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AN EXACT SERIES SOLUTION FOR FREE VIBRATION OF CYLINDRICAL SHELL WITH ARBITRARY BOUNDARY CONDITIONS

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Summary. *Simple accurate formulas for the natural frequencies of circular cylindrical shells are presented for modes in which transverse deflection dominates. Based on the Donnell-Mushtari thin shell theory the equations of motion of the circular cylindrical shell are introduced, using series expansion for axial coordinate and Fourier series for the circumferential direction, a simple explicit solution is obtained. Also, the influence of deformation component is investigated, it is shown that it can be neglected. Good agreement with experimental data and FEM is shown. The advantage of a current approach over the existing formulas is simplicity in programming.*

Key words*: Cylindrical shell, domain decomposition method, natural frequencies, free vibration.*

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Nomenclature

R, h, l	*Mean radius; Shell thickness; Shell length;*
ρ, E, μ	*Density of shell material; Young's modulus; Poisson ratio;*
N_x, N_φ	*Axial and circumferential normal forces;*
Q_x, Q_φ	*Axial and circumferential bending forces;*
L	*Tangential force;*
M_x, M_φ	*Axial and circumferential moments;*
$M_{x\varphi}$	*Tangential moment;*
u, v, w	*Axial, circumferential and radial displacements;*
$\varepsilon_x, \varepsilon_\varphi, \varepsilon_{x\varphi}$	*Median surface strains;*
$\chi_x, \chi_\varphi, \chi_{x\varphi}$	*Bending strains;*
n, m	*Wave number in circumferential and axial direction*

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Introduction. Cylindrical shells are the most investigated type of shells for which there are extensive experimental data and many analytical solutions have been obtained. To solve the shell equations expansion in Fourier series $\sum_n \cos(n\varphi)+\sin(n\varphi)$ along the circumferential coordinate φ is usually used, and then various methods for solving the obtained ordinary differential equation of the 8th degree is suggested. Nowadays, a large number of analytical and numerical methods for solving the problem of free and forced vibrations of an isotropic and composite cylindrical shell have been proposed: exact analytical solutions [1], the Rayleigh-Ritz method [2], the Bubnov-Galerkin method [3, 4], the domain decomposition approach [5], wave propagation method [6].

At the same time, the rapid development of universal computer programs based on the finite element method (FEM) raises the question of the need for further analytical studies. The problem with analytical methods lies in the fact that they aren't simple in realization, in proving their accuracy and further applications[1, 5]. On other hand, FEM solution is also difficult to analyze, it does not provide the clear engineering understanding of the range of natural frequencies and shape of natural forms. From an engineering point of view, the existence of simple formulas for estimating the natural frequencies of vibration is essential: firstly, for solving the problems of shell dynamic analysis [7], and secondly for testing and analyzing problems solved using FEM. The presence of such formulas also makes it possible to quickly estimate the frequency spectrum of a structure and, if necessary, carry out a refined calculation using an FEM.

The greatest interest in "engineering formulas" was in the 60-70s, when the first approximation expressions were proposed on the basis of the Bubnov-Galerkin method [3, 4]. Perhaps the best engineering estimates with the help of V. Vlasov's hypotheses and variational principles are obtained in the book of Kan [8]. The disadvantage of his solution is the incorrect application of hypotheses, as well as the absence of dependencies for all conditions of supporting the shell. Recent work in this area can be noted for membrane approximation [1], and a comparative analysis of formulas [9]. It should be noted that accurate formulas without any hypotheses and assumptions have a practical value if they are rather simple and understandably written.

The derivation of simple and accurate formulas for the natural frequencies of a cylindrical shell for various boundary conditions, including elastically supported edges, written explicitly is the main goal of this paper. The effect of the deformation component on natural frequencies is also analyzed and a comparison with the experimental data and the results of other researchers is given.

Equations of motion. Initial system of dynamic equations for the thin shell:

$$\frac{\partial N_x}{\partial x}+\frac{\partial L}{R\partial\varphi}+\rho h\ddot{u}=0\,, \tag{1a}$$

$$\frac{\partial N_\varphi}{R\partial\varphi}+\frac{\partial L}{\partial x}+\frac{Q_\varphi}{R}+\rho h\ddot{v}=0\,, \tag{1b}$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_\varphi}{R\partial\varphi}-\frac{N_\varphi}{R}+\rho h\ddot{w}=0\,, \tag{1c}$$

$$Q_\varphi=\frac{\partial M_\varphi}{R\partial\varphi}+\frac{\partial M_{x\varphi}}{\partial x}\,, \tag{1d}$$

$$Q_x=\frac{\partial M_x}{\partial x}+\frac{\partial M_{x\varphi}}{R\partial\varphi}\,, \tag{1e}$$

Inner forces are connected with deformations using physical equations:

$$N_{\varphi} = -H\left(\varepsilon_{\varphi} + \mu\varepsilon_{x}\right), \tag{2a}$$

$$N_{x} = -H\left(\varepsilon_{x} + \mu\varepsilon_{\varphi}\right), \tag{2b}$$

$$L = -G\varepsilon_{x\varphi}h\,, \tag{2c}$$

$$M_{x} = -H\delta\left(\chi_{x} + \mu\chi_{\varphi}\right), \tag{2d}$$

$$M_{\varphi} = -H\delta\left(\chi_{\varphi} + \mu\chi_{x}\right), \tag{2e}$$

$$M_{x\varphi} = -\frac{H\delta}{2}(1-\mu)\chi_{x\varphi}, \tag{2f}$$

Here we used notifications: $H = \dfrac{Eh}{1-\mu^{2}}, \quad \delta = \dfrac{h^{2}}{12}$.

Geometrical equations combine displacements with strains:

$$\varepsilon_{x} = \frac{\partial u}{\partial x}, \tag{3a}$$

$$\varepsilon_{\varphi} = \frac{1}{R}\frac{\partial v}{\partial \varphi} + \frac{w}{R}, \tag{3b}$$

$$\varepsilon_{x\varphi} = \frac{1}{R}\frac{\partial u}{\partial \varphi} + \frac{\partial v}{\partial x}, \tag{3c}$$

$$\gamma_{x} = -\frac{\partial w}{\partial x}, \tag{3d}$$

$$\gamma_{\varphi} = -\frac{\partial w}{R\partial \varphi} + \frac{v}{R}. \tag{3e}$$

Bending strains depend on displacements:

Without deformation component	With deformation component	
$\chi_{x} = -\dfrac{\partial^{2} w}{\partial x^{2}}$,	$\chi_{x} = -\dfrac{\partial^{2} w}{\partial x^{2}}$,	(4a)
$\chi_{\varphi} = \dfrac{\partial v}{R^{2}\partial \varphi} - \dfrac{\partial^{2} w}{R^{2}\partial \varphi^{2}}$,	$\chi_{\varphi} = -\dfrac{w}{R^{2}} - \dfrac{\partial^{2} w}{R^{2}\partial \varphi^{2}}$,	(4b)
$\chi_{x\varphi} = \dfrac{\partial v}{R\partial x} - \dfrac{2\partial^{2} w}{R\partial \varphi\partial x}$,	$\chi_{x\varphi} = \dfrac{\partial v}{R\partial x} - \dfrac{\partial u}{R^{2}\partial \varphi} - \dfrac{2\partial^{2} w}{R\partial \varphi\partial x}$.	(4c)

The system of equations given above is sufficient for solving the problem of free vibration of a cylindrical shell.

Numerical solution. Consider a numerical procedure for solving eighth-order differential equation, which follows from the governing Eqs. (1)–(4). This solution is the most accurate as it does not use essential simplifications at the stage of the problem statement. As the initial unknowns we use the expansions of eight parameters in trigonometric series, namely: the longitudinal displacement $u(x,\varphi,t)$, tangential displacement $v(x,\varphi,t)$, radial displacement $w(x,\varphi,t)$, rotation angle $\gamma_{x}(x,\varphi,t)$, axial stress resultant $N_{x}(x,\varphi,t)$, shearing stress resultant $L(x,\varphi,t)$, transverse stress resultant $Q_{x}(x,\varphi,t)$, and couple $M_{x}(x,\varphi,t)$.

From Eqs.(1a-1d)

$$\frac{\partial N_{x}}{\partial x} = -\frac{\partial L}{R\partial \varphi} - \rho h\frac{\partial^{2} u}{\partial t^{2}}, \tag{5a}$$

$$\frac{\partial L}{\partial x}=-\frac{\partial N_\varphi}{R\partial\varphi}-\frac{Q_\varphi}{R}-\rho h\frac{\partial^2 v}{\partial t^2}, \tag{5b}$$

$$\frac{\partial Q_x}{\partial x}=-\frac{\partial Q_\varphi}{R\partial\varphi}+\frac{N_\varphi}{R}-\rho h\frac{\partial^2 w}{\partial t^2}, \tag{5c}$$

$$\frac{\partial M_x}{\partial x}=Q_x-\frac{\partial M_{x\varphi}}{R\partial\varphi}, \tag{5d}$$

From Eqs.(2a), (2b) and (3a)

$$\frac{\partial u}{\partial x}=-\frac{N_x-\mu N_\varphi}{Eh}, \tag{5e}$$

From Eqs.(2c) and (3c)

$$\frac{\partial v}{\partial x}=\varepsilon_{x\varphi}-\frac{1}{R}\frac{\partial u}{\partial\varphi}=-\frac{L}{hG}-\frac{1}{R}\frac{\partial u}{\partial\varphi}, \tag{5f}$$

From Eq. (3d):

$$\frac{\partial w}{\partial x}=-\gamma_x, \tag{5g}$$

From Eqs. (4a), (3d) and (2d,2e):

$$\frac{\partial\gamma_x}{\partial x}=\chi_x=-\frac{M_x-\mu M_\varphi}{Eh\delta}, \tag{5h}$$

The quantities N_φ, Q_φ, M_φ, and $M_{x\varphi}$, which can be expressed through the accepted unknown parameters, appear in Eqs. (5). We obtain the expression for N_φ by excluding ε_φ from the physical equations for forces (2a-2d):

$$N_\varphi=\mu N_x-\varepsilon_\varphi Eh=\mu N_x-\frac{Eh}{R}\left(\frac{\partial v}{\partial\varphi}+w\right). \tag{6a}$$

The expression for M_φ entering this equation is written according to the physical equation (2e) and, with the account of the expression for χ_φ (4b), has the following form:

$$M_\varphi=\mu M_x-\chi_\varphi Eh\delta=\mu M_x-\frac{Eh\delta}{R^2}\left(\frac{\partial v}{\partial\varphi}-\frac{\partial^2 w}{\partial\varphi^2}\right), \tag{6b}$$

$$M_\varphi=\mu M_x-\chi_\varphi Eh\delta=\mu M_x+\frac{Eh\delta}{R^2}\left(w+\frac{\partial^2 w}{\partial\varphi^2}\right). \tag{6b'}$$

Here and below equations with «’» denote accounting for deformation component.

Eq. (2f) serves for the determination of $M_{x\varphi}$. By substituting the expression for $\chi_{x\varphi}$ (4c) into this equation, we have:

$$M_{x\varphi}=-\frac{H\delta(1-\mu)}{2R}\left(\frac{\partial v}{\partial x}-\frac{2\partial^2 w}{\partial\varphi\partial x}\right)=\frac{H\delta(1-\mu)}{2R}\left(\frac{L}{hG}+\frac{1}{R}\frac{\partial u}{\partial\varphi}-2\frac{\partial\gamma_x}{\partial\varphi}\right), \tag{6c}$$

$$M_{x\varphi}=-\frac{H\delta(1-\mu)}{2R}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{R\partial\varphi}-\frac{2\partial^2 w}{\partial\varphi\partial x}\right)=\frac{H\delta(1-\mu)}{2R}\left(\frac{L}{hG}+\frac{2}{R}\frac{\partial u}{\partial\varphi}-2\frac{\partial\gamma_x}{\partial\varphi}\right). \tag{6c'}$$

The fifth equilibrium equation (1e) serves to determine Q_φ:

$$Q_{\varphi} = \frac{\partial M_{\varphi}}{R\partial\varphi} + \frac{\partial M_{x\varphi}}{\partial x}. \tag{6d}$$

The unknown functions are written in the following way:

$$N_x(x,\varphi,t) = n_x(x)\cos(n\varphi)\sin(\omega t), \tag{7a}$$

$$L(x,\varphi,t) = l(x)\sin(n\varphi)\sin(\omega t), \tag{7b}$$

$$Q_x(x,\varphi,t) = q_x(x)\cos(n\varphi)\sin(\omega t), \tag{7c}$$

$$M_x(x,\varphi,t) = \frac{\delta}{R} m_x(x)\cos(n\varphi)\sin(\omega t), \tag{7d}$$

$$u(x,\varphi,t) = \frac{R}{Eh} u(x)\cos(n\varphi)\sin(\omega t), \tag{7e}$$

$$v(x,\varphi,t) = \frac{R}{Eh} v(x)\sin(n\varphi)\sin(\omega t), \tag{7f}$$

$$w(x,\varphi,t) = \frac{R}{Eh} w(x)\cos(n\varphi)\sin(\omega t), \tag{7g}$$

$$\gamma_x(x,\varphi,t) = \frac{1}{Eh} \gamma_x(x)\cos(n\varphi)\sin(\omega t). \tag{7h}$$

We write a system of eight ordinary deferential equations in terms of main variables:

$$\frac{dn_x(x)}{dx} = \frac{n}{R} l(x) - \Omega^2 u(x), \tag{8a}$$

$$\frac{dl(x)}{dx} = \left(\frac{\delta}{R^2} - \mu\right) n \frac{n_x(x)}{R} - \frac{n\delta}{R^3} m_x(x) + \left(\frac{n^2}{R} - \left[1 - \frac{\delta}{R^2}\right]\Omega^2\right) v(x) + \left(\frac{n}{R} + \frac{\delta n\left(n^2-1\right)}{R^3(1+\mu)}\right) w(x), \tag{8b}$$

$$\frac{dq_x(x)}{dx} = \left(n^2 \frac{\delta}{R^2} - \mu\right) \frac{n_x(x)}{R} - \frac{n^2\delta}{R^3} m_x(x) + \left(\frac{n}{R} + \frac{\Omega^2 \delta n}{R^2}\right) v(x) + \left(\frac{1}{R} - \Omega^2 + \frac{\delta n^2\left(n^2-1\right)}{R^3(1+\mu)}\right) w(x), \tag{8c}$$

$$\frac{dm_x(x)}{dx} = \frac{n}{R} l(x) - q_x(x) - \frac{1}{2}\frac{n^2}{(1+\mu)R} u(x) + \frac{n^2}{(1+\mu)R} \gamma_x(x), \tag{8d}$$

$$\frac{du(x)}{dx} = \frac{1-\mu^2}{R} n_x(x) + \mu \frac{n}{R} v(x) + \mu \frac{1}{R} w(x), \tag{8e}$$

$$\frac{dv(x)}{dx} = 2\frac{1+\mu}{R} l(x) - \frac{n}{R} u(x), \tag{8f}$$

$$\frac{dw(x)}{dx} = \frac{1}{R} \gamma_x(x), \tag{8g}$$

$$\frac{d\gamma_x(x)}{dx} = \frac{1-\mu^2}{R} m_x(x) + \mu \frac{n}{R} v(x) + \mu \frac{n^2}{R} w(x). \tag{8h}$$

Here we used notation: $\Omega^2 = \omega^2 \frac{\rho R}{E}$. The solution of Eqs. (8) can be easily found by series expansion, i.a. assuming that solution is:

$$\begin{aligned} & n_x(x) = n_{x0} + C_{11} \cdot x + C_{12} \cdot x^2 + C_{13} \cdot x^3 + \ldots \\ & \ldots \\ & \gamma_x(x) = \gamma_{x0} + C_{81} \cdot x + C_{82} \cdot x^2 + C_{83} \cdot x^3 + \ldots \end{aligned} \tag{9}$$

$C_{11}, C_{12} \ldots C_{82}, C_{83}$ – constants defined from Eqs. (9). We can use only several first terms in solution (9), for example accounting for coefficients of expansion x^0, x^1, x^2, x^3 gives a reliable accuracy. The coefficient matrix for x^0 is matrix of ones, and for x^1:

$$\begin{bmatrix} 0 & \frac{n}{R} & 0 & 0 & -\Omega^2 & 0 & 0 & 0 \\ -\frac{(R^2\mu-\delta)n}{R^3} & 0 & 0 & -\frac{\delta n}{R^3} & 0 & -\frac{\Omega^2 R^2-\Omega^2\delta-Rn^2}{R^2} & \frac{n(R^2\mu+\delta n^2+R^2-\delta)}{(1+\mu)R^3} & 0 \\ -\frac{R^2\mu-\delta n^2}{R^3} & 0 & 0 & -\frac{n^2\delta}{R^3} & 0 & \frac{(\Omega^2\delta+R)n}{R^2} & -\frac{\Omega^2R^3\mu+\Omega^2R^3-\delta n^4-R^2\mu+\delta n^2-R^2)}{(1+\mu)R^3} & 0 \\ 0 & \frac{n}{R} & -1 & 0 & -\frac{1}{2}\frac{n^2}{(1+\mu)R} & 0 & 0 & \frac{n^2}{(1+\mu)R} \\ -\frac{\mu^2-1}{R} & 0 & 0 & 0 & 0 & \frac{\mu\, n}{R} & \frac{\mu}{R} & 0 \\ 0 & \frac{2(1+\mu)}{R} & 0 & 0 & -\frac{n}{R} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R} \\ 0 & 0 & 0 & -\frac{\mu^2-1}{R} & 0 & \frac{\mu\, n}{R} & \frac{\mu\, n^2}{R} & 0 \end{bmatrix}$$

The proposed solution is easily programmed using the method of initial parameters. In this case, the shell is divided into small sections, which makes it possible not to increase the number of terms in the polynomial expansion (9). To complete the formulation of the problem, it is also necessary to address the boundary conditions.

Boundary conditions. At each edge of the shell, one of 16 types of homogeneous boundary conditions can be specified, they are determined by all possible combinations of the following four equations:

$$w = 0 \text{ or } Q_x + \frac{\partial M_{x\varphi}}{\partial \varphi} = 0\,, \tag{10a}$$

$$\gamma_x = 0 \text{ or } M_x = 0\,, \tag{10b}$$

$$u = 0 \text{ or } N_x = 0\,, \tag{10c}$$

$$v = 0 \text{ or } L - \frac{1}{R} M_{x\varphi} = 0\,. \tag{10d}$$

From Eq.(10) boundary conditions for an elastically restrained shell can be easily obtained:

$$w \cdot k_w = Q_x + \frac{\partial M_{x\varphi}}{\partial \varphi}\,, \tag{11a}$$

$$\gamma_x \cdot k_\gamma = M_x\,, \tag{11b}$$

$$u \cdot k_u = N_x\,, \tag{11c}$$

$$v \cdot k_v = L - \frac{1}{R} M_{x\varphi}\,. \tag{11d}$$

Here k_u, k_v, k_w, k_γ are axial, circumferential, radial, rotational spring stiffnesses.

Note that the most common boundary conditions are the simple support (the Navier boundary condition):

$$w = 0\,,\ M_x = 0\,,\ N_x = 0\,,\ v = 0\,. \tag{12a}$$

Clamped edge:

$$w = 0\,,\ \gamma_x = 0\,,\ u = 0\,,\ v = 0\,. \tag{12b}$$

Free edge:

$$Q_x + \frac{\partial M_{x\varphi}}{\partial \varphi} = 0\,,\ M_x = 0\ \ N_x = 0\,,\ L - \frac{1}{R} M_{x\varphi} = 0\,. \tag{12c}$$

To use Eq.(19a) and (19d), the boundary conditions $Q_x + \frac{\partial M_{x\varphi}}{\partial \varphi} = 0$ and $L - \frac{1}{R} M_{x\varphi} = 0$ should be rewritten in terms of the main variables:

$$Q_x + \frac{\partial}{\partial \varphi}\left(\frac{H\delta(1-\mu)}{2R}\left(\frac{L}{hG} + \frac{1}{R}\frac{\partial u}{\partial \varphi} - 2\frac{\partial \gamma_x}{\partial \varphi}\right)\right) = 0\,, \tag{13a}$$

$$L - \frac{H\delta(1-\mu)}{2R^2}\left(\frac{L}{hG} + \frac{1}{R}\frac{\partial u}{\partial \varphi} - 2\frac{\partial \gamma_x}{\partial \varphi}\right) = 0\,. \tag{13b}$$

Thus, we can use Eq.(13a) and (13b) directly in our numeric scheme.

Results and discussion. To illustrate how the frequency equation agrees with reality, its results are compared with experimental data[10]. The results of comparison are shown in Table 1, natural frequencies according to exact formulas of Xing[1], Smith[11] and Cammalleri [12], are also present. It should be noted that our results are obtain for two options: with deformation component and without it, the differences between them is negligible, thus only one value is present in Table 1. The obtained results allow us to state that our formula is in good agreement with the experimental data, as well as with other exact solutions, such as Xing [1], Smith [11] and Cammalleri [12]. Note, that Cammalleri [12] approach is less accurate, as far as it has some simplifying assumptions. The natural frequencies calculated using ANSYS are also presented, the results of which also converge well.

Table 1

Natural frequencies of a clamped shell:
l=305mm, R=76mm, h=0.254mm, E=207GPa, ρ=7833kg/m^3, μ=0.3

m	N	2	3	4	5	6	7
1	Experiment[10]	—	1025	700	559	525	587
	Present	1923	1157	765	578	535	596
	Xing et al[1]	1917	1154	764	580	538	598
	Smith[11]	1918	1145	765	580	538	597
	Cammalleri[12]	2017	1192	772	564	501	548
	ANSYS	1928	1163	770	584	542	607
2	Experiment[10]	—	—	1620	1210	980	838
	Present	3920	2546	1757	1289	1023	907
	Xing et al[1]	3903	2537	1752	1287	1022	907
	Smith[11]	3905	2538	1753	1287	1022	907
	Cammalleri[12]	4033	2614	1776	1274	980	839
	ANSYS	3932	2560	1772	1304	1037	924
3	Experiment[10]	—	—	—	—	1650	1395
	Present	5874	4071	2931	2198	1724	1434
	Xing et al[1]	5841	4052	2920	2191	1720	1431
	Smith[11]	5844	4054	2921	2192	1720	1431
	Cammalleri[12]	5669	4011	2892	2146	1651	1335
	ANSYS	5893	4097	2959	2227	1752	1462
4	Experiment[10]	—	—	—	—	—	1960
	Present	7350	5475	4122	3180	2527	2083
	Xing et al[1]	7299	5444	4102	3167	2518	2077
	Smith[11]	7303	5447	4104	3168	2516	2076
	Cammalleri[12]	6834	5205	3960	3052	2400	1940
	ANSYS	7370	5512	4166	3227	2574	2130

Fig. 1 and Fig. 2 show a comparison of our exact solution for cylindrical shells with experimental data[13]. Fig. 1 shows the frequency spectra for the supported shell, and in the Fig. 2 for the shell with a free edge. Also on Fig. 1 and Fig. 2 there are results calculated from the exact solution of the Soedel[14]. The agreement between present solution, experimental data and results of other researchers is obvious. A more complicated type of boundary conditions is presented in Table 2, i.e. one edge of the shell has elastic support, and the other edge is clamped. According to Eq.(11) when $k_u=k_v= k_w=k_\gamma=0$, we have clamped-free boundary conditions. Analysis of Table 2 shows than natural frequencies monotonically increase when the stiffness changes from 0 to 10e8.

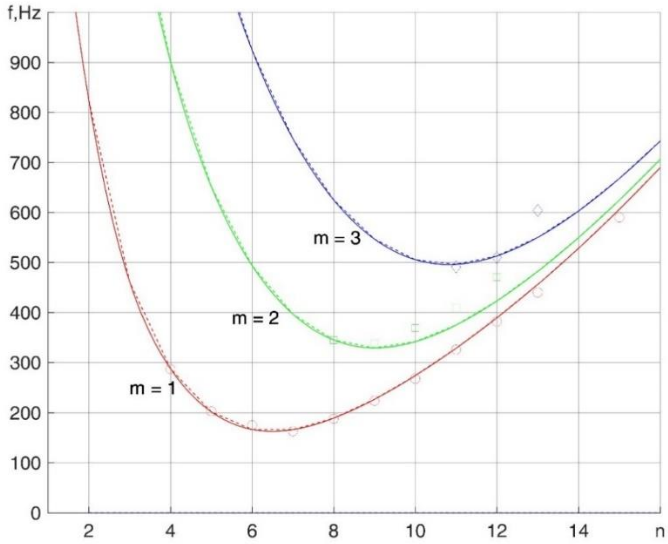

Figure 1. Natural frequencies of simply supported cylindrical shell: *l=610мм*, *R=242,3мм*, *h=0,648мм*, *E=68,9ГПа*, ρ*=2714,5кг/м*^{3}, μ*=0,315*; experimental data[13] (○) m=1, (□) m=2, (◊) m = 3; ——— exact solution ; ⋯⋯⋯ Soedel formula[14]

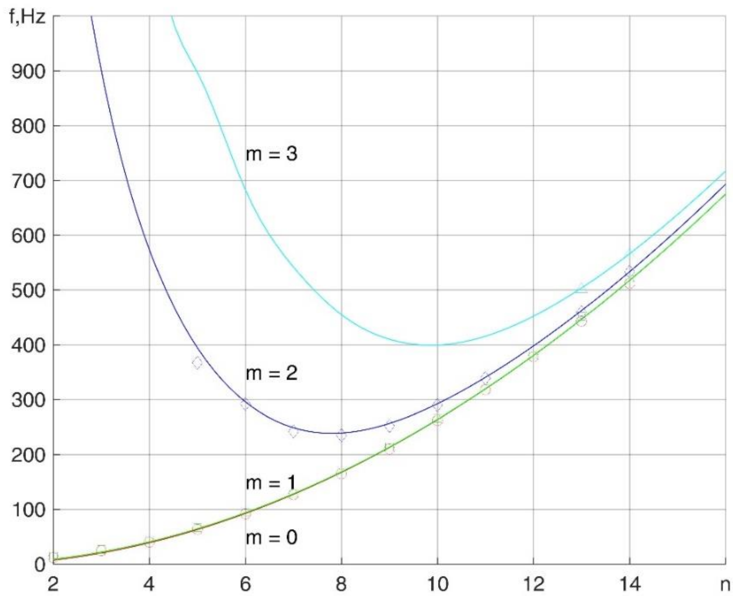

Figure 2. Natural frequencies of free edge cylindrical shell: *l=638мм*, *R=242,3мм*, *h=0,648мм*, *E=68,9ГПа*, ρ*=2714,5кг/м*^{3}; experimental data[13] (○) m=1, (◊) m=2, (Δ) m = 3; ——— exact solution ; ⋯⋯⋯ Soedel formula[14]

Table 2

Natural frequencies for a clamped–elastically supported shell:
l=1.25m, R=0.25m, h=0.008m, E=210GPa, ρ=7800kg/m³, μ=0.3, $k_u=k_v=k_\gamma=0$

Mode	$k_w/H=0$		$k_w/H=0.01$		$k_w/H=0.1$		$k_w/H=1$		$k_w/H=1e6$		$k_w/H=1e8$	
	[15]	Pres.	[15]	Pres.	[15]	Pres.	[15]	Pres.	[15]	Pres.	[15]	Pres.
1	132.0	131.6	183.0	183.4	298.0	299.1	315.1	316.0	315.2	316.6	315.2	316.6
2	249.8	247.8	278.2	278.2	310.1	310.8	339.9	340.7	343.4	345.9	343.4	345.9
3	262.8	263.0	279.8	279.9	365.0	365.5	473.3	476.1	491.4	492.1	491.4	492.1
4	377.0	374.8	404.1	402.9	490.4	490.8	491.6	492.0	501.1	505.4	501.1	505.4

Conclusions. The accurate numerical solution of free vibration frequencies of cylindrical shell, based on the Donell-Mushtari theory, is obtained in explicit form. Eight main variables are selected, they are used to write out all the equations and boundary conditions. This formulation allowed us to solve a system of partial differential equtions using series expansion. Also this formulation is suitable to adress elastically supported edges, which are generalization of classical boudary conditions. The system of equations is solved accounting for deformation component and without, it influence is negligible. A comparison is made with the experimental results and with the data of other researchers, the correctness of present approach is obvious.

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РОЗВ'ЯЗОК В РЯДАХ ЗАДАЧІ ПРО ВІЛЬНІ КОЛИВАННЯ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ З ДОВІЛЬНИМИ ГРАНИЧНИМИ УМОВАМИ

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Резюме. *У вигляді методу початкових параметрів записано точні формули для пошуку власних частот коливань циліндричної оболонки з довільними граничними умовами на основі теорії Донела-Муштарі. Також дано вирази з урахуванням деформаційної складової, показано що її вплив дуже незначний і ним можливо знехтувати. Наведено порівняння з літературними даними й показано, що отримані залежності добре описують експериментальні результати.*

Ключові слова*: циліндрична оболонка, розв'язок у рядах, власні частоти коливань.*

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