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DYNAMICS OF REGULAR MICRORELIEF FORMATION ON INTERNAL CYLINDRIC SURFACES

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Summary. *An analysis of modern literature sources to search for mathematical models describing the dynamics of the process of forming a regular microrelief on the inner cylindrical surface of parts, gas transmission equipment operating in severe operating conditions, in order to increase their life. It is established that there are no mathematical models describing this process and the peculiarities of its implementation under the point action of the deforming element on the workpiece surface. The molding movements accompanying the process of forming a regular microrelief on the inner cylindrical surface of the workpiece are considered and the driving forces that accompany this process are analyzed. A mathematical model of dynamic process of regular microrelief formation on internal cylindric surface of the part has been developed. The process of formation is a unique one as it occurs due to the concentrated force whose point of application varies in radial and axial directions relative to the part. Thus, the action has been described by the mathematical model with discrete right-hand side. This action is proposed to be simulated by Dirac delta functions of linear and time variables using the method of regularization of the specific features under discussion. These peculiar features have been described by the conventional methods of integrating of correspondent nonlinear mathematical models of longitudinal and lateral vibrations of the part. The analytical dependencies describing these vibrations have been obtained based on the initial data. Using Maple software, 3D changes in the torsion angle depending on different output values are constructed. The conducted researches will allow to consider torsional fluctuations that is especially actual for long cylindrical details, such as sleeves of hydraulic cylinders, details of drilling mechanisms and others.*

Key words: *technology, cylindrical surface, quality parameters, vibration processing, torsional vibrations, mathematical models.*

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Statement of the problem. The improvement of performance characteristics of machine parts functional surfaces has been an important task of mechanical engineering production. One of the most efficient methods of performance characteristics improvement is the method of plastic deformation, namely ball or roller burnishing.

Such treatment allows us to reduce the machining surface roughness considerably, to increase the surface microhardness, and in some cases to avoid the operation of surface quenching [1, 2, 3]

Analysis of the available investigations. A separate direction in this field is the methods of dynamic action on the processed surface. They involve a highly intensive action on the machining surface on the indenter's side which is mostly of aball shape. The dynamic action provides smaller deformation effort, allows to provide bigger support area of the surface, and, respectively better performance charactersitics described by the Abbott – Firestone curve [4]. The advantages of the method of dynamic action have been given in the paper including the construction of above-mentioned surface characteristics.

The assessment of the surface performance characteristics has been proposed to make by the parameters of this curve by other scientists as well. In particular, the method described in the paper [5], where the assessment of the surface performance characteristics is made not by the surface roughness but due to the parameters of the Abbot's- Firestone's curve. The

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authors claim that three parameters determined from the Abbott-Firestone curve, namely R_{pk}, R_k and R_{vk} characterize the ability of the surface to resist friction wear.

Some specific features of the surface state assessment by means of Abbott -Firestone curve parameters have been desribed in the paper as well [6]. The authors have found that the surface integrity to be the relationship between the required functional properties of the surface and the change in the properties of the new surface. The surface can be evaluated using two basic properties, namely the spatial arrangement of the surface (surface roughness) and the physical-chemical properties of the surface layer.

The dynamic action on the machining surface can be chaotic or ordered including the formation of the ordered microrelief on it. Shneider Y. G. is considered to be the founder of regular microrelief (RMR) using and formation methods. Some methods and ways of RMR formation, the tool design and its operation modes have been described in his paper [7]. He also classified the RMR formed on flat and cylindric surfaces and with different geometrical parameters.

The results of his study has been the basis for the GOST 24773-81 development [8], which specifies the parameters of regular microrelief formed on the flat and cylindric surfaces.

The development of principles of regular microrelief formation has been given in paper [9]. The advantages of the surfaces with formed microreliefs against the surfaces treated by other methods have been described.

Highly efficient instrumental complexes, their design and operation principle have been described in [10, 11]. Such complexes provide the microrelief formation on profiled surfaces of any complexity degree.

Some mathematical models describing regular microrelief on end surfaces of rotational bodies have neen given in paper [12] for the first time. Such microreliefs have also been classified and their features have been described mathematically.

The scheme, technological equipment and the instrument for regular microrelief formation on flat and spherical surfaces using 40 MPa pressure on 5-coordinates numericaly controlled milling machine have neen given in paper [13]. The surface microstructure with formed microrelief has been studied. Two approaches to regular microrelief formation on complex profiled surfaces have been compared in the article.

As the geometrical parameters of regular microrelief are quite small (only 1–3 mm), then the effect of any factors can skew it. But it is its regularity that provides fixed physical-mechanical properties of machine parts functionla surfaces.

Some theoretical investigation dealing with the dynamic characteristics of the vibration-centered strengthening (VCS) method of metal long cylindrical parts has been carried out in the paper [14]. The research methods have been described, a spacious principal scheme of a vibration-centered strengthened tool with electric magnetic drive and elastic systems has been given. Some empirical dependencies to determine main dynamic characteristics of the vibration-centered strengthening (VCS) method have been obtained and analyzed on the basis of the conducted research. An algorithm of analysis of dynamic characteristics of electric magnetic drive contact interaction with the elastic systems and the machining surface has been developed.

The Objective of the work is the development a mathematical model of the dynamic process of regular microrelief formation on internal cylindric surface of the part.

Main material statement. To provide the microrelief regularity it would be necessary to develop a dynamic model of its formation taking into account the following parameters:

physical-mechanical properties of the processed surface, external action value on the surface of the tool side (vibrating rolller burnishing machine), contact character and many others.

In the paper under discussion we are considering the RMR formation on the internal cylindrical surface of the workpiece. This process is being accompanied with the following movements: rotational motion D_n, feed motion D_s and reciprocal oscillation motion D_i (fig. 1).

The process of regular microrelief formation on the internal cylindric surface has some peculiar features, namely:

- cylindric machining surface is rotating around stationary axle;
- external action on the processed surface from the side of the body applying the microrelief has a point character, moreover, the contact point of the given bodies varies in longitudinal and radial directions.

The constituents of the above-mentioned motions and angular velocity of the cylindric surface rotation are making impact on the dynamic processes in an elastic cylindrical body accompanying the process of microrelief formation and determine the microrelief shape. As for the microrelief constituent, caused by the dynamic processes of a cylindrical body, first of all it is influenced by its longitudinal and torque vibrations and in this case it is determined by the elastic properties of the body, boundary conditions and external action.

All the above-mentioned findings have proved that an elastic cylindrical body on which the microrelief is applied is in complex motion under external force factors action [15]. So, to describe its dynamics in operation the following constituents should be taken into consideration:

- translational – rotational around horizontal axis as an abslutely solid body;
- relative – longitudinal vibrations of an elastic body.

The torque vibrations referred to the body relative motion require an absolute and irrespective consideration.

All the above-mentioned issues are the subject of investigation.

Study of the impact of external factors on translational rotational motion of the machining body.

The main assumptions underlying the description of the body translational motion are as follows (fig. 1):

- hollow body of cylinder shape on whose internal surface the microrelief is applied – homogeneous, its (body) external radius R_d, internal – r_d, length – l_d, mass – M_d;
- active and passive forces acting on the machining body:

a) point action of the body applying the microrelief on the internal cylindric surface. The constituents of this action are F_x, F_z. (horizontal axis OX coinsides with rotational axis, axis OZ is vertical, axis OY is horizontal);

б) actuating moment M_{gh}, which makes the body rotate around the horizontal axis, shearing moment (M_{ph}) of the vibration roller burnishing machine action on the internal surface and depends on the following factors – pressing force, shape of vibration roller burnishing machine, hardness of the processed surface material and others. We have assumed that the last factors are taken into account by the coefficient f_{ph} ($M_{ph} = F_z \cdot r_d \cdot f_{ph}$).

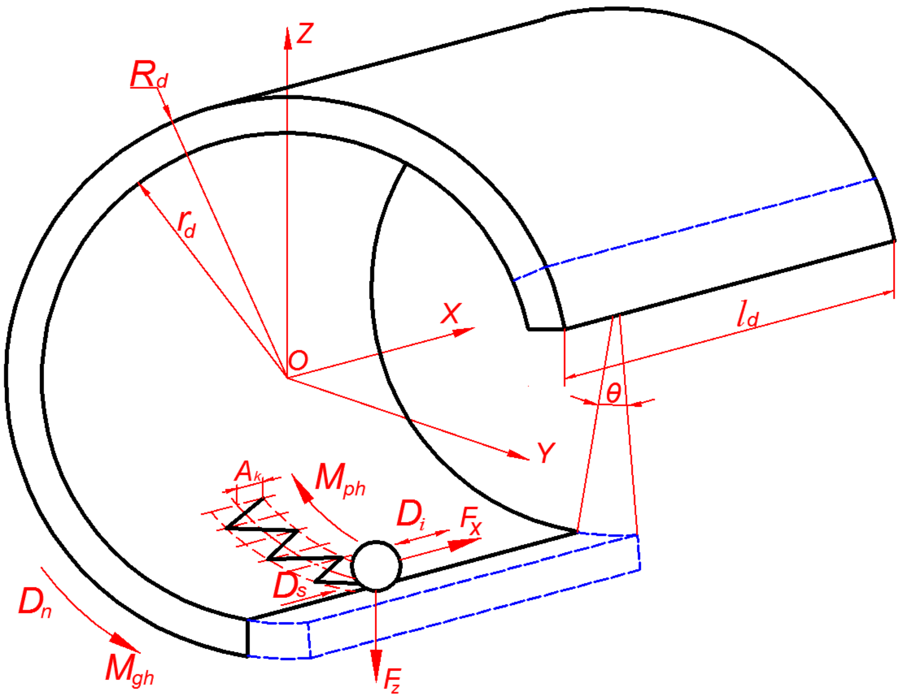

Figure 1. Scheme of regular microrelief formation on internal cylindric surface of the part

If the angle of turning of the machining body is marked as $\theta(t)$, then from the differential equation of the solid body rotational motion around the fixed axis [15, 16], we have

$$I_{ox}\frac{d^2\theta}{dt^2} = M_{gh} - M_{ph} \tag{1}$$

I_{ox} – moment of inertia of the machining body relative to the axis of rotation OX, i.e. $I_{ox} = M_d \frac{R_d^2 + r_d^2}{2}$, M_d – mass of the body, R_d, r_d – external and internal radius of cylindrical body.

As for the actuating moment, it will be equal to

$$M_{gh} = f(t) + F_z \cdot r_d \cdot f_{ph}, \tag{2}$$

where $f(t)$ – constituent of actuating moment which makes the workpiece do some relative oscillations (alternating function).

All the above-mentioned solutions allow to represent the differential equation (1) as

$$I_{ox}\frac{d^2\theta}{dt^2} = f(t). \tag{3}$$

Due to the dependence integration (3) we have obtained

$$\theta = A + Bt + \frac{1}{I_{ox}} \int_0^t \int_0^\tau f(\tau) d\tau dt, \tag{4}$$

where A, B – invariables of integration which can be found from the initial conditions.

In particular, if the function defining the cross-sectional constituent of the moment equals to $f(t) = A_k \cdot \sin(pt)$ (A_k – amplitude of actuating moment, p – frequency), then angular velocity of the body rotation ω equals to

$$\omega = \xi_0 - \frac{A_k}{pI_{ox}}(\cos pt - 1) \tag{5}$$

and respectively the angle of its rotation

$$\theta(t) = \omega_0 t - \frac{A_k}{p^2 I_{ox}}(\sin pt - pt), \tag{6}$$

ω_0 – initial angular velocity of the machining body.

Thus, the translational motion law of the machining body is described for the sinusoidal law of actuating moment by the dependence (6).

Dynamics of relative longitudinal motion of the machining cylindric body during the process of microrelief applying

Some relative vibrations of the machining elastic body in the process of microrelief applying on its surface have been studied in this section. Their longitudinal and torque constituents are considering. Their peculiarity is the fact that they are taking place under discrete external loading conditions. Moreover, this loading is changing its point of application with time. All these are making some difficulties in boundary value problems solutions construction descibing the above-mentioned vibrations.

It is known [17, 18], that the mathematical model of longitudinal vibrations of a homogeneous elastic body providing its elastic chracteristics satisfy nonlinear technical law of elasticity [19], and the viscous friction force is proportional to the speed, under certain boundary value conditions, is the differential equation

$$u_{tt}(x,t) - \beta^2 u_{xx}(x,t) = \varepsilon\left[\eta u_t(x,t) + \hat{\beta}\left(u_x(x,t)\right)^2 u_{xx}(x,t)\right] + \xi(x,t) \tag{7}$$

where $u(x,t)$ is the longitudinal displacement of the body cross section with coordinate x in an arbitrary moment of time t ; $\beta^2 = E\overline{A}/m_d$, $\overline{A} = \pi\left(R_d^2 - r_d^2\right)$ – cross sectional area of the part; $m_d = M_d / l_d$ – mass of the body length unit (mass per unit length); $m_d \xi(x,t)$ – external load distribution along the body); η – coefficient of proportionality in the viscous friction force; $\hat{\beta}$ coefficient characterizing the body elastic properties deviations from the linear law; ε – small parameter indicating a small value of the last two forces comparing to the addend of the left-hand side of the equation (7). We have a bit complex case for the process of microrelief applying on the maching body:

– firstly, external action is of point type;

– secondly, point of application of the given action is continuously moving along the generatrix of the machining body internal surface and the above-mentioned feature should be taken into account by the function $\xi(x,t)$ (fig. 2).

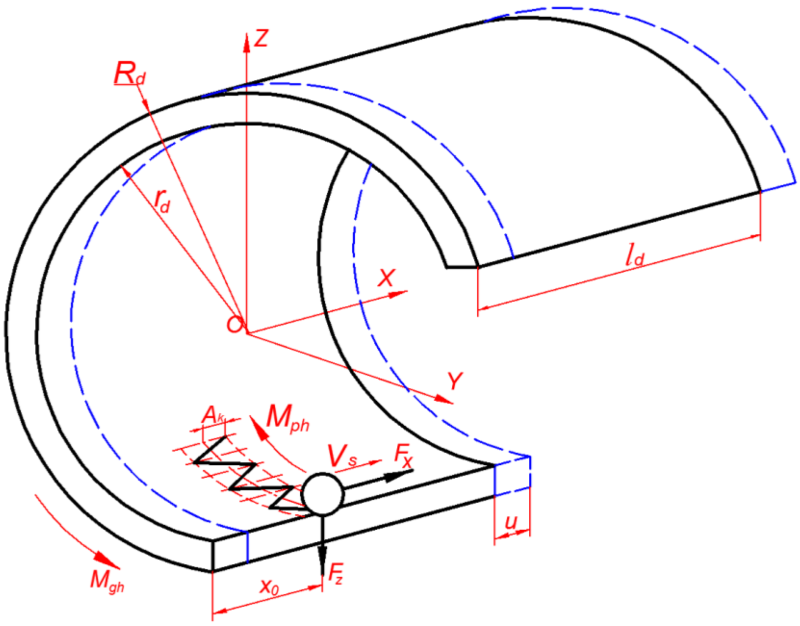

Figure 2. Design model for dynamic model construction of relative longitudinal motion of the machining body during the process of regular microrelief formation

All the above- mentioned allows us to claim that the external action on the machining body depends both on the time and linear variable as well. In the references [12–25] two approaches have been used to describe point action of the external load on elastic bodies. First of them represents the given action as a function value in the given point (for example, $\xi(x,t)_{|x=x_0}$, x_0 – force application point), the second one uses a delta function (for example $\xi(x,t)\delta(x-x_0)$, $\delta(x-x_0)$ is a delta function).

For the process of microrelief application on the machining body internal surface, as it was said above, we are considering a bit more complex case when the point of application of external action is varying in time and its position, i.e. relative position, is continuously changing. The above-mentioned specific features of the microrelief application dynamics on the machining body are proposed to be represented by means of delta function in the form $F_X \bar{f}_{ph}\delta(x-\bar{x}_0-V_s t)$, and V_s – is longitudinal constituent of the vibration roller burnishing machine motion speed along the generatrix of the body cylindric surface (feed rate), $\bar{x}_0$ – initial value of microrelief applying abscissa. Taking into account all the above-mentioned, the differential equation of relative longitudinal vibrations is becoming

$$u_{tt}-\beta^2 u_{xx}=\varepsilon\left[\eta u_t+\hat{\beta}\left(u_x\right)^2 u_{xx}\right]+F_X \bar{f}_{ph}\delta\left(x-\bar{x}_0-V_s t\right), \quad (8)$$

where $x_0+V_s\cdot t$ – coordinate of the vibration roller burnishing machine contact with the RMR formation surface (moving point), $\bar{f}_{ph}$ – coefficient analog f_{ph} for longitudinal motion.

As for the boundary values conditions for the equation (8), they are taking the form

$$u(x,t)_{|x=0} = 0,\ u(x,t)_{|x=l} = 0, \tag{9}$$

which are equivalent to the condition when there are no horizontal movements of the machining body ends.

The values F_x та F_z of action on the maching body are assumed to be constant.

Thus, the study of dynamics of of relative longitudinal motion of a cylindric body on whose internal surface the microrelief is being applied has been restricted by the construction and respectively by the study of the boundary value solution (8), (9) with the discrete right-hand side. The last one is not the restriction for the use of general ideas of disturbance method at construction of the above-mentioned boundary value problem. Thus, we will seek the general solution of nonlinear equation (8) under homogeneous boundary-value conditions (9) in the form [17, 23]

$$u(x,t) = u_0(x,t) + \tilde{u}_0(x,t) + \varepsilon U(x,t), \tag{10}$$

where $u_0(x,t)$ – the solution of homogeneous boundary-value problem, i.e.

$$u_{0tt}(x,t) - \beta^2 u_{0xx}(x,t) = 0\,, \tag{11}$$

$$u_0(x,t)_{|x=0} = 0,\ u_0(x,t)_{|x=l} = 0, \tag{12}$$

and $\tilde{u}_0(x,t)$ – a member solution of non homogeneous equation $\tilde{u}_{0tt} - \beta^2 \tilde{u}_{0xx} = F_X \bar{f}_{ph} \delta(x - \bar{x}_0 - V_s t)$; $U(x,t)$ – the function taking account the impact of viscous friction force and nonlinear constituent of restoring force on the process, and apparently it has to satisfy the homogeneous boundary-value conditions arising from (9)–(12). We will consider the process of its finding a bit lower.

The solution of a homogeneous boundary value problem can be presented [17] in the following form

$$u_0(x,t) = \sum_k X_k(x) S_k(t)\,, \tag{13}$$

where the system of proper functions $X_k(x)$ looks like $\{X_k(x)\} = \left\{\sin\frac{k\pi}{l}x\right\}$, and $S_k(t) = S_{0k}\sin(\omega_k t + \phi_{0k})$, S_{0k}, ϕ_{0k} - of steel, which can be found from the initial conditions of non-homogeneous equation, $\omega_k = \frac{k\pi}{l_d}\beta = \frac{k\pi}{l_d}\sqrt{\frac{E\bar{A}}{m_d}}$.

The system of proper functions $\{X_k(x)\}$ has a specific feature of completeness and orthonormalization, that is why the solution of the equation member (8), i.e. the function $\tilde{u}(x,t)$, should be found in the following form

$$\tilde{u}(x,t) = \sum X_k(x)\tilde{S}_k(t) \tag{14}$$

in this case the boundary-value conditions (9) will be for the general solution of boundary value problem (8), (9). So, functions $\tilde{S}_k(t)$ given in (14) must be the solution of of non-homogeneous equation

$$\ddot{\tilde{S}}_k(t)+\beta^2\left(\frac{k}{l_d}\right)^2\tilde{S}_k(t)=\frac{1}{l_d}\int_0^l X_k(x)F_X\,\overline{f}_{ph}\delta(x-\overline{x}_0-V_st)dx \qquad (15)$$

The properties of delta function [17, 25] allows the integral in the right-hand side side of the equation (15) be represented as $\int_0^l X_k(x)F_X\,\overline{f}_{ph}\delta(x-x_0-V_st)dx=F_X\,\overline{f}_{ph}\sin\frac{k\pi}{l_d}(\overline{x}_0+V_st)$, and in this case to find the partial solution of the equation (15) we obtain the dependence

$$\ddot{\tilde{S}}_k(t)+\beta^2\left(\frac{k}{l_d}\right)^2\tilde{S}_k(t)=\frac{1}{l_d}\overline{f}_{ph}F_X\sin\frac{k(\overline{x}_0+V_st)}{l_d} \qquad (16)$$

Thus, the representation of the external action on the maching body by means of delta function using the method of partial discretization has allowed us to solve the problem of the function $\tilde{u}(x,t)$ finding, as the partial solution of the equation (16) looks like

$$\tilde{S}_k(t)=\frac{1}{k\beta}\overline{f}_{ph}F_X\int_0^t\sin\frac{k(\overline{x}_0+V_s\tau)}{l_d}\sin\left(\frac{k}{l_d}\beta(t-\tau)\right)d\tau\,. \qquad (17)$$

As for the function $U(x,t)$, in this case from the fundamental equation and the results obtained above due to the of the left-hand side and right-hand side coefficients equalization at small parameter ε we obtain the formula

$$U_{tt}(x,t)-\beta^2U_{xx}(x,t)=\eta\left[u_{0t}(x,t)+\tilde{u}_{0t}(x,t)\right]+\left[\hat{\beta}\left(u_{0x}(x,t)+\tilde{u}_{0x}(x,t)\right)^2\left(u_{0xx}(x,t)+\tilde{u}_{0xx}(x,t)\right)\right] \qquad (18)$$

Its partial single-frequency solution satisfying homogeneous boundary-value conditions is similar to the one described above and we will give lower only pre-finish results: $U(x,t)=\sum_k\sin\frac{k\pi}{l}x\mathrm{H}_k(t)$, where the function $\mathrm{H}_k(t)$ is the solution of standard linear non-homogeneous differential equation

$$\begin{aligned}&\frac{d^2\mathrm{H}_k(t)}{dt^2}+\left(\beta\frac{k\pi}{l_d}\right)^2\mathrm{H}_k(t)=\eta\left\{S_{0k}\omega_k\cos(\omega_kt+\varphi_{0k})+\frac{d}{dt}\left[\frac{1}{l_d}\overline{f}_{ph}F_X\int_0^t\sin\frac{k(\overline{x}_0+V_s\tau)}{l_d}\sin\left(\frac{k}{l_d}\beta(t-\tau)\right)d\tau\right]\right\}+\\&+\left(\frac{k\pi}{l_d}\right)^4\hat{\beta}\int_0^l\cos^2\frac{k\pi}{l_d}x\sin^2\frac{k\pi}{l_d}xdx\left\{S_{0k}\sin(\omega_kt+\varphi_{0k})+\frac{1}{l_d}\overline{f}_{ph}F_X\int_0^t\sin\frac{k(\overline{x}_0+V_s\tau)}{l_d}\sin\left(\frac{k}{l_d}\beta(t-\tau)\right)d\tau\right\},\end{aligned} \qquad (19)$$

i.e.

$$\mathrm{H}_k(t)=\frac{l_d}{k\pi\beta}\int_0^t \cos\left(\beta\frac{k\pi}{l_d}(t-\tau)\right)\Theta(\tau)d\tau,$$

$$\Theta(t)=\eta\left\{S_{0k}\omega_k\cos(\omega_k t+\varphi_{0k})+\frac{d}{dt}\left[\frac{1}{l_d}\bar{f}_{ph}F_X\int_0^t \sin\frac{k(x_0+V_s\tau)}{l_d}\sin\left(\frac{k}{l_d}\beta(t-\tau)\right)d\tau\right]\right\}+$$

$$+\left(\frac{k\pi}{l_d}\right)^4\hat{\beta}\frac{\pi}{8}\left\{S_{0k}\sin(\omega_k t+\varphi_{0k})+\frac{1}{l_d}\bar{f}_{ph}F_X\int_0^t \sin\frac{k(x_0+V_s\tau)}{l_d}\sin\left(\frac{k}{l_d}\beta(t-\tau)\right)d\tau\right\}$$

In total, the obtained results have allowed us to represent the relative longitudinal vibrations of a cylindric body on whose internal surface the microrelief is being applied in the following form

$$u(x,t)=\sum_k \sin\frac{k\pi}{l_d}x\left\{S_{0k}\sin(\omega_k t+\varphi_{0k})+\frac{1}{l_d}\bar{f}_{ph}F_X\left(\frac{k}{l_d}\beta\right)^{-1}\int_0^t \sin\frac{k(\bar{x}_0+V_s\tau)}{l_d}\sin\left(\frac{k}{l_d}\beta(t-\tau)\right)d\tau+\right.$$
$$\left.+\frac{l}{k\pi\beta}\int_0^t\cos\left(\beta\frac{k\pi}{l_d}(t-\tau)\right)\Theta(\tau)d\tau\right\} \qquad (20)$$

We must admit, that the principle of single-frequency oscillations in the systems with many degrees of freedom and distributed parameters allows to use only one of the forms of «dynamic balance» of an elastic machining body for practical implementation. For the first one the dynamic process of the body under consideration looks like

$$u(x,t)=\sin\frac{\pi}{l_d}x\times\left\{S_{01}\sin\left(\frac{\pi}{l_d}\sqrt{\frac{E\bar{A}}{m}}t+\varphi_{01}\right)+\frac{1}{l_d\beta}\bar{f}_{ph}F_X\int_0^t \sin\frac{(\bar{x}_0+V_s\tau)}{l_d}\sin\left(\frac{1}{l_d}\beta(t-\tau)\right)d\tau+\right.$$
$$\left.+\varepsilon\frac{l_d}{k\pi\beta}\int_0^t\cos\left(\beta\frac{k\pi}{l_d}(t-\tau)\right)\Theta(\tau)d\tau\right\}. \qquad (21)$$

Simultaneously obtained result allowed us to record the relative longitudinal coordinate $\Xi(x,t)$ of the microrelief being applied on the internal surface of a cylindric body in the form close to the first form of «dynamic balance» and it looks like

$$\Xi(x,t)=x_0+V_s t+\sin\frac{\pi}{l_d}x\times\left\{S_{01}\sin\left(\frac{\pi}{l_d}\sqrt{\frac{E\bar{A}}{m}}t+\varphi_{01}\right)+\frac{1}{l_d\beta}\bar{f}_{ph}F_X\int_0^t \sin\frac{(x_0+V_s\tau)}{l_d}\sin\left(\frac{1}{l_d}\beta(t-\tau)\right)d\tau+\right.$$
$$\left.+\varepsilon\frac{l}{k\pi\beta}\int_0^t\cos\left(\beta\frac{k\pi}{l_d}(t-\tau)\right)\Theta(\tau)d\tau\right\}. \qquad (22)$$

Lower, according to the above-mentioned dependence, the time variations and in accordance with longitudinal coordinate of the curve applied on the internal cylindric surface taking into account its longitudinal vibrations with the following values of parameters $\varepsilon\hat{\beta}=10^3$, $\varepsilon\eta=10^2$, $\bar{x}_0=0$ (fig. 3).

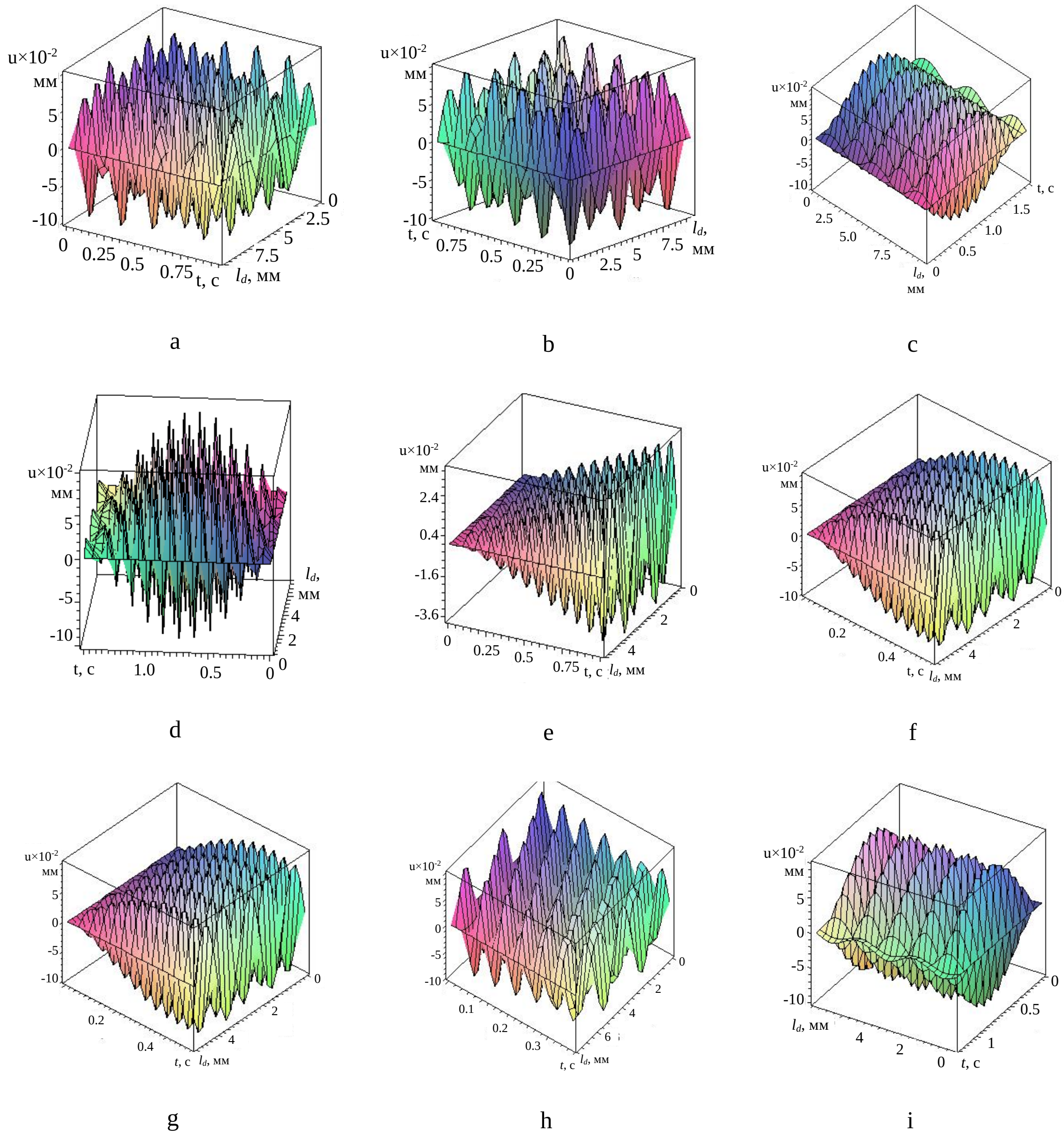

Figure 3. Change of the machining surface microrelief caused by longitudinal vibrations and vibration roller burnishing machine motion at:

a) $E = 2 \cdot 10^{11} H / м^2$, $l_d = 1$ м, $\bar{A} = 0,014$ м^2, $F_X = 300$ Н; $V_s = 0,01$ м/с; $\bar{f}_{ph} = 1$

b) $E = 2 \cdot 10^{11} H / м^2$, $l_d = 1$ м, $\bar{A} = 0,014$ м^2, $F_X = 300$ Н; $V_s = 0,01$ м/с; $\bar{f}_{ph} = 0,5$

c) $E = 2 \cdot 10^{11} H / м^2$, $l_d = 1$ м, $\bar{A} = 0,014$ м^2, $F_X = 300$ Н; $V_s = 0,05$ м/с; $\bar{f}_{ph} = 0,5$

d) $E = 2 \cdot 10^{11} H / м^2$, $l_d = 0.75$ м, $\bar{A} = 0,014$ м^2, $F_X = 300$ Н; $V_s = 0,05$ м/с; $\bar{f}_{ph} = 0,5$

e) $E = 2,4 \cdot 10^{11} H / м^2$, $l_d = 0.75$ м, $\bar{A} = 0,014$ м^2, $F_X = 300$ Н; $V_s = 0,05$ м/с; $\bar{f}_{ph} = 0,5$

f) $E = 2{,}4 \cdot 10^{11} H / м^2$, $l_d = 0.75$ м, $\bar{A} = 0{,}014$ м^2, $F_X = 400$ Н; $V_s = 0{,}05$ м/с; $\bar{f}_{ph} = 0{,}75$

g) $E = 2{,}4 \cdot 10^{11} H / м^2$, $l_d = 0.5$ м, $\bar{A} = 0{,}014$ м^2, $F_X = 400$ Н; $V_s = 0{,}01$ м/с; $\bar{f}_{ph} = 0{,}75$

h) $E = 2{,}4 \cdot 10^{11} H / м^2$, $l_d = 0.5$ м, $\bar{A} = 0{,}014$ м^2, $F_X = 400$ Н; $V_s = 0{,}02$ м/с; $\bar{f}_{ph} = 0{,}75$

i) $E = 2{,}4 \cdot 10^{11} H / м^2$, $l_d = 0.5$ м, $\bar{A} = 0{,}014$ м^2, $F_X = 400$ Н; $V_s = 0{,}005$ м/с; $\bar{f}_{ph} = 0{,}75$

The represented dependence has shown that the shape of microrelief generatrix caused by longitudinal vibrations depends not only on the speed of vibration roller burnishing machine motion, but on the physical-mechanical properties of the body, external action value, and the length of the surface under treatment.

Conclusions. The methodology of analytical description of technological process of microrelief applying on the cylindrical parts inner surface has been developed in the paper and the obtained calculating dependencies have made it possible to claim:

1. Micro irregularities configuration depends not only on the angular velocity of the body rotation and the actuating moment constituent making the processing body do not only some relative oscillations but elastic oscillations of the body as well;
2. The specific feature of the latter (both longitudinal and torque) is explained by the fact that they are affected by the body elastic properties and the external action of the force whose point of aplication changes its relative position on the internal cylindric surface, and respectively – relative motions of normal cross section of the processing body in the external action point depend on its location;
3. Amplitude-frequency characteristics of relative vibrations (longitudinal or torque) depends both on the external action value and on the physical-mechanical properties of the processing body; and for the body with more rigid characteristics the frequency of elastic vibrations is higher, but the amplitude is a bit smaller;
4. The obtained theoretical results of dynamics of microrelief formation process on the cylindrical surface can be the basis of more complex problems solving – study of external and internal resonance phenomena in elastic processing bodies.
5. The validity of the obtained calculating dependencies has been proved by obtaining in the boundary case the known values referred to the process of microrelief applying on the cylindrical surface without taking into consideration the elastic vibrations.

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ДИНАМІКА ПРОЦЕСУ ФОРМУВАННЯ РЕГУЛЯРНОГО МІКРОРЕЛЬЄФУ НА ВНУТРІШНІХ ЦИЛІНДРИЧНИХ ПОВЕРХНЯХ

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Резюме. *Проаналізовано сучасні літературні джерела на предмет пошуку математичних моделей, що описують динаміку процесу формування регулярного мікрорельєфу на внутрішній циліндричній поверхні деталей, газотранспортного обладнання, які працюють у важких умовах експлуатації, з метою збільшення їх ресурсу. Встановлено, відсутність математичних моделей, що описують даний процес та особливості його здійснення при точковій дії деформуючого елемента на поверхню заготовки. Розглянуто формувальні рухи, які супроводжують процес формування регулярного мікрорельєфу на внутрішній циліндричній поверхні заготовки та проаналізовано рушійні сили, що супроводжують цей процес. На основі проведеного аналізу розроблено математичну модель динамічного процесу формування регулярного мікрорельєфу на внутрішній циліндричній поверхні деталі. Особливістю цього процесу є те, що процес формування мікрорельєфу відбувається зосередженою силою, точка прикладання якої відносно деталі постійно змінюється в радіальному та осьовому напрямках, а відтак математична модель, яка описує цей процес, буде з дискретною правою частиною. Запропоновано таку дію моделювати за допомогою дельта функцій Дірака з лінійною та часовою змінними, використовуючи метод регуляризації вказаних особливостей, зокрема існуючі методи інтегрування відповідних нелінійних математичних моделей крутильних коливань деталі. Отримано аналітичні співвідношення, які описують ці коливання в процесі формування регулярного мікрорельєфу. Використавши програмне забезпечення Maple, побудовано 3D зміни кута закручування залежно від різних значень вихідних даних. Проведені дослідження дозволять враховувати крутильні коливання, що особливо актуально для довгомірних циліндричних деталей, таких, як гільзи гідроциліндрів, деталі бурових механізмів та ін.*

Ключові слова: *технологія, циліндрична поверхня, параметри якості, вібраційна обробка, крутильні коливання, математичні моделі.*

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