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CALCULATION OF THE STRESS – STRAIN STATE OF BLADES MADE OF POLYMER COMPOSITE MATERIALS OF STARTING TURBOEXPANDERS IN RESONANCE ZONES

Yurii Nakonetchnyi; Ihor Yarema; Vitalii Batiuk

Ternopil Ivan Pului National Technical University, Ternopil, Ukraine

***Summary**. An overview of the successful application of modern composite materials for the manufacturing of turbine blades of aircraft gas turbine engines, axial mine and blast furnace compressors. Their main advantages of these materials in comparison with metal are analyzed. Analytical calculations of stresses arising in the material of plastic blades of starting turboexpanders are carried out. The possibility of successful application of glass-filled polyamide for the manufacturing of moving and guide blades of starting turboexpanders and their successful operation at compressor stations of main gas pipelines is substantiated.*

***Key words**: composite materials, turbine blades, stresses, deformations, torsional oscillations, bending oscillations, resonance.*

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Statement of the problem. In order to start gas-turbine installations operating at compressor stations of the main gas pipelines, starting turboexpanders are used. In these turboexpanders the moving and guide blades can be made of polymer construction materials (PCM), for example, glass-filled polyamides. Turboexpanders operate on natural gas differences at temperatures from +20 to -40^{0}C. The use of plastics as the material for blades of such cold turbomachines is explained by the fact that despite the relatively low heat resistance and mechanical strength, they have other advantages: low specific gravity, which reduces the tensile stress from centrifugal forces; good damping properties; high manufacturability [1]. Since physical and mechanical characteristics of plastics significantly differ from the properties of metals [2–4], there is the need to calculate the strength of plastic blades [5].

Analysis of the available investigation results. Experience in the development of PCM blades has already been accumulated in foreign and domestic practice. For example, fiberglass blades of axial industrial compressors have been developed and implemented, They gave a good account of themselves in coal mines in Donetsk and other regions. There are examples such blades use in the first stages of blast furnace compressors. Investigations of the effective use of carbon-carbon PCM for the manufacturing of carbon fiber blades of modern gas-turbine engines in aircraft engineering are carried out [6, 7].

The experimental batch of moving and guide blades of turboexpanders made of polyamide glass-filled brand PA-66KS is developed and manufactured. The results of successful tests under operational conditions showed that the main advantage of such plastic blades is their manufacturability and low cost due to the high-tech method of injection molding on automatic molding machines, and less dangerous nature of their destruction. When foreign objects (metal lattice, electrode particles, etc.) get into the blade apparatus, the plastic blades break into very small particles at low temperatures and do not damage the turboexpander body, as is the case with metal blades.

The objective of the paper is to substantiate the possibility of PCM application for the manufacturing of plastic blades of cold turbomachines and to calculate their stress-strain state in resonant zones.

Calculation of stresses in the material of the blade airfoil. The results of analytical calculation of plastic blade airfoil made of glass-filled polyamide PA-66KS by injection molding and equipped with shroud platform is presented in this paper. It is assumed that the platforms do not come into contact with each other (i.e. the blades are fixed cantilevered). The calculation method is shown on the example of plastic blade of the first operating row. It is known [8] that the blade material operates on bending and stretching from centrifugal forces. Normal strengths from centrifugal forces are the easiest ones for determination. Since the ratio of the blade l height to the average radius R_s of the operating row $\frac{l}{R_s} = \frac{19}{130} = 0,127 \ll 1$, the airfoil can be considered as the concentrated mass rotating at distance R_s from the rotation axis. In this case, the normal force N_v at the base of the blade airfoil is determined by the following formula

$$N_v = m_n \cdot \omega^2 \cdot R_s \tag{1}$$

Here $m_{\text{п}}$ is the mass of plastic blade airfoil together with the shroud platform, ω is the angular velocity of the turboexpander rotor. Normal stresses from N_v at the base of the blade airfoil having the cross-sectional area F are calculated by the formula

$$\sigma_{str} = \frac{N_v}{F} \tag{2}$$

The results of the calculation of stresses at the base of the blade airfoil of the first operating row depending on the number of turboexpander revolutions are given in Table 1.

Table 1

Stresses at the base of blade airfoil of the first operating row depending on the number of turboexpander rotor revolutions

n, rpm	1000	3000	6000	9000
σ_{str}, MPa	0,048	0,430	1,720	3,871

When determining the stresses from the bending moment, we assume the constancy of axial component of the gas flow velocity before and after the operating blade. Then the bending moment at the base of blade airfoil equals [1]

$$M_{ben}^{\max} = \frac{P \cdot l}{2 \cdot \omega \cdot z \cdot R_s} \tag{3}$$

Here P is the power of turboexpander stage, z is the number of blades located in the nozzle apparatus sector.

The maximum normal stresses from $M_{ben}^{\max}$ are determined by the formula

$$\sigma_{str}^{\max} = \frac{M_{ben}^{\max}}{W_{yc}} = \frac{P \cdot l}{2 \cdot \omega \cdot z \cdot R_s \cdot W_{yc}} \qquad (4)$$

Here W_{yc}=29,43 mm^2 is the axial moment of resistance of the blade airfoil cross section relatively to the axis O_3Y_3 (Fig. 1).

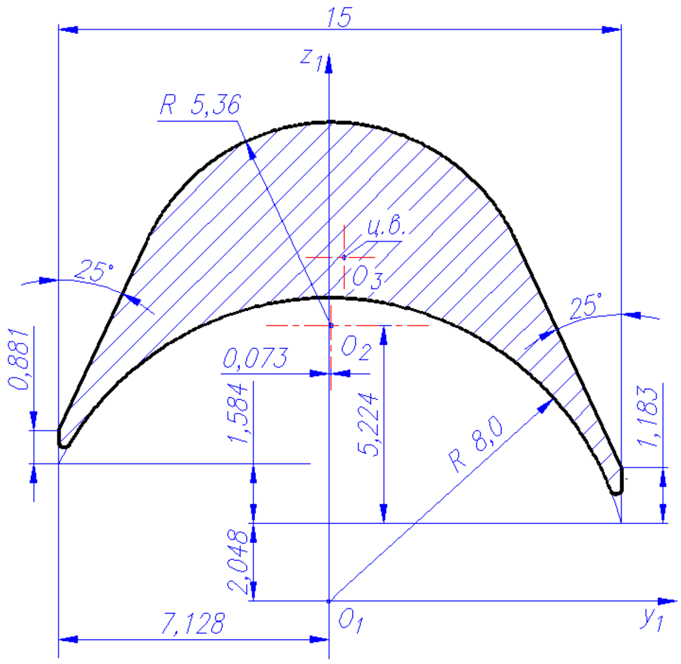

Figure 1. Profile of plastic blade airfoil

The calculation of axial moments of inertia and moments of resistance for cross sections with complex configuration is rather time-consuming task and is not considered in this calculation. For the blades of the first operating row at ω=942,5 rad/s (n=9000 rpm the normal bending stresses will be equal to $\sigma_{ben}^{\max} = \frac{12,7 \cdot 10^3 \cdot 19^{-3}}{2 \cdot 942,5 \cdot 43 \cdot 0,15 \cdot 29,43 \cdot 10^{-9}} = 6,7436\,MPa$ ..

Maximum total normal stresses at the dangerous point of the blade airfoil base are

$$\sigma^{\max} = \sigma^{str} + \sigma_{ben}^{\max} = 3,871 + 6,745 = 10,616\,MPa \qquad (5)$$

Since the tensile strength for glass-filled polyamide PA-66-KS is equal to $\sigma_в = 90\,MPa$, and also taking into account the fact that the load on the blade increases slowly over time (acceleration of the turbine occurs on average over 30 minutes), then the safety factor K can be taken as 2. Then the allowable stress with the total action of stretching and bending equals $[\sigma] = \frac{\sigma_в}{K} = \frac{90}{2} = 45\,MPa$ being larger than the total normal stresses occuring in the cross section at the blade airfoil base. That is, the strength of the plastic blade airfoil is provided.

Calculation of stresses in the locking part. The blades of turboexpanders have the locking part in the form of triangular teeth, which is mounted in the annular grooves of the rotor steel disk (Fig. 2). Since the operating temperature of the plastic blades is negative, in order to eliminate the possibility of loosening the locking connection, they are pressed with pre-tension during installation. In order to check the strength of the plastic blade locking part, you should find the total distribution of contact stresses on the teeth surface from the pre-tension and from the change in temperature.The results of the calculation of contact stresses from the pre-tension are given below.

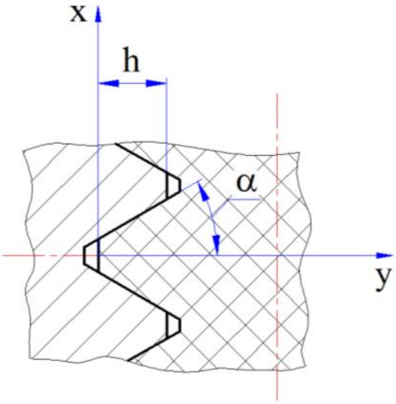

Figure 2. The profile of the tooth of the blade locking part

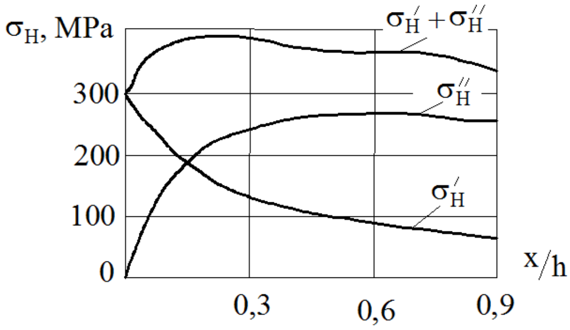

Figure 3. Distribution of contact stresses in the blade locking part

In real plastic blade structures constrution of, the inclination angle α of the teeth of the tail part is greater than the friction angle f in the pair «plastic-metal» (α> arctg f = f). Therefore, when pressing the blade lock into the metal holder, the relative slip of the surfaces of the contact teeth of the holder and the blade occur. Due to the presence of friction forces, the part of the tension along the axis «y» is not compensated. According to the calculations, the contact stresses from this uncompensated tension $\sigma_H^{/}$ are greater than those at the top of the tooth and decrease in the direction of its base.

The second part of the total contact stresses from the compensated tension $\sigma_H^{//}$, on the contrary, increases from the top of the tooth to its base. The compensation of tension that causes this part of the contact stresses is due to the relative slippage of the teeth of the holder and the blade, as well as due to the deformation of the connection parts along the axis «*y*» and the tooth of the holder along the axis «*x*».

Fig. 3 shows the graph of the distribution of contact stresses from uncompensated $\sigma_H^{/}$ and compensated $\sigma_H^{//}$ tensions along the tooth of plastic blade made of polyamide glass-filled PA-66 KS for the case when α = 30^0; height of teeth h = 1,8mm; the number of teeth – three on each side and the total tension along the axis «*y*» is 0,2 mm.

Determination of natural frequency of torsional oscillations of plastic blades. While calculating the strength, it is important to know the magnitude of the blades natural oscillation frequency, because at some (resonant) speeds of the rotor, the amplitudes of the blades oscillations can be so large that their destruction occurs.

The calculation scheme of the blade airfoil is presented in the form of a rod, one end of which is clamped, and the other is free (Fig. 4, a).

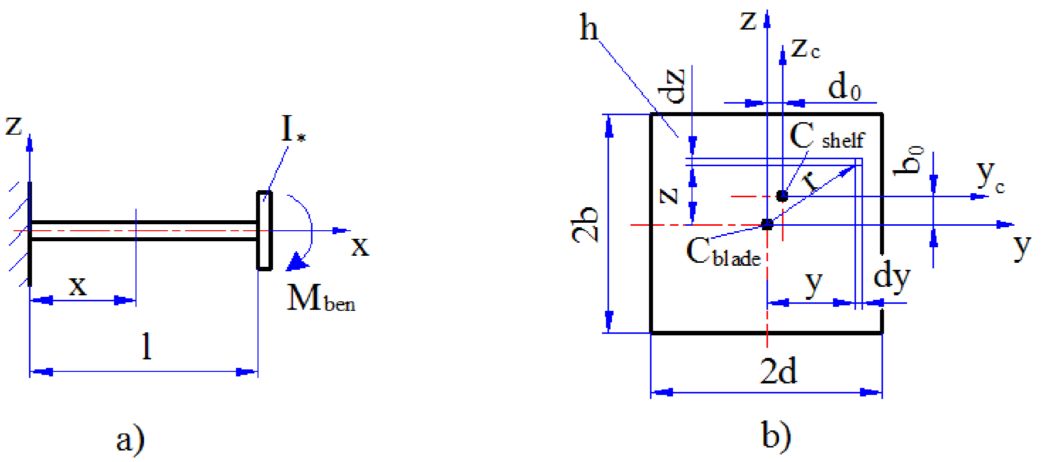

Figure 4. Scheme for calculating the frequency of torsional and bending oscillations of the plastic blade (a) and the flywheel moment of the shroud platform inertia (b)

On the free side of the airfoil (x=l) the reduced moment of inertia I_*, consisting of the rotative moment of the shroud platform and airfoil is concentrated.

The rotative moment of the platform relative to its own center of gravity C_{shelf} (Fig. 4, b) is equal to

$$I_{C_{sh}} = \int_m r^2 dm = \iint_S \left(z^2 + y^2\right)\rho_м \cdot h \cdot dz \cdot dy, \text{ here } I_{C_{sh}} = \frac{4}{3}\rho_м \cdot h \cdot d \cdot b \cdot \left(d^2 + b^2\right) \qquad (6)$$

Here $\rho_м$ is specific weight of the blade material.

The rotative moment of the platform relatively to the center of gravity of the airfoil C_{blade} is equal to

$$I'_* = I_{C_{sh.}} + 4\rho_м \cdot h \cdot d \cdot b\left(d_0^2 + b_0^2\right) = 4\rho_м \cdot h \cdot d \cdot b\left(\frac{d^2 + b^2}{2} + \left(d_0^2 + b_0^2\right)\right) \qquad (7)$$

Taking into account mathematical difficulties caused by complex configuration of the airfoil cross section, in order to calculate its rotative moment of inertia, the following simplifications are assumed. With proportional dimensions along the y and z axes of the airfoil, it can be replaced by d_{cyl} cylinder with equal cross-sectional area. Then the rotative moment of airfoil inertia is

$$I''_* \approx \frac{m_{blade} \cdot d_{cyl}^2}{8} = \frac{\pi \cdot d_{cyl}^4}{32}\rho_м \cdot l \qquad (8)$$

It is known [8] that the rotative moment of inertia of the oscillating system reduced to the free end of the rod can be assumed as approximately equal to half calculated by (8). Then the rotative moment of inertia of the whole system, reduced to the free end of the airfoil, is equal to

$$I_* \approx I'_* + \frac{1}{2} I''_* = 4\rho_м \cdot h \cdot d \cdot b\left(\frac{d^2+b^2}{2} + \left(d_0^{\ 2} + b_0^{\ 2}\right)\right) + \frac{\pi \cdot d_{cyl}^4}{64}\rho_м \cdot l \qquad (9)$$

It is also known [3] that the natural form of torsional oscillations of the rod in the arbitrary cross section x has the form

$$\varphi(x) = \frac{M_0 \cdot a}{P \cdot G \cdot I_{tor}}\rho_м \frac{P \cdot x}{a} + \varphi_0 \cos\frac{P \cdot x}{a} \qquad (10)$$

Her M_0 and φ_0 are respectively, the torque and rotation angle of the rod cross section at the pinch; P is natural oscillation frequency; G is modulus of elasticity of the second kind for the blade material; I_{tor} is the moment of the blade torque; $a = \sqrt{G/\rho_м}$ is internal torque in an arbitrary cross section x

$$M(x) = M_0 \cdot \cos\frac{P \cdot x}{a} - \frac{\varphi_0 \cdot P \cdot G \cdot I_{tor}}{a}\sin\frac{P \cdot x}{a} \qquad (11)$$

Taking into account that at the free end of the blade (x=l), where the rotative moment is I_*, torque is $M(l) = 0$, and in the place of pinching the rotation angle is $\varphi_0 = 0$ we obtain the equation for determination of blade airfoil natural frequency

$$\cos\frac{P \cdot l}{a} - \frac{I_* \cdot P \cdot a}{G \cdot I_{tor}}\sin\frac{P \cdot l}{a} = 0;\ tg\frac{P \cdot l}{a} = \frac{\chi}{\frac{P \cdot l}{a}}, \qquad (12)$$

where $\chi = \dfrac{G \cdot I_{tor} \cdot l}{I_* \cdot a}$

The results of the calculation of the operating plastic blades of turboexpander GT-700-5, made by this method, are given in Table 2.

Table 2

The first (lower) oscillation frequency of the operating plastic blades

Name	Dimension	Operating blades	
		1 degree	2 degree
Rotor speed, n	rpm	9000	9000
The rotative moment of inertia (airfoil-blade), I_*	kg·m^{2}x10^{-9}	22,063	50,474
The first (lower) frequency of torsion oscillations, $P_{1(tor)}$	s^{-1}	34723	17351

As can be seen, the lower (resonant) frequencies of torsion oscillations of the plastic blades are much higher than the operating range of the turbine rotor angular velocity (9000 rpm, або ω=942,5 s^{-1}), so there is no threat of resonant phenomena.

Determination of the natural frequency of bending oscillations of plastic blades. The calculation procedure is shown on the example of the plastic blade of the first operating row. For the calculation scheme given in Fig. 4, and for the determination of natural bending oscillations the solutions given in [9] are used.

Concentrated mass of plastic blade (mass of the shroud platform, which is poured together with the operating part of the plastic blade) is as follows

$$M_{\text{shelf}} = 2b \cdot 2d \cdot h \cdot \rho_{м} = 2 \cdot 7,5 \cdot 2 \cdot 5,8 \cdot 2 \cdot 10^{-9} \cdot 1350 = 46,98 \cdot 10^{-5} kg \tag{13}$$

Here $\rho_{м}$ is the specific weight of the blade material. It is known that the unit mass of blade airfoil length is $m = 6,914 \cdot 10^{-2}$ kg/ m . Then the total airfoil mass is $ml = 131,37 \cdot 10^{-5} kg$.

If the ratio is $\frac{M_{shelf}}{ml} = 0,3805$, then according to the graph [9, p.302] $\alpha l = 1,870$.

Since $\alpha = \sqrt[4]{\frac{mP_{bend}^2}{EI_{yc}}} = \frac{1,870}{l}$ [9, p. 294], then the first lower frequency of the natural bending oscillations of the blade airfoil is

$$P_{1bend} = \frac{3,497}{l^2}\sqrt{\frac{EI_{yc}}{m}} \tag{14}$$

Here $I_{yc} = 198,24 \, m^4$ is central axial moment of inertia relatively to axis y_c. Calculation of the axial moments of inertia for profiles with complex configuration is rather time-consuming task and is not considered in this calculation.

For the blades of the first operating row

$$P_{1bend} = \frac{3,497}{\left(19 \cdot 10^{-3}\right)^2}\sqrt{\frac{6 \cdot 10^9 \cdot 198,24 \cdot 10^{-12}}{6,914 \cdot 10^{-2}}} = 40179 s^{-1} \tag{15}$$

Similar calculations are carried out for the blades of the first and second rows of the turboexpander and guide blades. The results of the calculation of plastic blades, carried out by this method, are given in Table 3.

Table 3

The first (lower) frequency of bending oscillations of the plastic blade

Name	Dimension	Operating blades		Guide blades
		1 degree	2degree	
Rotor speed, n	rpm	9000	9000	0
Central axial moment of inertia, I_{yc}	mm^4	198,24	100,1	135,2
First (lower) frequency of natural bending oscillations, P_{1bend}	s^{-1}	40179	17351	24496

As can be seen from the results of the calculations, the lower natural (resonant) frequencies of bending oscillations for plastic blades of the starting turboexpanders are beyond the operating range of the rotor angular velocities. Therefore, there is no need to determine the coefficient of dynamism, which takes into account dynamic phenomena, when determining the stresses in the blade material.

Conclusion. The carried out calculations of the stress-strain state of the lock part and the plastic blade airfoil of the cold turboexpander of the gas turbine unit, as well as the calculation of resonant frequencies of torsion and bending oscillations of these blades indicate that stresses in the plastic blade material do not exceed the permissible values and lower (resonant) frequencies are much lower than the operating range of rotor angular velocities. Therefore, plastic blades of cold turboexpanders made of glass-filled polyamide, for example, brand PA-66KS can be successfully applied at compressor stations of main gas pipelines.

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УДК 678.5

РОЗРАХУНОК НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ЛОПАТОК З ПОЛІМЕРНИХ КОМПОЗИЦІЙНИХ МАТЕРІАЛІВ ПУСКОВИХ ТУРБОДЕТАНДЕРІВ У РЕЗОНАНСНИХ ЗОНАХ

Юрій Наконечний; Ігор Ярема; Віталій Батюк

Тернопільський національний технічний університет імені Івана Пулюя, Тернопіль, Україна

Резюме. *Приведено огляд успішного застосування сучасних композиційних матеріалів для виготовлення лопаток турбін в авіаційних газотурбінних двигунах, осьових шахтних та доменних компресорах. Проаналізовано основні їх переваги порівняно з металевимии: невелика питома вага, що зменшує напруження розтягування від відцентрових сил; добріі демпфуючі властивості; висока технологічність; менш небезбечний характер їх руйнування, який пояснюється тим, що за низьких температур пластмасові лопатки руйнуються на дуже дрібні частинки й не пошкоджують корпус та інші деталі, як це буває з металевими лопатками. Виконано аналітичні розрахунки напружень від згинання та розтягування відценровими силами, які виникають у матеріалі пера та замкової частини пластмасових лопаток пускових турбодетандерів, що працюють на перепадах тиску природного газу за температур від +20 до -40*^{0}*С і виготовлені зі склонаповненого поліаміду ПА-66КС високопродуктивним методом лиття під тиском. При розрахунку напружень у замковій частині лопаток враховано можливість послаблення замкового з'єднання внаслідок зміни величини контактних напружень на поверхні зубців замкової частини від попереднього натягу та від зміни температури, отримано графік сумарного розподілу цих контактних напружень. Розраховано, також перші (найнижчі й найнебезпечніші) частоти власних згинальних та крутних коливань пластмасових лопаток, оскільки при деяких (резонансних) числах обертів ротора амплітуди коливань лопаток можуть бути настільки великими, що відбудеться їх руйнування. Результати розрахуків показують, що власні частоти коливань пластмасових лопаток пускових турбодетандерів лежать за робочим діапазоном кутових швидкостей ротора, отже немає загрози резонансних явищ. Обгрунтовано можливість успішного застосування склонаповненого поліаміду для виготовлення робочих та напрямних лопаток пускових турбодетандерів та їх успішної експлуатації на компресорних станціях магістральних газопроводів.*

Ключові слова: *композиційні матеріали, лопатки турбін, напруження, деформації, крутні коливання, згинальні коливання, резонанс.*

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