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BIAXIAL TENSION OF THE PLATE SOFTENED BY THE GRIFFITH CRACK

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***Summary**. According to the elastic approach the biaxial loading problem of the plane with an elliptical defect was solved. Using the obtained solution and the deformation approach [1, 3] the solution for the cracked plane under the biaxial load in the elastic-plastic statement has been obtained. Thereupon a deformation criterion for the critical crack tip opening has been proposed. It was shown, that the tensile load applied parallelly to the major defect axis results in the increase of the crack tip opening and the decrease of the crack shores displacement along the minor axis.*

***Key words**: elliptical defect, crack tip opening, elastic approach, elastic-plastic approach, biaxial load, deformation criterion.*

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Statement of the problem. Thin-walled structural elements are widely applied in the industrial machinery products. As a rule, they operate under sufficient stress state, that is why there is a need to have theoretical solutions for the plates softened by the crack-like defects under complicated load, as well as the criteria for estimation of their strength.

Analysis of the available investigation results. The normal separation crack model (the Dagdail model) was defined for the plane stress state (1, 2). Then it was generalised for the plane stress state and plane deformation for any plasticity (3, 4). Taking into account the approximation approach for the estimation of the two-dimentional plastic area is presented in the papers [5, 6]. But in the obtained solutions of the tasks the deformation criteria of strength are not stated. The development of the Griffith model for the biaxial loading in the elastic-plastic statement has been proposed and the deformation criterion of strength has been stated.

The Objective. To solve the problem on the biaxial loading of the plane with the elliptical defect within the elastic-plastic approach and to propose the deformation criterion of the crack tip critical opening. To show the effect of the tensile forces under biaxial loading on the shores notch displacement and on the opening in the crack tip.

Statement of the task. Let us analyse the infinite plate softened by the Griffith crack tensiled into infinities by the distributed forces p and q (Fig. 1). To solve this task at the elastic-plastic approach, that is, taking into account the plasticity at the crack tip, threshold let us solve first of all the auxiliary task, the task on the biaxial tension of the plate softened by the elliptical defect with the semi-axis a (major axis) and b (minor axis), when $a>>b$ (Fig. 2). To find the ellipse shores displacement caused by the forces p and q let us use the potentials $\varphi(\zeta)$ and $\psi(\zeta)$ by the Muskhelyshvyly paper [7].

Here such signs are used

$$\varphi(\zeta)=\frac{pR}{4}\left(\zeta+\frac{2e^{2i\alpha}-m}{\zeta}\right)$$

$$\psi(\zeta) = -\frac{pR}{2}\left[e^{-2i\alpha}\zeta + \frac{e^{2i\alpha}}{m\zeta} - \frac{(1+m^2)(e^{2i\alpha}-m)}{m}\frac{\zeta}{\zeta^2-m}\right] \quad (1)$$

$$a = R(1+m); \quad R = \frac{a+b}{2}; \quad m = \frac{a-b}{a+b}; \quad b = R(1-m) \quad (2)$$

α – the forces direction angle p with the abscissa axis (as it is seen in Fig. 2 $\alpha = 90^o$).

In the case, when $\alpha = 0$ (the forces q are directed along the major axis, $p = 0$), the potentials will look like

$$\varphi(\zeta) = \frac{qR}{4}\left(\zeta + \frac{2-m}{\zeta}\right);$$

$$\psi(\zeta) = -\frac{qR}{2}\left[\zeta + \frac{1}{m\zeta} - \frac{(1+m^2)(1-m)}{m}\frac{\zeta}{\zeta^2-m}\right]. \quad (3)$$

If the forces q are directed along the abscissa axis, then in the formulas (3) p is changed into q. There are such ralations between the variables z and ζ

$$z = x+iy, \quad z = \omega(\zeta) = R\left(\zeta + \frac{m}{\zeta}\right) \quad (4)$$

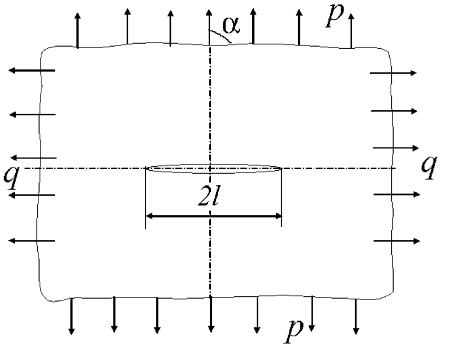

Figure 1. Biaxial tension of a plate with the Griffith crack

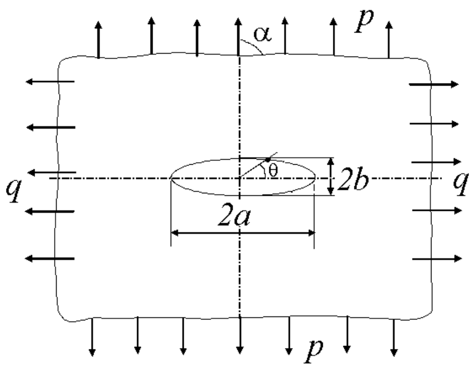

Figure 2. Biaxial tension of a plate with an elliptical notch

Horisontal u and vertical υ displacements of the notch shores for this case ($\alpha = 0$, p=0) are found from the relation [7]

After transformations and calculations we will obtain:

$$2G(u+i\upsilon) = \chi\varphi(z) - z\overline{\varphi'(z)} - \psi(\bar{z}) \quad . \quad (5)$$

After transformations and calculations we will obtain:

$$2Gu=\chi\frac{qR}{8}\left(\zeta+\frac{2-m}{\zeta}\right)-\frac{qR}{8}\left(\zeta+\frac{m}{\zeta}\right)\left(\frac{\bar{\zeta}^2-2+m}{\bar{\zeta}^2-m}\right)-$$
$$-\frac{qR}{4}\cdot\left[\bar{\zeta}+\frac{1}{m\bar{\zeta}}-\frac{(1+m^2)(1-m)}{m}\frac{\bar{\zeta}}{\bar{\zeta}^2-m}\right]+\chi\frac{qR}{8}\left(\bar{\zeta}+\frac{2-m}{\bar{\zeta}}\right)-$$
$$-\frac{qR}{8}\cdot\left(\bar{\zeta}+\frac{m}{\bar{\zeta}}\right)\frac{\left(\zeta^2-2+m\right)}{\left(\zeta^2-m\right)}-\frac{qR}{4}\left[\zeta+\frac{1}{m\zeta}-\frac{(1+m^2)(1-m)\zeta}{m\left(\zeta^2-m\right)}\right]. \qquad (6)$$

$$2G\upsilon=\chi\frac{qR}{8i}\left(\zeta-\frac{2-m}{\zeta}\right)-\frac{qR}{8i}\left(\zeta+\frac{m}{\zeta}\right)\cdot\frac{\left(\bar{\zeta}^2-2+m\right)}{\left(\bar{\zeta}^2-m\right)}-$$
$$-\frac{qR}{4i}\left[\bar{\zeta}+\frac{1}{m\bar{\zeta}}-\frac{\left(1+m^2\right)(1-m)}{m}\frac{\bar{\zeta}}{\bar{\zeta}^2-m}-\right]-\chi\frac{qR}{8i}\left(\bar{\zeta}+\frac{2-m}{\bar{\zeta}}\right)+$$
$$+\frac{qR}{8i}\left(\bar{\zeta}+\frac{m}{\bar{\zeta}}\right)\frac{\left(\zeta^2-2+m\right)}{\left(\zeta^2-m\right)}+\frac{qR}{4i}\left[\zeta+\frac{1}{m\zeta}-\frac{\left(1+m^2\right)(1-m)}{m}\frac{\zeta}{\zeta^2-m}\right]. \qquad (7)$$

where G – the displacement modulus; ν – the Poisson's ratio, $\chi=\frac{3-\nu}{1+\nu}$

Under the distributed forces p ($q = 0$) in the direction perpendicular to the ellipse major axis $\left(\alpha=\frac{\pi}{2}\right)$ the potentials $\varphi(\zeta)$ and $\psi(\zeta)$ will look like

$$\varphi(\zeta)=\frac{pR}{4}\left(\zeta-\frac{2+m}{\zeta}\right),$$

$$\psi(\zeta)=-\frac{pR}{2}\left[-\zeta-\frac{1}{m\zeta}+\frac{(1+m^2)(1+m)}{m}\frac{\zeta}{\zeta^2-m}\right]. \qquad (8)$$

Then let us write the expressions for the displacements u and υ for this case

$$2Gu=\chi\frac{pR}{8}\left(\zeta-\frac{2+m}{\zeta}+\bar{\zeta}-\frac{2+m}{\bar{\zeta}}\right)-\frac{pR}{8}\left[\left(\zeta+\frac{m}{\zeta}\right)\cdot\frac{\left(\bar{\zeta}^2+2+m\right)}{\left(\bar{\zeta}^2-m\right)}+\left(\bar{\zeta}+\frac{m}{\bar{\zeta}}\right)\frac{\left(\zeta^2+2+m\right)}{\left(\zeta^2-m\right)}\right]+$$
$$+\frac{pR}{4}\left[-\bar{\zeta}-\frac{1}{m\bar{\zeta}}+\frac{\left(1+m^2\right)(1+m)}{m}\frac{\bar{\zeta}}{\bar{\zeta}^2-m}-\zeta-\frac{1}{m\zeta}+\frac{\left(1+m^2\right)(1+m)\zeta}{m\left(\zeta^2-m\right)}\right]; \qquad (9)$$

$$2G\upsilon=\chi\frac{pR}{8i}\left(\zeta-\frac{2+m}{\zeta}\right)-\frac{pR}{8i}\left(\zeta+\frac{m}{\zeta}\right)\cdot\frac{\left(\bar{\zeta}^2-2+m\right)}{\left(\bar{\zeta}^2-m\right)}+$$
$$+\frac{pR}{4i}\left[-\bar{\zeta}-\frac{1}{m\bar{\zeta}}+\frac{\left(1+m^2\right)(1+m)}{m}\frac{\bar{\zeta}}{\bar{\zeta}^2-m}-\right]-\chi\frac{pR}{8i}\left(\bar{\zeta}-\frac{2+m}{\bar{\zeta}}\right)+$$
$$+\frac{pR}{8i}\left(\bar{\zeta}+\frac{m}{\bar{\zeta}}\right)\frac{\left(\zeta^2+2+m\right)}{\left(\zeta^2-m\right)}-\frac{pR}{4i}\left[-\zeta-\frac{1}{m\zeta}+\frac{\left(1+m^2\right)(1+m)}{m}\frac{\zeta}{\zeta^2-m}\right]. \qquad (10)$$

Under the simultaneous action of the distributed forces q ($\alpha = 0$) and $p\left(\alpha = \dfrac{\pi}{2}\right)$ the resultant displacements u and υ can be found according to the sum of the obtained relations (See (6), (7) and (9), (10)).

Having substituted in these formulas instead of ζ and $\bar{\zeta}$ the expressions

$$\begin{aligned}\zeta &= \rho(\cos\theta + i\sin\theta),\\ \bar{\zeta} &= \rho(\cos\theta - i\sin\theta)\end{aligned} \tag{11}$$

after transformations and simplifications, and taking into account the fact, that on the ellipse contour ρ = 1 we will obtain the expressions for υ and u from the simultaneous action of the forces p and q

$$\begin{aligned}2G\upsilon = {} & \chi\frac{qR}{4}(-1+m)\sin\theta - \\ & -\frac{qR}{4}\left[\frac{\left(1+m^2(m-2)\right)\sin\theta + ((m-2)+m^2)\sin\theta\left(3\cos^2\theta - \sin^2\theta\right)}{1-2m\left(\cos^2\theta - \sin^2\theta\right)+m^2}\right] + \\ & +\frac{qR}{2}\left[\left(1-\frac{1}{m}\right)\sin\theta + \frac{(1+m^2)(1-m^2)\sin\theta}{m[1-2m\left(\cos^2\theta-\sin^2\theta\right)+m^2]}\right] + \\ & +\chi\frac{pR}{4}(3+m)\sin\theta - \\ & -\frac{pR}{4}\left[\frac{\left(1+m^2(m+2)\right)\sin\theta + ((2+m)+m^2)\sin\theta\left(3\cos^2\theta - \sin^2\theta\right)}{1-2m\left(\cos^2\theta - \sin^2\theta\right)+m^2}\right] + \\ & +\frac{pR}{2}\left[\left(1-\frac{1}{m}\right)\sin\theta + \frac{(1+m^2)(1+m)^2\sin\theta}{m[1-2m\left(\cos^2\theta-\sin^2\theta\right)+m^2]}\right]\end{aligned} \tag{12}$$

$$\begin{aligned}2Gu = {} & \chi\frac{qR}{8}(3-m)2\cos\theta - \\ & -\frac{qR}{4}\left[\frac{\left(1-m^2(m-2)\right)\cos\theta + ((m-2)-m^2)\cos\theta\left(\cos^2\theta - 3\sin^2\theta\right)}{1-2m\left(\cos^2\theta - \sin^2\theta\right)+m^2}\right] - \\ & -\frac{qR}{2}\left[\left(1+\frac{1}{m}\right)\cos\theta - \frac{(1+m^2)(1-m)^2\cos\theta}{m[1-2m\left(\cos^2\theta-\sin^2\theta\right)+m^2]}\right] + \\ & +\chi\frac{pR}{4}(-1-m)\cos\theta - \\ & -\frac{pR}{4}\left[\frac{\left(1-m^2(2+m)\right)\cos\theta + ((2+m)-m^2)\left(\cos^2\theta - 3\sin^2\theta\right)\cos\theta}{1-2m\left(\cos^2\theta - \sin^2\theta\right)+m^2}\right] - \\ & -\frac{pR}{2}\left[\left(1+\frac{1}{m}\right)\cos\theta - \frac{(1+m^2)(1-m^2)\cos\theta}{m[1-2m\left(\cos^2\theta-\sin^2\theta\right)+m^2]}\right].\end{aligned} \tag{13}$$

To find the displacements u and υ of the elliptic notch shores, let us use the following relations [7]

$$x = \mathrm{Re}(w(\zeta)) = \frac{R}{2}(\zeta + \bar{\zeta} + \frac{m}{\zeta} + \frac{m}{\bar{\zeta}}),$$

$$y = \mathrm{Im}(w(\zeta)) = \frac{R}{2i}(\zeta - \bar{\zeta} + \frac{m}{\zeta} - \frac{m}{\bar{\zeta}}). \quad (14)$$

Here we will obtain the ellipse contour coordinate points:

$$x = R(1+m)\cos\theta,$$

$$y = R(1-m)\sin\theta. \quad (15)$$

Let us analyse the ellipse with the axis a = 0,012 m and b = 0,003 m, for which R = 0,0075 m; m = 0,6. Being found according to the formulas (12), (13) at p = 250 MPa; σ_0 = 250 MPa; $E = 0{,}71 \cdot 10^5$ MPa, $G = 0{,}27 \cdot 10^5$ МПа; ν = 0,34; χ = 3,9; q = 0; 0,5p; p (Д16Т) horizontal u and vertical υ displacements of the ellipse contour points in the first quarter (θ = 0...90°) are presented in Fig. 3, 4.

As it is seen from Fig. 3 under similar conditions the horizontal displacements of the ellipse contour points u are minus, that is, the ellipse contour is displaced to the left for the contour points, at $\theta = 90°$ the displacement being zero for different relations p and q, and in the tip ($\theta = 0$) while q approaching p they being directed to 0 (here θ – the angle with the top in the ellipse centre, which specifies the position of the ellipse contour points).

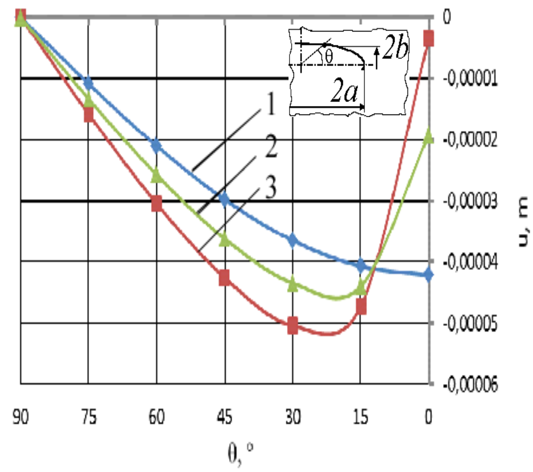

Figure 3. Horizontal displacement u of the ellipse contours points (a = 0.012 m, b = 0.003 m) along the major semi axis under biaxial tension (13): *1* – p = 250 MPa, q = 0; *2* – p = 250 MPa, q = 125 MPa; *3* – p = q = 250 MPa

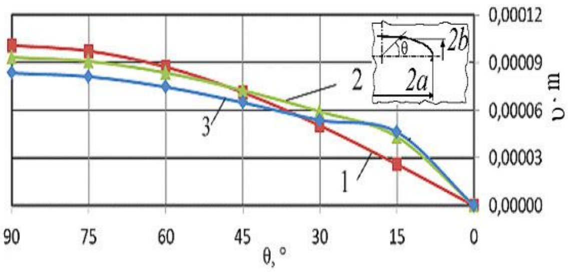

Figure 4. Vertical displacement v of the ellipse contours points (a = 0.012 m, b = 0,003 m) along the minor semiaxis under biaxial tension (12): *1* – p = 250 MPa, q = 0; *2* – p = 250 MPa, q = 0,5 and *3* – p = 250 MPa, q = p

In Fig. 4 the results of calculation of the vertical displacements υ of the ellipse contour points at $0 \le \theta \le 90°$ according to the formula (12) are presented. Here the curve 1 – the displacement υ caused by the force p (q = 0), the curve 2 – total action of forces p and

$q = 0{,}5p$ and the curve $3 - p = q$. As it is seen, when the longitudinal forces q are applied, the displacements υ in the minor threshold of the sharp ellipse top ($x = a$) are increased, and those close to the ellipse middle part – decrease, that is, they are less than when q is not available ($q = 0$).

The ultimate equilibrium state of the plate with the small – diameter tip rounding of the elliptic hole. The solution of the task on the elliptic hole (Fig. 2) has been obtained in the plastic approach (formulas (12), (13)), and the tasks on the cracked plate – in the elastic-plastic one (taking into account the plasticity area). That is why to obtain the solution for the cracked plate under the biaxial loading let us take advantage of the combined solutions for the crack and the ellipse (opened crack).

In the paper [1] the formula for the crack tip opening obtained for the biaxial plate tension has been presented (Fig. 1). For the plate with the similar crack under single-axial tension by the intensity forces p in the paper [8] the relations have been obtained

$$\delta_p = -8 l_0 \sigma_0 C \ln \cos \frac{\pi p}{2\sigma_0}, \tag{16}$$

where σ_0 – adhesion stress in the pre-fracture area modeled by the additional crack Δl;

$\sigma_0 = \sigma_t$ – for the ideal elastic-plastic materials and $\sigma_0 = \frac{\sigma_t + \sigma_B}{2}$ – for the hardened materials, where σ_t and σ_B – the material ultimate yield and strength correspondingly;

$C = \frac{1}{\pi \cdot E}$ – for the plane stress state and $C = \frac{1-\nu^2}{\pi \cdot E}$ – for the plane deformation.

According to the expressions [1] and (16) the values of the crack tip opening are presented on Table 1 depending on the applied forces p and q (δ_Γ). In the last column the opening calculated according to the formula (16) is presented. As it is seen from the Table, when the longitudinal force q is increased from *0* to *0,7p* under the constant value of the force *p*, the crack tip opening calculated according to the [1], decreases. Under the further increase of q (from $0{,}7p$ till p) the crack tip opening increases and at $q = p$ it coinsides with the opening calculated according to the formula (16) (single-axial tension) [5]. The obtained results, at $q = 0$; $q = 0{,}1p$; $q = 0{,}5p$, $q = 0{,}7p$ and $q = p$ in particular are presented graphically in Fig.5. As it was mentioned above for the case $q = p$ the opening calculated according to the formula [1] coinsides with the opening calculated according to the formula (16).

Table 1. Dependence of the crack tip opening on the applied load

q/p / *p*,MPa	δ_Γ, m (formula [1])					δ_p, m (formula (16))
	0	0,1	0,5	0,7	1,0	
0	0	0	0	0	0	0
30	$1{,}772 \cdot 10^{-6}$	$1{,}776 \cdot 10^{-6}$	$1{,}802 \cdot 10^{-6}$	$1{,}838 \cdot 10^{-6}$	$1{,}848 \cdot 10^{-6}$	$1{,}848 \cdot 10^{-6}$
60	$7{,}12 \cdot 10^{-6}$	$7{,}117 \cdot 10^{-6}$	$7{,}198 \cdot 10^{-6}$	$7{,}434 \cdot 10^{-6}$	$7{,}519 \cdot 10^{-6}$	$7{,}519 \cdot 10^{-6}$
90	$1{,}691 \cdot 10^{-5}$	$1{,}669 \cdot 10^{-5}$	$1{,}647 \cdot 10^{-5}$	$1{,}712 \cdot 10^{-5}$	$1{,}743 \cdot 10^{-5}$	$1{,}743 \cdot 10^{-5}$
120	$3{,}442 \cdot 10^{-5}$	$3{,}282 \cdot 10^{-5}$	$3{,}044 \cdot 10^{-5}$	$3{,}162 \cdot 10^{-5}$	$3{,}241 \cdot 10^{-5}$	$3{,}241 \cdot 10^{-5}$
150	$7{,}866 \cdot 10^{-5}$	$6{,}456 \cdot 10^{-5}$	$5{,}087 \cdot 10^{-5}$	$5{,}227 \cdot 10^{-5}$	$5{,}409 \cdot 10^{-5}$	$5{,}409 \cdot 10^{-5}$
180	–	–	$8{,}192 \cdot 10^{-5}$	$8{,}173 \cdot 10^{-5}$	$8{,}575 \cdot 10^{-5}$	$8.575 \cdot 10^{-5}$
210	–	–	$1{,}364 \cdot 10^{-4}$	$1{,}261 \cdot 10^{-4}$	$1{,}357 \cdot 10^{-4}$	$1{,}357 \cdot 10^{-4}$

240	–	–	$3{,}230 \cdot 10^{-4}$	$2{,}048 \cdot 10^{-4}$	$2{,}368 \cdot 10^{-4}$	$2{,}368 \cdot 10^{-4}$
250						$3{,}140 \cdot 10^{-4}$

According to the results of carried out analysis (Table 1 and Figure 5) there arises the need to obtain the alternative solution of the task for the biaxial tension of the cracked plate (Fig. 1).

Under the biaxial tension of the cracked plate the effect of the forces q on the stress-strain state at the crack tip threshold will be in the case of:

a) initial opening in the crack tip caused by the method of its initiation marked as $2\rho_0$;

b) crack opening caused by the forces $p - \delta_p$ ([8]).

Thus, the total tip radius rounding of the loaded crack will be found from the relation

$$\rho_1 = \rho_0 + \rho_p , \tag{17}$$

where $\rho_p = \dfrac{\delta_p}{2}$.

Let us note, that the parameter ρ_p depends on the applied force p similar to δ_p.

As the result we will obtain the ellipse-crack with such parameters:
the major semiaxis

$$a_1 = l_0 + \rho_1, \tag{18}$$

where ρ_1 is found according to the relation (17),
the minor semiaxis

$$b_1 = \sqrt{\rho \cdot (l_0 + \rho)}. \tag{19}$$

Vertical displacements υ of the shores along the crack contour caused by the forces p (at q= 0) are found according to the δ_c – model [8] and according to the formula

$$\upsilon(x,0) = c\sigma_0 \left[(x - l_0)\Gamma(l,x,l_0) \right] - \left[(x + l_0)\Gamma(l,x,-l_0) \right], \quad -l \le x \le l. \tag{20}$$

To find the additional vertical displacements under q forces let us substitute in the formula (12) the values from the (18) and (19) (ρ_0 = 0,00005 m, $\rho_p = 1{,}57 \cdot 10^{-4}$ m, a_1 = 0,012207 m, b_1 = 0,0015898 m) and other parameters, which these relations include.

In Fig. 6 the results of calculation of the displacement υ of the ellipse-crack from its middle part (a_1 = 0) till the crack tip (a_1 = 0,012 m) according to the formula (12) taking into account the mentioned above.

Here, the curve 1 – the contour displacement according to the formula (12) p = 0, q = 250 MPa, the curve 2 – the contour displacement at p = 0, q = 250 MPa, the curve 3 – the p and q total forces action (p = q = 250 MPa).

As it is seen from Fig. 6 under the longitudinal forces q the vertical displacements at the crack tip threshold increase and being approached to its middle part they decrease.

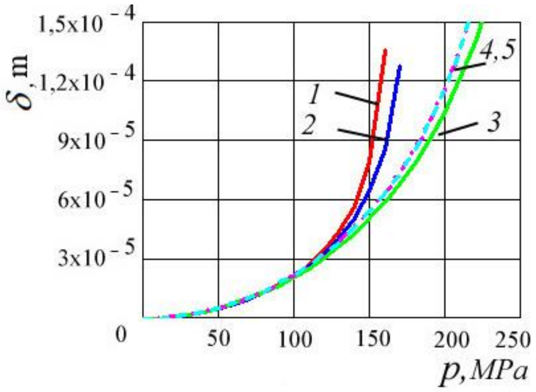

Figure 5. Dependence of the crack tip opening on the applied forces under biaxial load (p * = 250 MPa): *1* – q / p = 0; *2* – q / p = 0,1; *3* – q / p = 0,7; *4* – q / p = 1; *5* – by the δ_c-model (16), *1-4* – by the formula [1]

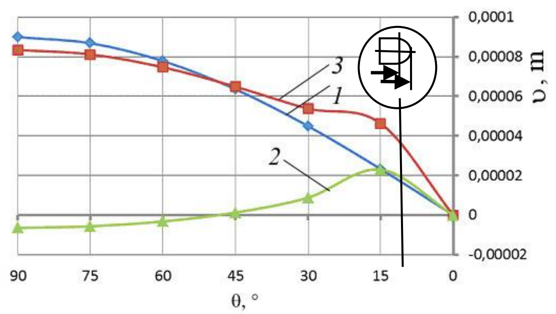

Figure 6. Dependence of the vertical displacements v of the crack contour on the angle θ: *1* – p = 250 MPa, q = 0; *2* – p = 0 MPa, q = 250 MPa; *3* – p = q = 250 MPa. Vertical line – the measuring line of the opening in the crack tip

Thus, the criterion of the crack tip ultimate opening under the biaxial loading of the elliptical cracked plane will look like

$$\delta_c = \delta_{p*} + 2 \cdot \upsilon(0, q^*), \tag{21}$$

where δ_{p*} – the crack tip opening according to the formula (16) at $p = p*$;

$2 \cdot \upsilon(0, q^*)$ – the elliptical crack tip opening according to the formula (12) at $p = 0,\ q = q*$.

Conclusions. The task on the biaxial loading of the plane with the elliptic defect has been solved in the plastic statement. The deformation criterion of the critical opening of the crack tip basing on the solution on the biaxial loading of the plane with the elliptic hole within the plastic statement and the task on the biaxial loading of the cracked plate in the elastic-plastic statement, has been proposed. It was shown, that the tensile forces q parallel to the crack major axis result in the increase of the contour displacements in the crack tip threshold and their decrease along the minor axis as compared with those under only the tensile forces p.

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ДВОВІСНИЙ РОЗТЯГ ПЛАСТИНИ, ПОСЛАБЛЕНОЇ ТРІЩИНОЮ ГРІФФІТСА

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Резюме. *В рамках пружної постановки розв'язано задачу про двовісне навантаження площини з еліпсоподібним дефектом. Використовуючи цей розв'язок та деформаційний підхід [1, 6], отримано розв'язок для площини, послабленої тріщиною за умов двовісного навантаження в пружно-пластичній постановці. На цій основі запропоновано деформаційний критерій критичного розкриття вершини тріщини. Показано, що наявність розтягуючих зусиль, паралельних більшій осі дефекта, призводить до збільшення розкриття вершини тріщини та зменшення переміщень берегів тріщини уздовж меншої осі.*

Ключові слова: *еліпс, тріщина, розкриття вершини тріщини, двовісне навантаження, деформаційний критерій.*

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