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FINDING PHYSICALLY JUSTIFIED PARTIAL SOLUTIONS OF THE EQUATIONS OF THE THERMOELASTICITY THEORY IN THE CYLINDRICAL COORDINATE SYSTEM

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Summary. *The paper considers the linear model of three-dimensional isotropic body of theories of thermoelasticity in the cylindrical coordinate system. We consider the case when the stationary temperature satisfies the Laplace equation. After substituting thermoelastic stresses into the equilibrium equation, the system of Navier's differential equations were obtained. Its general solution is presented as the sum of homogeneous and partial solutions. The partial solution of the system of Navier's equations, which is clearly determined by the stationary temperature and does not contain elastic displacements, is called the temperature solution. The physical and mathematical features of the thermoelastic stress state were taken into account and it was proved that in the temperature solution the sum of normal stresses is zero and the volume deformation is equal to* $e = 3\alpha T$ *. The found dependencies were used and the new temperature solution of the system of Navier's equations were constructed in the cylindrical coordinate system, when the temperature does not depend on the axial variable. Simple formulas for expressing temperature stresses are given. The general solution of the equations of the theory of thermoelasticity by four harmonic functions is recorded.*

Key words: *Navier's equations, thermoelastic body, cylindrical coordinate system, characteristics of temperature state, stresses, displacements.*

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Introduction. Thermoelastic materials, under the influence of various temperature fields, are used in aerospace and other fields of engineering [1–3]. Elastic bodies subjected to force and temperature loads are widely used in power engineering, technological and engineering structures [4, 5].

Review of the available static solutions of elasticity and thermoelasticity equations. Methods for solving static boundary-value problems in elastic three-dimensional body are based mainly on the construction and application of various representations of the general solution of elasticity equations [1, 2, 5, 6], where the temperature is predetermined. In the investigation of three-dimensional static problems of thermoelasticity [1, 3, 4], known solutions of the equations of elasticity where added specific temperature distribution of stresses, determined by thermoelastic potential are used [2, 4]. The vast majority of solutions of the boundary value problems of the thermoelasticity theory are constructed by means of thermoelastic potential [3–6]. Thus, in paper [7], analytical solutions of the plane thermoelasticity problem by means of generalized functions and Fourier transformation are proposed, and in paper [8], the thermoelastic potential is used and three-dimensional thermoelastic state of the plate is reduced to the solution of two-dimensional boundary value problem. In paper [9], method for analytical and numerical evaluation of temperature stresses in the hollow cylinder of finite length was developed on the basis of direct integration method. However, in paper [10], new physically substantiated partial solutions of the thermoelasticity theory in Cartesian coordinate system without thermoelastic potential application were found. These solutions take into account the effect of temperature field on the stress state of thermoelastic bodies more accurately. Ibid, on the basis of paper [11], new presentation of the general solution of thermoelasticity equations by four harmonic functions is proposed.

The objective of the paper is to find physically substantiated partial temperature solutions of Navier equations in the cylindrical coordinate system, when the temperature does not depend on the axial variable.

1. Statement of the problem and presentation of the static thermoelasticity equations. Let us consider the general formulation of three-dimensional static problem of the thermoelasticity theory for three-dimensional isotropic body in the cylindrical coordinate system. The initial temperature, when there are no stresses, is set as equal to zero. The temperature in the body varies within such limits that the elastic and heat-conducting coefficients of the material can be modeled as constant.

Let us use Duhamel-Neumann relation to represent thermoelastic stresses [3, 6] in homogeneous solid body in the cylindrical coordinate system

$$\sigma_r = 2G\left[\varepsilon_r + \frac{\nu}{1-2\nu}e - \frac{1+\nu}{1-2\nu}\alpha T\right],\ \ \sigma_\varphi = 2G\left[\varepsilon_\varphi + \frac{\nu}{1-2\nu}e - \frac{1+\nu}{1-2\nu}\alpha T\right],$$

$$\sigma_z = 2G\left[\varepsilon_z + \frac{\nu}{1-2\nu}e - \frac{1+\nu}{1-2\nu}\alpha T\right], \qquad (1)$$

$$\tau_{r\varphi} = G\gamma_{r\varphi},\ \tau_{rz} = G\gamma_{rz},\ \tau_{\varphi z} = G\gamma_{\varphi z},$$

where $G = E/2(1+\nu)$, E are shear and Young's modules, $\varepsilon_r = \frac{\partial u_r}{\partial r}$, $\varepsilon_\varphi = \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r}$, $\varepsilon_z = \frac{\partial u_z}{\partial z}$ are elongation deformations, u_r, u_φ, u_z are elastic displacements, $\gamma_{r\varphi} = r\frac{\partial}{\partial r}\frac{u_\varphi}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \varphi}$, $\gamma_{rz} = \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}$, $\gamma_{z\varphi} = \frac{1}{r}\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_\varphi}{\partial z}$ are relative shear strains, $e = \varepsilon_r + \varepsilon_z + \varepsilon_\varphi$ is volume deformation, ν is Poisson's ratio, α is coefficient of thermal expansion.

Let us substitute relation (1) into the equilibrium equations of thermoelastic body in the cylindrical coordinate system and write the system of Navier's differential equations with partial derivatives for elastic displacements [1, 2].

$$\nabla^2 u_r + \frac{1}{1-2\nu}\frac{\partial e}{\partial r} - \frac{2}{r^2}\frac{\partial u_\varphi}{\partial \varphi} - \frac{u_r}{r^2} = 2\frac{1+\nu}{1-2\nu}\alpha\frac{\partial T}{\partial r},$$

$$\nabla^2 u_\varphi + \frac{1}{1-2\nu}\frac{\partial e}{r\partial \varphi} + \frac{2}{r^2}\frac{\partial u_r}{\partial \varphi} - \frac{u_\varphi}{r^2} = 2\frac{1+\nu}{1-2\nu}\alpha\frac{1}{r}\frac{\partial T}{\partial \varphi}, \qquad (2)$$

$$\nabla^2 u_z + \frac{1}{1-2\nu}\frac{\partial e}{\partial z} = 2\frac{1+\nu}{1-2\nu}\alpha\frac{\partial T}{\partial z},$$

where $\nabla^2 = [\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}]$ is Laplace operator [12]. Let us assume that the body has stationary temperature field without internal heat release that satisfies Laplace equation

$$\nabla^2 T = 0. \qquad (3)$$

The system of Navier's equation of the thermoelasticity theory (2) will be considered as the system of three-dimensional differential equations with known non-zero right-hand parts, which are determined by the harmonic temperature (3). The general solution of the system of equations (2) is represented as the sum of homogeneous and partial solution. It is known that partial solution is not usually determined uniquely, but with accuracy up to the unknown homogeneous solution.

While solving many practical and scientific problems of thermoelasticity, there is the need to find uniquely defined stress-strain state that depends only on temperature and does not contain elastic displacements. Therefore, we will make an appropriate separation of displacements in Cartesian coordinate system

$$u_j = u_j^e + u_j^\tau, \; j = \overline{1,3}, \tag{4}$$

where u_j^e are components of the elastic displacement vector (with index e), u_j^τ are components of temperature displacements (with index τ), which are uniquely determined by the temperature (3) and do not contain elastic displacements. Here, we do not investigate the effect of temperature on the stress state of the particular body as the result of taking into account the boundary conditions on its outer surface [6].

Definition. *A partial solution* u_j^τ, $j = \overline{1,3}$ *of the system of thermoelastic equations is called the temperature solution if it does not contain elastic displacements.*

Known partial solutions of the system of equations (2) in Cartesian coordinate system that do not contain elastic displacements are analyzed in paper [10]. It is determined that the following simple dependencies are true for these temperature solutions:

$$e^\tau = 3\alpha T, \; \Theta^\tau = 0, \tag{5}$$

where $\Theta^\tau = \sigma_1^\tau + \sigma_2^\tau + \sigma_3^\tau$ is the sum of normal stresses.

Theorem 1. *Equalities (5) are true for all temperature solutions of the system (2), (3).*

Proof. *Lit us write for the thermoelastic distribution of displacements (4)*

$$e = e^\tau + e^e, \; \Theta = \Theta^\tau + \Theta^e. \tag{6}$$

Let us present the known dependences between the elastic volume deformation and the sum of normal stresses, when the influence of temperature is not taken into account [6]

$$e^e = \frac{1-2\nu}{E}\Theta^{\mathrm{e}}, \tag{7}$$

and when the effect of temperature is taken into account [2, 4]:

$$e = \frac{1-2\nu}{E}\Theta + 3\alpha T.$$

Let us substitute relations (6), (7) into the last dependence and obtain

$$e^\tau = \frac{1-2\nu}{E}\Theta^\tau + 3\alpha T. \tag{8}$$

Since equality (8) includes dependencies (5) as a special case, the simultaneous fulfillment of conditions (5), (8) is possible only when the sum of normal stresses is zero. From this condition it follows: $\Theta^{\tau}=0$, $e^{\tau}=3\alpha T$. These equations are also true in the cylindrical coordinate system. The end of the proof.

Temperature solution of the system of equations (2) in Cartesian coordinate system when the temperature does not depend on the variable z was found in paper [10]: $T = T(x, y)$

$$u_j^t = \frac{\partial \vartheta}{\partial x_j} + \chi_1 \Omega_j, \; j = \overline{1,2}, \; u_3^t = \alpha z T, \; e^t = 3\alpha T, \tag{9}$$

where T, $\Omega_j = \int \mathrm{T} dx_j$, $j = \overline{1,2}$ are harmonic functions of two variables x, y, $\vartheta(x, y) = \chi(x\Omega_1 + y\Omega_2)$ is the main biharmonic function, $\chi = -\alpha/4$, $\chi_1 = 3\alpha/2$. It is also defined that the volume deformation and the sum of normal stresses of temperature solutions in Cartesian coordinate system satisfy relation (5).

2. Construction of the partial solution of the equations of thermoelasticity theory in the cylindrical coordinate system when the temperature does not depend on coordinate z.

Since we are considering the isotropic material, dependencies (5) will also be true in the cylindrical coordinate system. If the temperature does not depend on the variable z, then, it follows from physical considerations [10], that the strains and stresses in axis Oz direction are

$$\varepsilon_z^{\tau} = \alpha T, \; \sigma_z^{\tau} = 0. \tag{10}$$

In order to construct the partial solution of the system of equations (2), we take into account that the volume deformation from the temperature solution is invariant in orthogonal coordinate systems. Let us take into account dependencies (10) and write relation (5) in the cylindrical coordinate system:

$$\sigma_r^{\tau} + \sigma_{\varphi}^{\tau} = 0, \; \varepsilon_r^{\tau} + \varepsilon_{\varphi}^{\tau} = 2\alpha T. \tag{11}$$

Since the temperature and the partial solution of the system of equations (2) do not depend on the variable z, we introduce two-dimensional Laplace operator

$$\Delta_r = [\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}]. \tag{12}$$

2.1. Conversion of temperature displacements (9) into the cylindrical coordinate system. Let us write down the relationship between coordinates and displacements in the Cartesian and cylindrical coordinate systems [12].

$$x = r\cos\varphi, \; y = r\sin\varphi, \; x_3 = z,$$
$$\mathbf{i} = \cos\varphi e_r - \sin\varphi e_{\varphi}, \; \mathbf{j} = \cos\varphi e_{\varphi} + \sin\varphi e_r, \tag{13}$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$; $\mathbf{e}_r, \mathbf{e}_{\varphi}, \mathbf{e}_z$ are unit vectors in the Cartesian and cylindrical coordinate systems, respectively. Let us use dependencies (13) and write down the partial derivatives of the transition between these coordinate systems.

$$\frac{\partial r}{\partial x} = \cos\varphi, \quad \frac{\partial r}{\partial y} = \sin\varphi, \quad \frac{\partial \varphi}{\partial x} = -\frac{\sin\varphi}{r}, \quad \frac{\partial \varphi}{\partial y} = \frac{\cos\varphi}{r}. \tag{14}$$

The following expressions for displacements in the cylindrical coordinate system follow from relations (13):

$$u_r = \mathrm{u}\cos\varphi + v\sin\varphi, \; u_\varphi = v\cos\varphi - u\sin\varphi. \tag{15}$$

Relations (12)–(15) make it possible to transform the displacements expression (9) from the Cartesian coordinate system to the cylindrical coordinate system. Let us use relations (13), (14) and present integrals Ω_j in the cylindrical coordinate system

$$\Omega_1 = \int \mathrm{T}(r,\varphi) d(r\cos\varphi), \; \Omega_2 = \int \mathrm{T}(r,\varphi) d(r\sin\varphi). \tag{16}$$

Due to relations (9), (13), (15), (16), we define the functions that are included in the displacements (15) without taking into account the gradient part $\frac{\partial \vartheta}{\partial x_j}$, $j = \overline{1,2}$ in the representation (9)

$$\Omega_r = [\cos\varphi\Omega_1 + \sin\varphi\Omega_2], \; \Omega_\varphi = [\cos\varphi\Omega_2 - \sin\varphi\Omega_1]. \tag{17}$$

Let us use relations (9), (13), (16), (17), and find the expression of the basic function in the cylindrical coordinate system

$$\vartheta(r,\varphi) = \chi r \Omega_r(r,\varphi). \tag{18}$$

Let us take into account that components $\frac{\partial \vartheta}{\partial x_j}$ of the gradient vector in expressions (9) are invariant while replacing the orthogonal coordinate system [12]. Due to relations (9), (13), (15), (16)–(18), we determine the displacements in the cylindrical coordinate system

$$u_r^\tau = \chi r \frac{\partial \Omega_r}{\partial r} + \frac{5}{4}\alpha\Omega_r, \; u_\varphi^\tau = \chi r \frac{\partial \Omega_r}{\partial \varphi} + \frac{3}{2}\alpha\Omega_\varphi, \; u_3^t = \alpha z T. \tag{19}$$

Expressions (18), (19) include functions Ω_r, Ω_φ expressed by integrals (16), (17), which are difficult to calculate in the cylindrical coordinate system. In order to determine these functions, we are to solve the equilibrium equations of the thermoelastic body (2).

2.2. Solving the equations of thermoelasticity theory in the cylindrical coordinate system. To construct the partial solution of the system of equations (2), we take into account that the value of the volume deformation of the temperature solution is the invariant value in orthogonal coordinate systems. Let us substitute the value of the volume deformation (5) into system (2) and obtain the system of two dependent equations for the desired partial solution

$$\Delta_r u_r^\tau - \frac{2}{r^2}\frac{\partial u_\varphi^\tau}{\partial \varphi} - \frac{u_r^\tau}{r^2} = -\alpha \frac{\partial T}{\partial r},$$
$$\Delta_r u_\varphi^\tau + \frac{2}{r^2}\frac{\partial u_r^\tau}{\partial \varphi} - \frac{u_\varphi^\tau}{r^2} = -\alpha \frac{1}{r}\frac{\partial T}{\partial \varphi}. \tag{20}$$

The third equation of system (2) is satisfied by the displacements (19) identically. From the last equation (11) and the expression of deformations (1), we define

$$\frac{1}{r}\frac{\partial u_\varphi^\tau}{\partial \varphi} = -\frac{\partial u_r^\tau}{\partial r} - \frac{u_r^\tau}{r} + 2\alpha T. \tag{21}$$

Let us substitute relation (21) into the first equation (20) and get the following equation for determining the displacement u_r^τ

$$\Delta_r u_r^\tau + \frac{2}{r}\frac{\partial u_r^\tau}{\partial r} + \frac{u_r^\tau}{r^2} = \alpha \frac{4}{r} T - \alpha \frac{\partial T}{\partial r}. \tag{22}$$

It has been determined by mathematical calculations that the partial solution of equation is expressed by displacement (19), where the function $\Omega_r(r, \mathrm{z})$ has the following expression

$$\Omega_r = \int \mathrm{T} dr. \tag{23}$$

Let us use function (23) and present the expression of the radial displacement (19)

$$u_r^\tau = -\frac{1}{4}\alpha r T + \frac{5}{4}\alpha \int T dr. \tag{24}$$

Let us substitute expression (24) into equation (21) and find

$$\frac{\partial u_\varphi}{\partial \varphi} = \frac{5}{4}\alpha(rT - \int T dr) + \frac{1}{4}\alpha r^2 \frac{\partial}{\partial r} T. \tag{25}$$

Using representations (19), (25) we can write the expression of circular displacements

$$u_\varphi^\tau = -\frac{1}{4}\alpha \frac{\partial}{\partial \varphi}\int \mathrm{T} dr + \chi_1 \Omega_\varphi. \tag{26}$$

Let us substitute expression (26) into equation (25), and after mathematical transformations we find the integral function

$$\Omega_\varphi = r\int T d\varphi - \iint T dr d\varphi. \tag{27}$$

Let us present formulas (9), (24), (26), (27) together and after mathematical transformations express the temperature displacements in the cylindrical coordinate system

$$u_r^\tau = \frac{\alpha}{4}[5\int T dr - rT],\ u_\varphi^\tau = \frac{\alpha}{4}[6r^2 \int \frac{\partial T}{\partial r} d\varphi + 5\frac{\partial}{\partial \varphi}\int \mathrm{T} dr],\ u_3^t = \alpha z T. \tag{28}$$

Use relations (1), (10), (11), (28) we can find the temperature deformations

$$\varepsilon_r^\tau = \frac{\alpha}{4}(4T - r\frac{\partial T}{\partial r}),\ \ \varepsilon_\varphi^\tau = \frac{\alpha}{4}(4T + r\frac{\partial T}{\partial r}),\ \varepsilon_z^\tau = \alpha T,$$

$$\gamma_{r\varphi} = -\frac{\alpha}{2}\frac{\partial T}{\partial \varphi},\ \gamma_{rz} = \alpha z\frac{\partial T}{\partial r},\ \gamma_{z\varphi} = \frac{\alpha}{r} z\frac{\partial T}{\partial \varphi}. \qquad (29)$$

Let us substitute functions (29) into expressions (1) and determine the temperature stresses in the cylinder

$$\sigma_r^\tau = -G\frac{\alpha}{2} r\frac{\partial T}{\partial r},\ \sigma_\varphi^\tau = G\frac{\alpha}{2} r\frac{\partial T}{\partial r},\ \ \sigma_z^\tau = 0,$$

$$\tau_{r\varphi}^\tau = -\frac{\alpha}{2}G\frac{\partial T}{\partial \varphi},\ \ \tau_{zr}^\tau = \alpha G z\frac{\partial T}{\partial r},\ \ \tau_{z\varphi}^\tau = G\frac{\alpha}{r} z\frac{\partial T}{\partial \varphi}. \qquad (30)$$

We have determined two important patterns of temperature stress distribution in the cylindrical body: $\sigma_r^\tau + \sigma_\varphi^\tau = 0$, $\varepsilon_r^\tau + \varepsilon_\varphi^\tau = 2\alpha T$.

Theorem 2. *The general solution of the system of equations of thermoelasticity theory in the cylindrical coordinate system, when the temperature does not depend on the axial variable, can be represented as follows:*

$$u_r = \frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial Q}{\partial \varphi} + u_r^\tau,\ \ \ u_z = \frac{\partial P}{\partial z} - 4(1-\nu)\Phi + \alpha z T,$$

$$u_\varphi = \frac{1}{r}\frac{\partial P}{\partial \varphi} - \frac{\partial Q}{\partial r} + u_\varphi^\tau, \qquad (31)$$

where $P = z\Phi + \Psi$; $\Phi(r,\varphi,z)$, $\Psi(r,\varphi,z)$, $Q(r,\varphi,z)$ are harmonic functions of displacements [11].

Proof. Representation of the general stress state in the cylindrical coordinate system in elastic body is given in [11], let us add temperature solution (18), (19), (23), (27) to it and obtain formulas (31). The end of the proof.

Using displacement (31) and temperature stresses (30), we find the expression of the total thermoelastic stresses in the cylindrical coordinate system

$$\sigma_r = 2G[\frac{\partial^2 P}{\partial r^2} - 2\nu\frac{\partial \Phi}{\partial z} + \frac{\partial}{\partial r}\frac{\partial Q}{r\partial \varphi}] - G\frac{\alpha}{2} r\frac{\partial T}{\partial r},$$

$$\sigma_\varphi = \frac{2G}{r}[\frac{1}{r}\frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial P}{\partial r} - 2\nu r\frac{\partial \Phi}{\partial z} - r\frac{\partial}{\partial r}\frac{\partial Q}{r\partial \varphi}] + G\frac{\alpha}{2} r\frac{\partial T}{\partial r},$$

$$\sigma_z = 2G[\frac{\partial^2 P}{\partial z^2} - 2(2-\nu)\frac{\partial \Phi}{\partial z}],$$

$$\tau_{rz} = G\{\frac{\partial}{\partial r}[2\frac{\partial P}{\partial z} - \chi\Phi] + \frac{1}{r}\frac{\partial}{\partial\varphi}\frac{\partial Q}{\partial z} + \alpha G z\frac{\partial T}{\partial r}\},$$

$$\tau_{r\varphi} = G\{\frac{2}{r}\frac{\partial^2 P}{\partial r\partial\varphi} - \frac{2}{r^2}\frac{\partial P}{\partial\varphi} - r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial Q}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Q}{\partial\varphi^2} - \frac{\alpha}{2}\frac{\partial T}{\partial\varphi} - \frac{\alpha}{2}\frac{\partial T}{\partial\varphi}\},$$

$$\tau_{z\varphi} = G\{\frac{1}{r}\frac{\partial}{\partial\varphi}[2\frac{\partial P}{\partial z} - \chi\Phi] - \frac{\partial^2 Q}{\partial r\partial z} + \frac{\alpha}{r}z\frac{\partial T}{\partial\varphi}\},$$

where $\chi = 4(1-\nu)$

Conclusions. It has been determined that in the cylindrical coordinate system, the sum of normal temperature stresses is zero, and the volume deformation is equal $e = 3\alpha T$; if the temperature does not change in some spatial direction, then in this direction the normal temperature stresses are equal to zero. Simple dependences (29), (30) are obtained for determining the temperature normal deformations and stresses in the cylindrical coordinate system. The expression of the total thermoelastic displacements and stresses in the cylindrical coordinate system is found. The obtained formulas make it possible to solve the problems of determining the thermoelastic state of bodies.

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ЗНАХОДЖЕННЯ ФІЗИЧНО ОБҐРУНТОВАНИХ ЧАСТКОВИХ РОЗВ'ЯЗКІВ РІВНЯНЬ ТЕОРІЇ ТЕРМОПРУЖНОСТІ В ЦИЛІНДРИЧНІЙ СИСТЕМІ КООРДИНАТ

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Резюме. *Знайдено нові розв'язки теорій термопружності в циліндричній системі координат. Для описування термопружного стану використано лінійну статичну модель тривимірного ізотропного тіла під дією стаціонарного температурного поля за відсутності об'ємних сил. Розглянута модель деформованого тіла базується на поданні переміщень і напружень через чотири гармонічні функції, три функції описують пружний стан, а температура описує чисті температурні переміщення. Використано співвідношення Дюамеля–Неймана для подання термопружних напружень в однорідному твердому тілі. Розглянуто випадок, коли стаціонарна температура задовольняє рівняння Лапласа. Після підстановки термопружних напружень у рівняння рівноваги термопружного тіла отримано систему диференціальних рівнянь Нав'є в частинних похідних другого порядку на пружні й температурні переміщення. Загальний розв'язок подано у вигляді суми однорідного й часткового розв'язків. Частковий розв'язок системи рівнянь Нав'є, який явно визначається стаціонарною температурою і не містить у собі пружних переміщень, названо температурним. Переміщення, деформації й напруження, які визначаються цими температурними розв'язками, названо температурними. Використано фізичні й математичні особливості термопружного напруженого стану й доведено, що для температурних розв'язків сума нормальних напружень дорівнює нулю, а об'ємне розширення рівне* $e = 3\alpha T$ *. Використано знайдені залежності й записано новий температурний розв'язок системи рівнянь термопружності в циліндричній системі координат, коли температура не залежить від осьової змінної. Отоимано прості формули для вираження температурних напружень. Побудовано загальний розв'язок рівнянь теорії термопружності через чотири гармонічних функції, коли температурне поле задається двовимірною гармонічною функцією.*

Ключові слова: *циліндрична система координат, термопружний стан тіла, фізичні характеристики температурного стану, температурні напруження й переміщення.*

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