Вісник Тернопільського національного технічного університету
https://doi.org/10.33108/visnyk_tntu
Scientific Journal of the Ternopil National Technical University
2021, № 3 (103) https://doi.org/10.33108/visnyk_tntu2021.03
ISSN 2522-4433. Web: ***visnyk.tntu.edu.ua***

UDC 539.3

CONTACT OF THE EDGES OF THE INTERPHASE CUT ON THE ARC OF THE CIRCLE BETWEEN THE ISOTROPIC PLATE AND THE CLOSED ELASTIC RIB

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Summary*. In the conditions of the general flat stress state created by uniformly distributed effects of tension (compression) at infinity, the mixed contact problem for an infinite isotropic plate with a circular hole, which contour reinforced by a closed elastic rib in the presence of a symmetrical interfacial section at the boundary of their connection and the edges of cut in the process of deformation is smoothly contacted, is considered. The components of the deformation tensor (unit elongation, the angle of rotation of the normal and the curvature) at the point of the contour of the hole of the plate are represented by integral dependences on the contact forces. By modeling the reinforcement of a closed elastic rod of a stable rectangular cross of large curvature and using the basic equations of linear theory of curvilinear rods the mathematical model of problems is constructed in the form of systems of three singular integral equations with Hilbert cores to find contact forces between plates and rib. To determine the initial parameters of a closed static indeterminate rod, the conditions of unambiguous displacement and angles of rotation at the point of its axis and the equilibrium conditions are used. The approximate solution of the problem is constructed by the method of mechanical quadrature and collocations, which investigated the influence on the stress state of the plate and the reinforcing rib and on the size of the area of smooth contact of stiffness factor of rib.*

Key words*: interphase insection, isotropic plate, reinforcing rib, contact forces, singular integral equations, smooth contact area.*

https://doi.org/10.33108/visnyk_tntu2021.03.088 *Received 28.07.2021*

Introduction. Plates, weakened by holes, are widely used in modern engineering and construction. To reduce high concentration of stresses near the holes, their contours are reinforced with closed or open reinforcing ribs. The stress-strain state of such plates significantly depends on the model of the reinforcing rib. Such problems are most fully studied for cases when reinforcement is modeled by an elastic line of constant or variable stiffness (compression) and bending [1] or a curved rod of constant cross-section, whose line of actual connection with the plate does not coincide with its geometric axis [2]. In modern scientific literature, the model of a curvilinear bar of big curvature of constant rectangular cross-section became widespread [3, 4]. Using this model in [5], a number of problems on the contact interaction between an infinite isotropic (orthotropic) plate with a curved (elliptical) hole and a closed elastic rib when they are completely connected by press fit method with guaranteed tension or by welding.

In the process of production or operation of piecewise homogeneous plate structures, such defects as interphase cuts (cracks) may occur at the interface of materials, which will cause high stress concentration and further interphase failure under force or heat load. A systematic study of such problems for rectilinear and arc cracks was conducted in [6–11].

The case of one or two symmetrical interphase sections, the edges of which do not come into contact during deformation, between an isotropic (orthotropic) plate with a curved (elliptical) hole and a closed elastic rib of large curvature is considered in [5, 12]. The case of one or two symmetrical interphase sections, the shores of which do not come into contact during deformation, between an isotropic (orthotropic) plate with a curved (elliptical) hole and a closed elastic rib of large curvature is considered in [5, 12]. The method of mechanical squaring and collocation for different loads is used to study the effect of the anisotropy of the plate material, the hole shape, the size of interfacial section and physical and geometric parameters of the rib on the stress state of the plate structure. In the presence of smooth contact of the edges of the interphase section (sections), such tasks remain unexplored.

It is suggested to use numerical-analytical method for solving a mixed contact problem for an infinite isotropic plate with a circular hole reinforced with a closed elastic rib under conditions of generalized flat stress state.

Statement of the problem. Supposing, an infinite isotropic plate of thickness $2h$ is weakened by a circular hole of unit radius, the contour γ of which is reinforced by a closed elastic rib of constant cross-section. The plate structure is in the conditions of the generalized plane stress state created by evenly distributed on infinity forces p and q acting in two mutually perpendicular directions.

The common medium plane of the plate and the edge is assigned to the system of rectangular (x, y) and polar (ρ, λ) coordinates with the pole in the center of the hole. The reference systems are chosen so that the coordinate axes determine the directions of the external load action (Fig. 1).

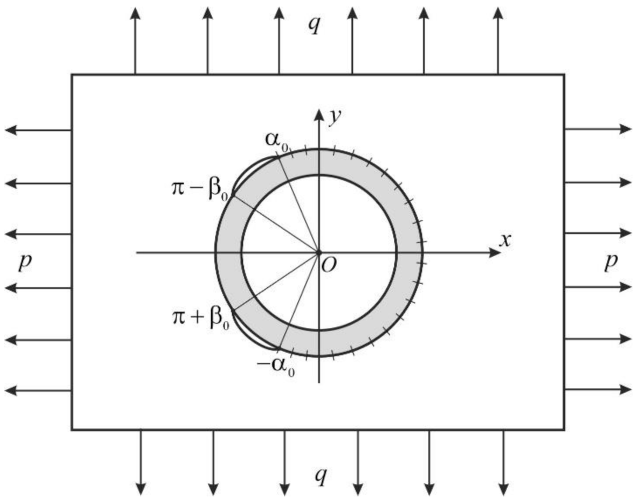

Figure 1. Design diagram of the plate

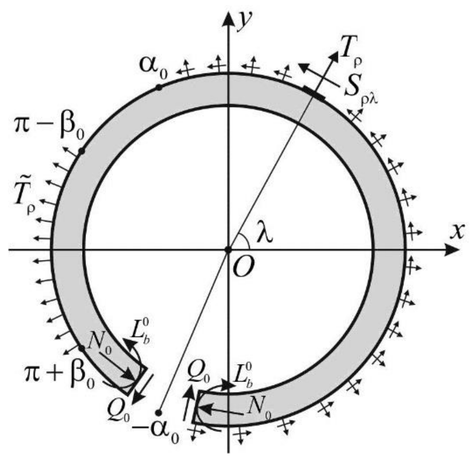

Figure 2. Design diagram of the reinforcing rib

Assume that on the line connecting the plate and the ribs outside the area $[-\alpha_0^*; \alpha_0^*]$ there is a symmetrical interfacial section in relation to the axis Ox, the edges of which under the action of a given load on the area $[\pi - \beta_0^*; \pi + \beta_0^*]$ are in smooth contact.

Objective of the research ids to determine the component of stress state on the contour γ in the plate, reinforcing rib and to find out their dependence on size of the interphase section, the size of smooth contact area and the external load.

Main equations of the problem. Stress-strain state of the plate is determined by contact forces T_ρ^0, $S_{\rho\lambda}^0$ on the area $\left[-\alpha_0; \alpha_0\right]$, contact forces $\tilde{T}_\rho$ on the area $\left[\pi - \beta_0; \pi + \beta_0\right]$ applied to the contour γ from the side of reinforcing rib, and external load on infinity.

At given load, the deformation parameters at the points of contour γ are determined from the relations [12], which can be represented as

$$\varepsilon_\lambda(\lambda)=\frac{1}{2Eh}\left[(1-v)T_\rho(\lambda)-\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}T_\rho^0(t)\,dt+\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}S_{\rho\lambda}^0(t)\operatorname{ctg}\frac{\lambda-t}{2}dt-\right.$$
$$\left.-\frac{1}{\pi}\int_{\pi-\beta_0}^{\pi+\beta_0}\tilde{T}_\rho(t)\,dt+(p+q)-2(p-q)\cos 2\lambda\right];$$
$$V(\lambda)=\frac{1}{2Eh}\left[(1-v)S_{\rho\lambda}(\lambda)-\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}T_\rho^0(t)\operatorname{ctg}\frac{\lambda-t}{2}dt-\right. \tag{1}$$
$$\left.-\frac{1}{\pi}\int_{\pi-\beta_0}^{\pi+\beta_0}\tilde{T}_\rho(t)\operatorname{ctg}\frac{\lambda-t}{2}dt+2(p-q)\sin 2\lambda\right];$$
$$k(\lambda)=1-\varepsilon_\lambda(\lambda)+\frac{dV(\lambda)}{d\lambda};\quad \lambda\in[-\alpha_0;2\pi-\alpha_0].$$

Here the nomenclature is introduced: E, v are Young's modulus and Poisson's ratio of the plate material; ε_λ, V, k are relative elongation at the points of the hole contour, the rotation angle of normal to it and the curvature;

$$T_\rho=\begin{cases}T_\rho^0, & \lambda\in[-\alpha_0;\,\alpha_0];\\ \tilde{T}_\rho, & \lambda\in[\pi-\beta_0;\,\pi+\beta_0];\end{cases}\qquad S_{\rho\lambda}=\begin{cases}S_{\rho\lambda}^0, & \lambda\in[-\alpha_0;\,\alpha_0];\\ 0, & \lambda\in[\pi-\beta_0;\,\pi+\beta_0].\end{cases}$$

If the contact forces become known, the ring forces on γ can be determined by the formula

$$T_\lambda(\lambda)=vT_\rho(\lambda)+2Eh\varepsilon_\lambda(\lambda). \tag{2}$$

Closed reinforcing rib is in elastic equilibrium under the action of contact forces T_ρ^0, $S_{\rho\lambda}^0$, $\tilde{T}_\rho$ transmitted to its outer side surface from the plate. It is modeled by a closed curved rod of large curvature with rectangular cross-section [4].

According to the accepted model, the stress state of the reinforcement is determined by the longitudinal N and cross-sectional Q forces and bending moment L_b which act in the cross-sections of the rod and are related to its axis. Deformations of the longitudinal fibers of the reinforcing rib are characterized by relative elongation $\varepsilon_\lambda^{(c)}$, angle of rotation of the normal θ_b and curvature $k^{(c)}$.

The problem of calculating a closed rib is statically indeterminate. Using the idea of the method of forces [4], we conventionally cut the edge with a plane $\lambda=-\alpha_0$, and to the ends of the section apply statically balanced forces N_0, Q_0 and bending moments L_b^0 (Fig. 2). As a result, we obtain a statically determinate open rod, which is in equilibrium under the action of the load on the ends and the contact forces transmitted to it from the plate.

From the conditions of equilibrium of the part of the rib between the cross-sections $\lambda = -\alpha_0$ and $\lambda = \lambda$ find

$$N(\lambda) = N_0 \cos(\lambda + \alpha_0) - Q_0 \sin(\lambda + \alpha_0) + f_1(\lambda)\cos\lambda + f_2(\lambda)\sin\lambda;$$
$$Q(\lambda) = N_0 \sin(\lambda + \alpha_0) + Q_0 \cos(\lambda + \alpha_0) + f_1(\lambda)\sin\lambda - f_2(\lambda)\cos\lambda; \tag{3}$$
$$L_b(\lambda) = L_b^0 + (1-\eta) N_0 - \int_{-\alpha_0}^{\lambda} S_{\rho\lambda}(t)\,dt - (1-\eta) N(\lambda), \quad \lambda \in [-\alpha_0; 2\pi - \alpha_0],$$

where

$$f_1(\lambda) = \begin{cases} -\int_{-\alpha_0}^{\lambda} \left[T_\rho^0(t)\sin t + S_{\rho\lambda}^0(t)\cos t\right]dt, & \lambda \in [-\alpha_0; \alpha_0]; \\ 0, & \lambda \in [\alpha_0; \pi - \beta_0]; \\ -\int_{\pi-\beta_0}^{\lambda} \tilde{T}_\rho(t)\sin t\,dt, & \lambda \in [\pi - \beta_0; \pi + \beta_0]; \end{cases}$$

$$f_2(\lambda) = \begin{cases} \int_{-\alpha_0}^{\lambda} \left[T_\rho^0(t)\cos t - S_{\rho\lambda}^0(t)\sin t\right]dt, & \lambda \in [-\alpha_0; \alpha_0]; \\ 2(N_0 \sin\alpha_0 + Q_0 \cos\alpha_0), & \lambda \in [\alpha_0; \pi - \beta_0]; \\ 2(N_0 \sin\alpha_0 + Q_0 \cos\alpha_0) - \int_{\pi-\beta_0}^{\lambda} \tilde{T}_\rho(t)\cos t\,dt, & \lambda \in [\pi - \beta_0; \pi + \beta_0]; \end{cases} \tag{4}$$

$$f_1(\pm\alpha_0) = f_1(\pi \pm \beta_0) = 0; \quad f_2(-\alpha_0) = f_2(\pi + \beta_0) = 0;$$
$$f_2(\alpha_0) = f_2(\pi - \beta_0) = 2(N_0 \sin\alpha_0 + Q_0 \cos\alpha_0).$$

Functions $f_1(\lambda)$, $f_2(\lambda)$ that determine the projections on the coordinate axes Ox and Oy, according to the external force load applied to the rib on the area $[-\alpha_0; \lambda]$. In the case of balanced load, these functions are one-valued.

Deformations of the outer longitudinal fiber of the rib in contact with the plate are determined by the formulas [4]

$$\varepsilon_\lambda^{(c)}(\lambda) = \frac{1}{E_0 F_0}\left[N(\lambda) + \frac{\eta + \eta_c}{\eta_c} L_b(\lambda)\right]; \quad \frac{d\theta_b(\lambda)}{d\lambda} = \frac{1}{E_0 F_0}\left[N(\lambda) + \frac{L_b(\lambda)}{\eta_c}\right];$$

$$k^{(c)}(\lambda) = 1 - \varepsilon_\lambda^{(c)}(\lambda) + \frac{d\theta_b(\lambda)}{d\lambda}, \quad \lambda \in [-\alpha_0; \pi + \beta_0], \tag{5}$$

in which $2h_0$, 2η are the height and width of reinforcing rib; E_0 is Young's modulus of the rib material; $F_0 = 2h_0 \times 2\eta$ is the cross-section area; $E_0 F_0$ is stiffness of the rib in tension (compression); η_c is the distance from the axis of the rib to the neutral for pure bending longitudinal fiber with a radius of curvature $r_0 = 1-\eta-\eta_c$.

The initial parameters N_0, Q_0, L_b^0 are determined from the equilibrium condition of reinforcing rib as a rigid whole

$$\int_{-\alpha_0}^{\alpha_0} \left[T_\rho^0(t)\cos t - S_{\rho\lambda}^0(t)\sin t\right]dt + \int_{\pi-\beta_0}^{\pi+\beta_0} \tilde{T}_\rho(t)\cos t\,dt = 0 \qquad (6)$$

and the conditions of unambiguity of the displacement vector components of the points of contour γ and the angle of rotation of the normal to it

$$\int_0^\pi L_b(t)\cos t\,dt = 0; \quad \theta_b(\pi) = 0. \qquad (7)$$

Normal stresses in the external and internal fibers of the rib and the highest tangential stresses in the axial fiber are determined by the formulas [4, 5, 12]

$$\sigma^{(1)}(\lambda) = \frac{1}{F_0}\left[N(\lambda) + \frac{\eta+\eta_c}{\eta_c} L_b(\lambda)\right]; \quad \sigma^{(2)}(\lambda) = \frac{1}{F_0}\left[N(\lambda) + \frac{\eta_c-\eta}{\eta_c}\frac{L_b(\lambda)}{1-2\eta}\right];$$

$$\tau_{\rho\lambda}^{(c)}(\lambda) = \frac{3}{2}\frac{Q(\lambda)}{F_0}. \qquad (8)$$

Relations (3)–(8) form a complete system of equations for determining the stress-strain state of statically indeterminate rib of large curvature.

Mathematical model of the problem. Boundary conditions of compatible deformation of the plate and the reinforcing rib at the area of their connection are formulated as the conditions of ideal mechanical contact in differential form

$$\varepsilon_\lambda(\lambda) = \varepsilon_\lambda^{(c)}(\lambda); \quad V(\lambda) = \theta_b(\lambda), \quad \lambda \in [-\alpha_0; \alpha_0], \qquad (9)$$

and in the area of smooth contact as the equality of curves

$$k\ (\lambda) = k^{(c)}(\lambda). \qquad (10)$$

Absence of contact forces of lag of the rib from the plate is taken into account when writing relations (1), (3), (4).

Substituting (1), (5) taking into account (3), (4) in boundary conditions (9), (10), after certain transformations, we obtain a system of three singular integral equations with Hilbert nuclei to determine the functions T_ρ^0, $S_{\rho\lambda}^0$, $\tilde{T}_\rho$

$$
\begin{aligned}
&(1-\nu)T_\rho^0(\lambda)-\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}T_\rho^0(t)\,dt+\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}S_{\rho\lambda}^0(t)\operatorname{ctg}\frac{\lambda-t}{2}dt-\\
&-\frac{1}{\pi}\int_{\pi-\beta_0}^{\pi+\beta_0}\tilde{T}_\rho(t)\,dt+(p+q)-2(p-q)\cos 2\lambda\Bigg]=\frac{2Eh}{E_0F_0}\left[N(\lambda)+\frac{\eta+\eta_c}{\eta}L_b(\lambda)\right];\\
&(1-\nu)S_{\rho\lambda}^0(\lambda)-\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}T_\rho^0(t)\operatorname{ctg}\frac{\lambda-t}{2}dt-\frac{1}{\pi}\int_{\pi-\beta_0}^{\pi+\beta_0}\tilde{T}_\rho(t)\operatorname{ctg}\frac{\lambda-t}{2}dt+\\
&+2(p-q)\sin 2\lambda=\frac{2Eh}{E_0F_0}\int_0^\lambda\left[N(t)+\frac{L_b(t)}{\eta_c}\right]dt,\quad \lambda\in[-\alpha_0;\alpha_0];\qquad (11)\\
&\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}T_\rho^0(t)\,dt+\frac{1}{2\pi}\int_{-\alpha_0}^{\alpha_0}T_\rho^0(t)\frac{dt}{\sin^2\frac{\lambda-t}{2}}-\frac{1}{\pi}\int_{-\alpha_0}^{\alpha_0}S_{\rho\lambda}^0(t)\operatorname{ctg}\frac{\lambda-t}{2}dt-\\
&-(1-\nu)\tilde{T}_\rho(\lambda)+\frac{1}{\pi}\int_{\pi-\beta_0}^{\pi+\beta_0}\tilde{T}_\rho(t)\,dt-\frac{1}{\pi}\frac{d}{d\lambda}\int_{\pi-\beta_0}^{\pi+\beta_0}\tilde{T}_\rho(t)\operatorname{ctg}\frac{\lambda-t}{2}dt-\\
&-(p+q)+6(p-q)\cos 2\lambda=\frac{2Eh}{E_0F_0}(1-\eta)r_0N(\lambda),\quad \lambda\in[\pi-\beta_0;\pi+\beta_0].
\end{aligned}
$$

We supplement system (11) with conditions (6), (7), which are used to determine constants N_0, Q_0, L_b^0.

Relations (11), (6), (7) determine the mathematical model of the formulated problem. Note that the magnitude of the contact area of the edges of the interphase section $(2\beta_0)$ is unknown in advance and needs to be determined.

Approximate solution of the problem. The exact solution of the system (11), (6), (7) cannot be found. To find an approximate solution, it is necessary to determine the structure of the required functions at the ends of the sections $[-\alpha_0;\alpha_0]$ and $[\pi-\beta_0;\pi+\beta_0]$.

In the area of smooth contact of the plate and the rib, function $\tilde{T}_\rho(\lambda)$ is limited and continuous, and equals zero at its ends ($\tilde{T}_\rho(\pi\pm\beta_0)=0$).

Since the problem under consideration belongs to the mixed contact problems of the theory of elasticity, the contact forces T_ρ^0, $S_{\rho\lambda}^0$ at the ends $\lambda=\pm\alpha_0$ of the junction of the plate and the reinforcement have a root feature with local oscillation. Neglecting its influence, the approximate solution of the system (11), (6), (7) can be constructed by a combined method of mechanical squaring and collocation. Quadrature formulas of this method for the case of absence of contact of the interphase section edges are given in [5].

In the numerical implementation of the problem, the size of the area of smooth contact was determined by the method of half-division of the segment proposed in [5].

Analysis of numerical results. For an infinite isotropic plate with a circular ($\rho_0=1$) hole and a reinforcing rib with physical and geometric parameters

$$\alpha_0=120°;\quad h_0/h=1;\quad \eta=0.1$$

the influence of smooth contact of the relative stiffness of the reinforcement E_0/E on their stress state and the size of the area is investigated.

Results of numerical calculation of values $T_\rho = 2h\sigma_\rho$, $S_{\rho\lambda} = 2h\tau_{\rho\lambda}$, $T_\lambda = 2h\sigma_\lambda$, $F_0\sigma^{(1)}$, $F_0\sigma^{(2)}$, $F_0\tau_{\rho\lambda}^{(c)}$ when $p = 0$; $q \neq 0$ are shown in Fig. 3–5. Continuous lines are drawn for the case $E_0 / E = 2$, dashed for $E_0 / E = 5$, dot-and-dashed for $E_0 / E = 10$.

The values of the angle β_0 which determines the size of the smooth contact area, respectively, are equal to

$$\beta_0 = 24.067°; \quad \beta_0 = 35.600°; \quad \beta_0 = 34.964°.$$

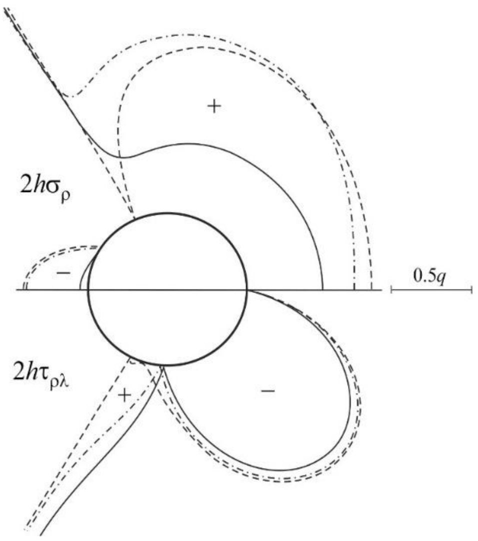

Figure 3. Distribution diagram of contact forces on the contour γ in the plate

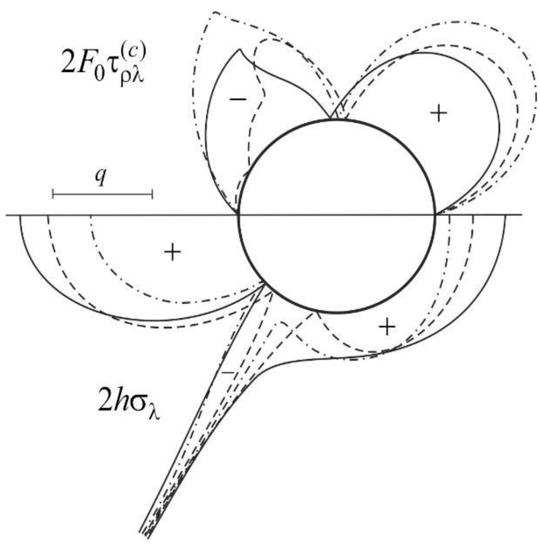

Figure 4. Distribution diagram of the largest tangential stresses in the rib and annular forces on the contour γ in the plate

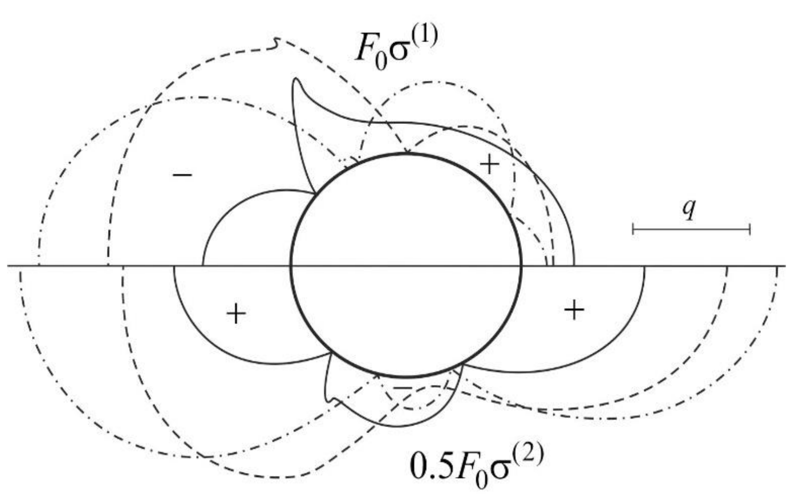

Figure 5. Distribution diagram of normal stresses in the extreme longitudinal fibers of the rib

Table 1 shows the results of calculating the maximum values of the components of the stress state in the plate and rob with contact of the section edges (in the numerator) and without contact (in the denominator).

Table 1

Comparative table of maximum values of stress components in the plate and the reinforcing rib

$\frac{E_0}{E}$	$T_\rho(\lambda)$		$S_{\rho\lambda}(\lambda)$	$T_\lambda(\lambda)$		$F_0\sigma^{(1)}(\lambda)$		$F_0\sigma^{(2)}(\lambda)$		$F_0\tau_{\rho\lambda}^{(c)}(\lambda)$
	$\lambda=0^\circ$	$\lambda=180^\circ$	$\lambda=46.34^\circ$	$\lambda=0^\circ$	$\lambda=180^\circ$	$\lambda=0^\circ$	$\lambda=180^\circ$	$\lambda=0^\circ$	$\lambda=180^\circ$	$\lambda=46.34^\circ$
2	0.481 / 0.449	–0.053 / 0	0.976 / 0.983	0.726 / 0.695	2.217 / 2.144	0.465 / 0.448	–0.764 / –0.843	2.169 / 2.125	2.021 / 2.399	0.534 / 0.543
5	0.784 / 0.499	–0.405 / 0	1.050 / 1.076	0.374 / 0.246	1.948 / 1.897	0.279 / 0.197	–1.591 / –1.267	3.585 / 3.158	2.928 / 3.368	0.631 / 0.725
10	0.676 / 0.435	–0.386 / 0	1.043 / 1.039	0.145 / 0.079	1.518 / 1.750	–0.231 / –0.202	–2.199 / –1.604	4.447 / 3.963	4.711 / 4.123	0.773 / 0.859

Conclusions. Within the refined theory of curvilinear rods of great curvature and the basic integral relations of the plane problem of theory of elasticity, the problem is set, dealing with partial smooth contact of interphase section edges between an infinite isotropic plate with a circular hole and a closed elastic rib, which are in the conditions of homogenous ultimate plane stressed state created by forces evenly distributed on infinity and acting in two mutually perpendicular directions. The boundary conditions of the problem at the junction of the plate and the rib are formulated as the conditions of ideal mechanical contact in differential form, and at the smooth contact as the equality of curves. Using the integral dependences between the deformation components of the plate (of reinforcing rib) and the contact forces arising at the interface of materials, the mathematical model of the problem is created as a system of three singular integral equations with Gilbert nuclei. To determine the initial parameters of statically indeterminate rib, it is proposed to use the conditions of unambiguous displacements, the angle of rotation of the normal and the equilibrium condition of reinforcement. The structure of contact forces at the ends of the joint of the plate and the ribs as well as at the ends of the section of their smooth contact is established. Numerical implementation of the problem is carried out by the method of mechanical squaring and collocation. As a result of numerical experiments, it was found that the tangential stresses at the junction of the plate and the rib do not depend on the stiffness of reinforcement and the area of smooth contact. The presence of a smooth contact area leads to an increase in normal stresses in the center of the joint and a slight decrease in the ring stresses in the plate. With increasing stiffness, the maximum values of stresses in the rib increase significantly. In the vicinity of the ends of the interphase section, the normal stresses in the longitudinal fibers of the rib are limited and continuous, and the contact and ring stresses in the plate take unlimited values. The developed approach and obtained results can be transferred to orthotropic plates with a circular hole and generalized for plates with curved holes other than circular one without specific changes.

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УДК 539.3

КОНТАКТ БЕРЕГІВ МІЖФАЗНОГО РОЗРІЗУ ПО ДУЗІ КОЛА МІЖ ІЗОТРОПНОЮ ПЛАСТИНКОЮ І ЗАМКНЕНИМ ПРУЖНИМ РЕБРОМ

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***Резюме**. В умовах узагальненого плоского напруженого стану, створеного рівномірно розподіленими зусиллями розтягу (стиску) на нескінченності, розглянуто мішану контактну задачу для нескінченної ізотропної пластинки з круговим отвором, контур якого підсилено замкненим пружним ребром, за наявності на межі їх сполучення симетричного міжфазного розрізу, береги якого у процесі деформації гладко контактують. Компоненти тензора деформації (відносне видовження, кут повороту нормалі й кривина) в точках контуру отвору пластинки представлені інтегральними залежностями від контактних зусиль. Моделюючи підсилення замкненим пружним стрижнем сталого прямокутного поперечного перерізу великої кривини та використовуючи основні рівняння лінійної теорії криволінійних стрижнів, математичну модель задачі побудовано у вигляді системи трьох сингулярних інтегральних рівнянь з ядрами Гільберта для знаходження контактних зусиль між пластинкою і ребром. Для визначення початкових параметрів замкненого статично невизначеного стрижня використано умови однозначності зміщень і кутів повороту в точках його осі та умову рівноваги. Встановлено структуру шуканих функцій на ділянці сполучення пластинки й підсилювального ребра та ділянці гладкого контакту. Наближений розв'язок задачі побудовано комбінованим методом механічних квадратур і колокації, яким досліджено вплив на напружений стан пластинки і підсилювального ребра та величину ділянки гладкого контакту відносної жорсткості ребра. Встановлено, що в околі торців міжфазного розрізу нормальні напруження в поздовжніх волокнах ребра обмежені йі неперервні, а контактні й кільцеві зусилля в пластинці набувають необмежені значення. Отримані результати порівнюються з відповідними результатами для міжфазного розрізу, береги якого в процесі деформації не контактують.*

***Ключові слова:** міжфазний розріз, ізотропна пластинка, підсилювальне ребро, контактні зусилля, сингулярні інтегральні рівняння, ділянка гладкого контакту.*

https://doi.org/10.33108/visnyk_tntu2021.03.088 *Отримано 28.07.2021*