a = const; \quad \varepsilon_{ine,i} = \varepsilon_a - \sigma_{a,i}/E;\\ \varepsilon_a = \left(\dfrac{\sigma_{a.i}}{K}\right)^{1/m} \cdot \left(L_0\left(\sigma_{a,i}\right) + \dfrac{\sigma_{T,i} - \sigma_{T,0}}{\sigma_{T,cr} - \sigma_{T,0}} \cdot \left(L_K\left(\sigma_{a.i}\right) - L_0\left(\sigma_{a,i}\right)\right)\right) + \dfrac{\sigma_{a,i}}{E};\end{cases} \tag{14}$$

$$\begin{cases}\varepsilon_{ine} = const; \quad \sigma_{a,i} = E\left(\varepsilon_{a,i} - \varepsilon_{ine}\right);\\ \varepsilon_{ine,i} = \dfrac{E\left(\varepsilon_{a,i} - \varepsilon_{ine}\right)}{K}\left(L_0\left(E\left(\varepsilon_{a,i} - \varepsilon_{ine}\right)\right) + \dfrac{\sigma_{T,i} - \sigma_{T,0}}{\sigma_{T,cr} - \sigma_{T,0}}\begin{pmatrix}L_K\left(E\left(\varepsilon_{a,i} - \varepsilon_{ine}\right)\right) - \\ -L_0\left(E\left(\varepsilon_{a,i} - \varepsilon_{ine}\right)\right)\end{pmatrix}\right).\end{cases} \tag{15}$$

The fatigue curves for the 45 and 1X2M steels have been calculated according to the UECP model for the “rigid” loading modes (Fig. 3). The calculated fatigue curves show good correspondence to the experimental curves (Fig. 3). The comparison of the experimental and calculated fatigue curves shows that the calculative curves describe good enough the experimental ones. The maximum error in the description of the lifetime is 31% and that is at the level of endurance limit. For the transition point from high-cycle to low-cycle fatigue domain, the maximum error in lifetime calculation is 22%. The average error does not exceed 11%.



a

b

Figure 3. Calculating (dotted lines) and experimental (solid lines) strain-life curves for "rigid" loading modes under total strain amplitude control (1) and inelastic strain range control (2) for steels 45 (a) and 1X2M (b):

dashed lines – scattering boundary; points – experimental data

Taking into account the loading parameters calculated according to the equation system (14) – (15) under the conditions described above, the stress-strain state kinetics and damages at every loading semi-cycle are determined. The results for the "rigid" loading modes are presented on Fig. 4 (under $\Delta\varepsilon_{ine}$ =*const*) and on Fig. 5 *(*under ε_a =*const)*. As it is seen from the Fig. 4 during the cycling of the hardening materials (steel 45) the stress amplitude and the total strain amplitude increase are revealed, and for the softening steels (1X2M) – vice versa. As it is seen from the Fig. 5, the greater is the hardening materials running, the greater are stress amplitudes and the smaller are the inelastic strain rates, for the softening steels – vice versa.

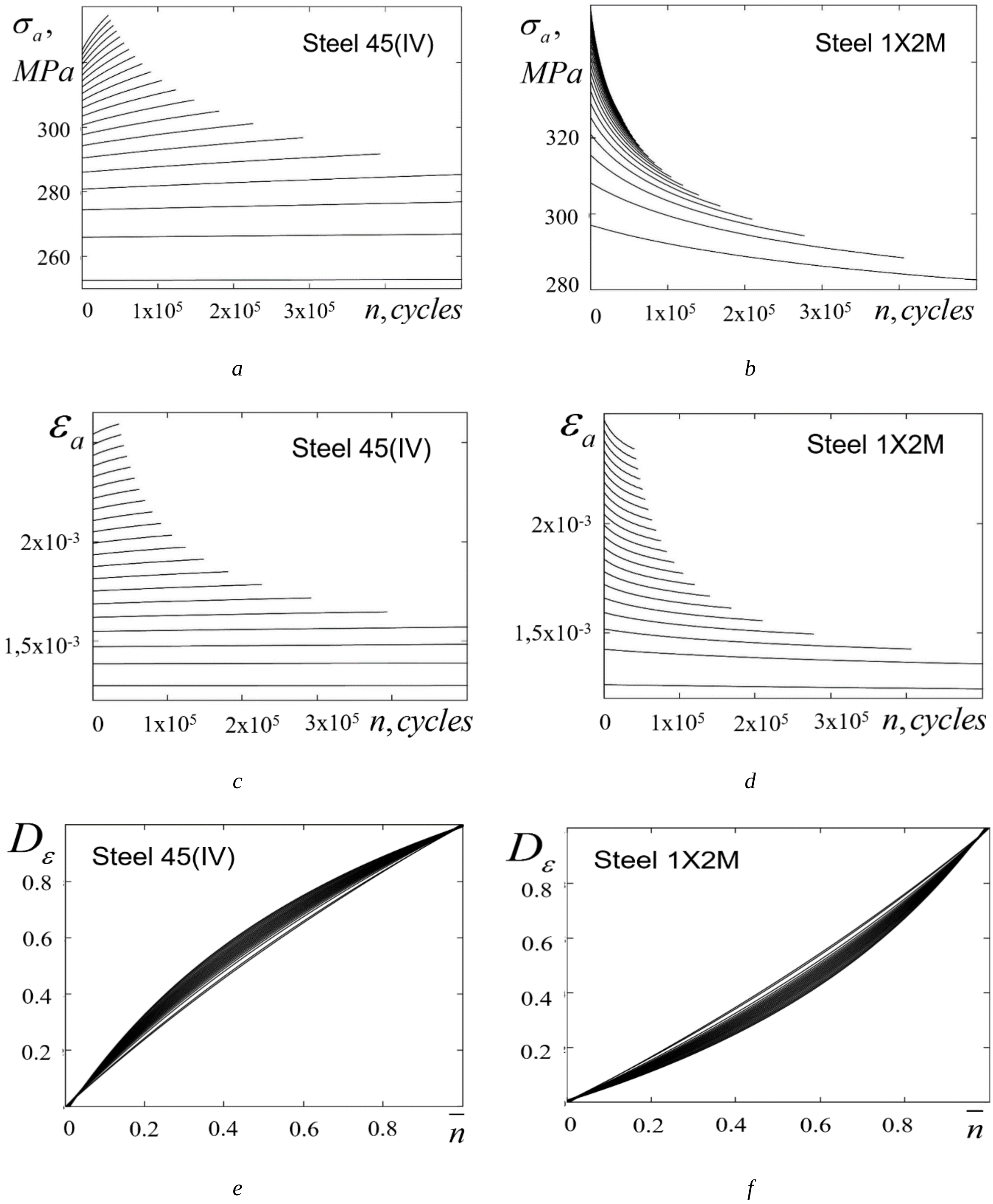


Figure 4. Kinetics of stresses (*a*, *b*), total strain amplitudes (*c*, *d*) and damage (*e*, *f*) for “rigid” loading mode under inelastic strain range $\Delta\varepsilon_{ine}$ control for steels 45 (*a, c, e*) and 1X2M (*b, d, f*). The plotted data are given for discrete $\Delta\varepsilon_{ine}$ values corresponding to high cyclic fatigue

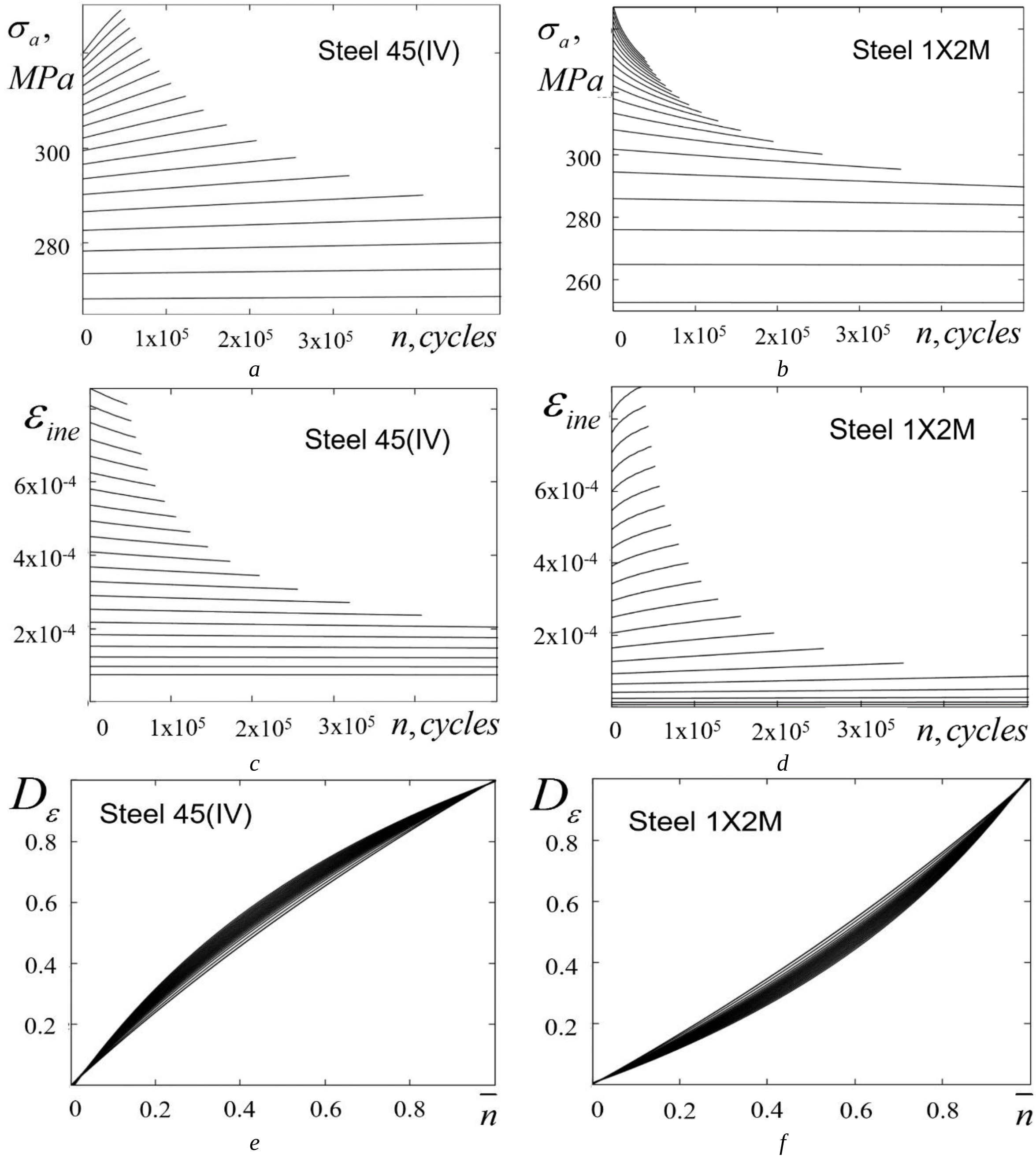


Figure 5. Kinetics of stresses (a, b), inelestic strain amplitudes (c, d) and damage (e, f) for "rigid" loading mode under total strain amplitude ε_a control for steels 45 (a, c, e) and 1X2M (b, d, f). The plotted data are given for discrete values ε_a corresponding to high cyclic fatigue

If the summation coefficient *S* is introduced, then taking account the non-linear stress-strain state kinetics calculated according to the UECP model for the hardening materials it equals 1,07 and for the softening materials it equals 0,93 as compared with calculated according to the linear summation hypothesis. If the summation coefficient *S* is introduced calculated according to the stabilized values of the inelastic strains, it equals 1,25 for the hardening materials and 0,73 for the softening materials as compared with calculated according to the linear summation hypothesis. That is, taking into account the inelastic strains kinetics makes it possible to demonstrate the differences in the fatigue damages accumulation for the hardening and softening materials.

Conclusion. The developed UECP model can be used to sum up fatigue damage and estimate fatigue lifetime, as well as to describe the kinetics of a stress-strain state under “rigid” cyclic loading regimes (with control of inelastic ε_{ine} = const or total ε_a = const strains).

Examples of fatigue curves (σ_a – N) constructions for two steels corresponding to the “rigid” modes of cyclic loading ε_{ine} = const and ε_a = const based on the data obtained under “soft” load mode (σ_a = const), show good correspondence of experimental and calculated data.

Using the UECP model to take into account the kinetics of inelastic deformations under conditions of the “rigid” cyclic loads can justify the difference in the accumulation of fatigue damage for materials with different kinetics of inelastic strains. For materials, which under cyclic loading show instability of inelastic strains, the coefficient of fatigue damage summation is different from unit. That is to say, the kinetics of fatigue damage for these materials under conditions of irregular loading differs from the same, which is determined by the linear hypothesis of damage summation and its consideration allows one to describe the nonlinear nature of fatigue damage accumulation.

In the case of cyclic loading of hardening materials, the inelastic strain kinetics trajectories of which are convex or concave, “rigid” loading modes result in an increase in the coefficient of fatigue damage summation as compared with linear hypothesis of damage summation. For softening materials with a concave inelastic strain kinetics trajectory this statement will be reversed

Application of the UECP model to the fatigue damage summation under “rigid” cyclic loading modes makes it possible to estimate the materials lifetime with less error as compared with application of stabilized inelastic strains and formal hypotheses of fatigue damage summation. Application of linear damage summation hypothesis results in lifetime underestimation or overestimation up to 30% as compared with the values obtained by the UECP model.

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ВЗАЄМОЗВ’ЯЗОК І КІНЕТИКА ВТОМНОГО ПОШКОДЖЕННЯ ЗА УМОВ М’ЯКОГО І ЖОРСТКОГО РЕЖИМІВ НАВАНТАЖЕННЯ

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Резюме. *Наведено розрахункове оцінювання кінетики втомного пошкодження та пружно-пластичного напружено-деформованого стану (НДС) сталей за жорстких режимів циклічного навантаження з використанням моделі граничного вичерпання циклічної пластичності (ГВЦП). Для цього використано розроблену раніше методику підсумовування втомних пошкоджень, яка дозволяє розраховувати кінетику втомних пошкоджень при нерегулярному циклічному навантаженні. Показано відмінність у накопиченні втомних пошкоджень та кінетики пружно-пластичного НДС для жорстких режимів по повній та непружній деформаціях на прикладі двох матеріалів: циклічно зміцнюваної сталі 45 та циклічно знеміцнюваної сталі 1Х2М. Розрахунки за допомогою ГВЦП моделі показують, що для матеріалів, які при циклічному навантаженні мають нестабільність непружної деформації, коефіцієнт підсумовування втомного пошкодження значно відрізняється від оцінки за лінійною гіпотезою підсумовування. Наведені результати можуть бути використані для розрахункового оцінювання довговічності елементів конструкцій, які працюють у режимі жорсткого навантаження, з меншою похибкою у порівнянні з застосуванням стабілізованих значень непружних деформацій і відомих гіпотез підсумовування втомних пошкоджень. Це пов’язано з тим, що запропонована розрахункова модель описує відмінність у накопиченні втомних пошкоджень за змінних амплітуд циклічного навантаження для матеріалів з різною кінетикою непружних деформацій. У свою чергу, це дає можливість теоретично обґрунтувати необхідність урахування особливостей кінетики непружних деформацій в умовах жорсткого режиму навантаження та описати відмінність і нелінійний характер накопичення втомних пошкоджень у цьому режимі для вказаних груп матеріалів.*

Ключові слова*: втома, жорстке навантаження, пошкодження, вичерпання пластичності, пружно-пластичний напружено-деформований стан, непружна деформація, розрахункова модель.*

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