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## MATHEMATICAL MODELING OF THE CLARIFIER PERFORMANCE FOR WATER COAGULATION

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**Abstract.** This study explores the performance optimization of clarifiers by examining geometric parameters, flow velocity, and sludge characteristics. Mathematical models predict system behavior under various conditions, aiding in stable operation through optimized temperature, reagent dosing, and sludge management. An experimental approach identifies optimal reagent and temperature settings for water purification, ensuring consistent results. Key performance indicators-water turbidity, suspended solids concentration, and sludge settling velocity-are linked to operational variables. Maintaining stable hydraulics is critical to prevent sludge carryover. The proposed equations facilitate performance evaluation and guide further clarifier design and automation improvements.

**Key words:** Clarifier, coagulation unit, dosing pump, reagents.

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### 1. INTRODUCTION

Special clarifiers are used to implement chemical and hydrodynamic regimes in the precipitation processes when treating water with coagulants, slaked lime, soda, and magnesite. This paper presents a mathematical model for calculating the performance of a water clarifier for coagulation, which takes into account the main physical and chemical characteristics of water and sludge.

In Ukraine, scientists are actively researching water treatment issues, focusing on the development of mathematical models that take into account the physical and chemical characteristics of water and sludge. For example, Yuriy Vlasenko from the National University of Water and Environmental Engineering is researching water treatment processes, particularly in the context of their application in real-world conditions. At the Kyiv Polytechnic Institute, Olena Kovaleva specializes in water treatment technologies, working to improve models that optimize technological processes.

In Europe, research in this area is led by scientists such as Christian Kazner in Germany, who focuses on coagulation processes and water treatment technologies. His work is aimed at developing integrated approaches to water treatment that take into account complex physical and chemical interactions. In France, Luc Brissot works on the analysis of filtration processes, and in the Czech Republic, Jiri Wanner is engaged in modeling wastewater treatment processes, including the use of mathematical models to improve the efficiency of water treatment systems. These scientists are investing significant efforts in the development of theoretical and practical aspects of water treatment, developing models that allow not only to understand but also to predict the behavior of water treatment systems in constantly changing conditions. Their research contributes to improving existing technologies and creating new solutions that make the coagulation process more efficient and economically viable.

A clarifier is an apparatus in which chemical reactions associated with the introduction of reagents, as well as physical processes of formation of sediments (sludge) in the clarifier

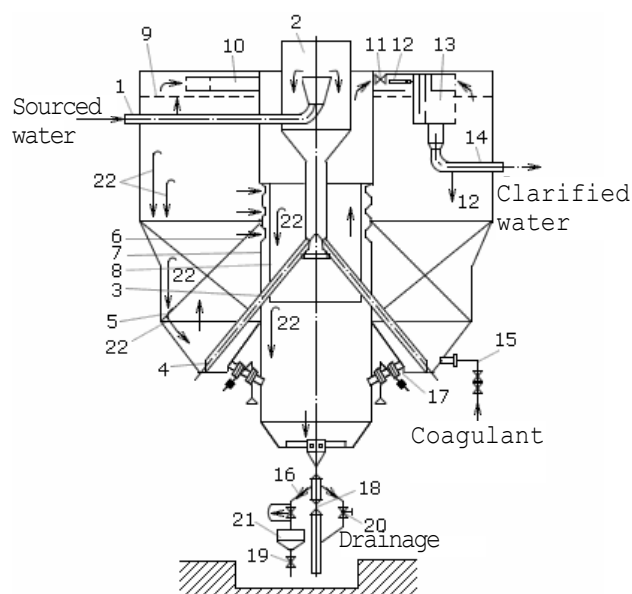
water volume and filtering of treated water through their layer, simultaneously take place. The contact medium in the clarifier, called a sludge filter, is formed from sludge particles that are suspended due to the action of the outgoing water flow. The water that passes through the sludge filter is freed from coarse particles contained in the source water and formed as a result of chemical reactions in the clarifier. Therefore, their residual concentration is usually in the range of 5–10 mg/l, if the chemical and hydraulic conditions in the clarifier are not disturbed.

The design of the clarifier takes into account the hydraulic processes in it, which include the following components:

- maintaining suspended solids that form the contact medium by the upward flow of water;
- removal of excess particles from the contact medium zone;
- modes of water movement in the contact medium, as well as in the inlet and outlet parts of the clarifier.

In the clarifier, water and reagents are mixed, sludge is formed and separated from water, i.e. clarification. From the clarifier, water flows into the coagulated water tank (CWT), and the separated sludge into the sludge water tank, from where it is pumped to a sludge outlet or sludge dewatering plant. From the BWT, pumps supply water to clarifying (mechanical) filters, where it is finally freed from those small amounts of suspended solids (5–10 mg/dm<sup>3</sup>) that came from the clarifier to the coagulated water. The retained suspended solids are removed from the mechanical filters during the next flushing with a reverse water flow. Rinse water is collected in a special tank (BWMF) and pumped evenly into the clarifier throughout the day.

In addition to saving water, heat, and reagents, such measures allow for the intensification of flaking in low turbidity feed water due to the favorable effect of the suspension contained in the return wash water.



**Figure 1.** Diagram of the clarifier for coagulation of the TsNDI-2 type:

- 1 – water inlet; 2 – air separator; 3 – distribution pipes; 4 – nozzles; 5 – vertical perforated partitions; 6 – sludge inlet windows; 7 – sludge traps; 8 – «glass»; 9 – upper distribution grate; 10 – prefabricated ring gutter; 11 – cut-off valve; 12 – cut-off pipe; 13 – distributing device with a calming baffle; 14 – exit of treated water; 15 – injection of coagulant; 16 – continuous blowing line; 17 – periodic blowing line; 18 – liquidator emptying valve; 19 – drainage valve; 20 – crane with servo; 21 – tare funnel; 22 – sampling points

At present, many types of clarifiers are used at various water treatment plants. Basically there are no fundamental differences in the designs, but the water velocities in different zones

of the clarifier are different. This is due to the characteristics of the sludge formed in clarifiers: during coagulation, it is lighter, so the speed of the upward movement of water should be lower than during liming, since the speed of the outward movement of water in the clarifier should not exceed the speed of particle deposition to avoid carrying them out of the clarifier by the water flow. Clarifiers for liming, such as of the VTI type, can be used for coagulation with aluminum sulfate, but taking into account the above calculation, the capacity of such clarifiers is reduced to 0.7 of the nominal capacity for liming. The scheme of the clarifier for coagulation is shown in Fig. 1.

The total capacity of clarifiers, clarified water tanks and pumps should be selected with a margin of 10% of the calculated capacity of the treatment plant. In addition to the requirement to stabilize the temperature of the treated water within  $+1^{\circ}\text{C}$ , the operation of VTI-type clarifiers for liming requires the following conditions:

- limiting the content of suspended solids in the source water during floods to 800 mg/l, and at other times of the year to 200 mg/l with a total amount of sedimentation of up to 1500 mg/l;

- maintaining the mass ratio  $\alpha_m$ , which is the content of magnesium compounds in the sludge in terms of  $\text{Mg}(\text{OH})_2$  to the content of calcium compounds in terms of  $\text{CaCO}_3$ , within 8–15%. The mass amount of  $\text{Mg}(\text{OH})_2$  in the sludge is calculated as (when substituting the corresponding concentrations in equivalent units):

$$m(\text{Mg}) = \frac{\mu(\text{Mg}(\text{OH})_2)(C_{\text{start}}(\text{Mg}) - C_{\text{end}}(\text{Mg}))}{n(\text{Mg})} \approx 29(C_{\text{start}}(\text{Mg}) - C_{\text{end}}(\text{Mg}))$$

and  $\text{CaCO}_3$  as

$$m(\text{Ca}) = \mu(\text{CaCO}_3)(C_{\text{start}}(\text{Alc}) - C_{\text{end}}(\text{CO}_2)) \approx 100 C_{\text{start}}(\text{Ca}) - 50 C_{\text{end}}(\text{Ca})$$

where  $\mu(\text{Mg}(\text{OH})_2)$   $\mu(\text{CaCO}_3)$  – molar mass of  $\text{Mg}(\text{OH})_2$  and  $\text{CaCO}_3$  respectively,  $n(\text{Mg})$   $n(\text{Ca})$  – oxidation state of magnesium and calcium.

Hence.

$$\alpha_m = \frac{m(\text{Mg})}{m(\text{Ca})} = \frac{29(C_{\text{start}}(\text{Mg}) - C_{\text{end}}(\text{Mg}))}{100 C_{\text{start}}(\text{Alc}) + 50 C_{\text{end}}(\text{CO}_2)} \times 100 = \frac{58 \Delta(\text{Mg})}{2 C_{\text{start}}(\text{Alc}) + C_{\text{end}}(\text{CO}_2)}$$

where  $\Delta(\text{Mg})$  is the number of ions in the  $\text{Mg}_2+$  precipitate in mg-eq/l,

$$\Delta(\text{Mg}) = \alpha_m \frac{2 C_{\text{start}}(\text{Alc}) + C_{\text{end}}(\text{CO}_2)}{58}$$

The ratio allows us to calculate the magnesium component of water hardness  $C_{\text{end}}(\text{Mg})$ , and from the calculated value of  $C_{\text{in}}(\text{Mg})$  to determine the value of  $C_{\text{in}}(\text{Ca})$ . Using the data and calculation results, taking the value  $\alpha_m = 10\%$ , we obtain

$$\Delta(\text{Mg}) = 10 \frac{2 \cdot 3.0 + 0.16}{58} = 1.06$$

So, taking into account that the water hardness is almost completely equal to the sum of the hardness of magnesium and calcium, the water hardness for magnesium is equal to

$$F(Mg) = F_{start} - F(Ca) = 3.8 - 3.0 = 0.8 \text{ mg-eq/l},$$

the liming regime should be carried out with complete magnesium precipitation:

$$F(Ca) = F_{end} = 0.8 \text{ mg-eq/dm}^3$$

## 2. REAGENT DOSING

Recommended C values for reagents used for pretreatment are given in Table 1. To ensure reliable operation of the dosing pumps and the required dosing accuracy, it is necessary to maintain an uninterrupted supply, ensure the consistency of the concentration of dosed liquids and regularly flush the dosing pumps with clarified water for 5–7 minutes.

Dosing pumps of the NDE series are characterized by automatic flow control, other designations are similar to those for the ND series. Automatic control of the reagent dosing into the clarifier is carried out by dosing pumps with different control systems based on the flow rate of the treated water within the clarifier load from 50 to 100%. To increase the accuracy, a pH correction signal is introduced into the lime dosing scheme based on the treated water flow rate.

**Table 1**

Values of working concentrations of reagents for pretreatment

Reagent	Solution or suspension concentration	
	mg-eq/dm <sup>3</sup>	% by anhydrous product (approximate)
Lime milk	200–2000	0.6–6 for CaO
Ferrous sulfate iron	200–2000	1.5–15
Aluminum sulfate	250–2500	1.3–13
Polyacrylamide	10000	1

**Table 2**

Approximate change in water quality and seasonal doses of reagents for pretreatment

Indicator	Winter	Spring (flood period)	Summer
Total hardness, mg-eq/l	7.8	2.0	4.5
Sodium concentration, mg/l	12	5	8
Total alkalinity, mg-eq/l	4.8	1.8	3.2
Sulfate concentration, mg/l	110	70	95
Chloride concentration, mg/l	13	6	10
Silicon content, mg/l	12	4	5
Oxidizability, mg O <sub>2</sub> /l	4	25	5
Iron content, mg/l	0.6	0.8	0.4
Salt content, mg/l	500	280	380
Coagulant dose, mg-eq/l	0.2–0.5	1.0	0.2–0.8
Lime dose, mg-eq/l	7.0	3.0	5.0

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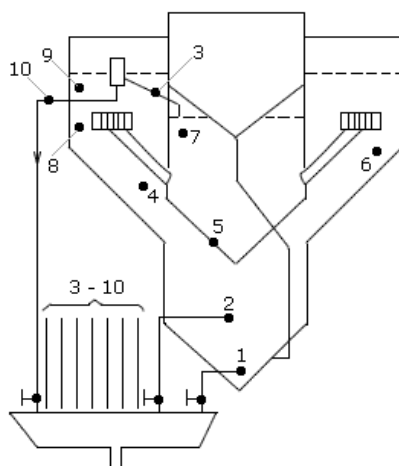
In case of seasonal changes in surface water quality (Table 2), the dose of reagents supplied to the clarifier can be adjusted, in addition to changing the stroke of the plunger of the dosing pump, by changing the concentration of the dosing reagent in the outlet tanks (mixers). The combination of the three possible types of reagent dose control simultaneously allows you to change the dose within a very wide range.

### 3. OPERATION OF INSTALLATIONS WITH CLARIFIERS

Two types of control are carried out during the operation of the clarifiers: technological and chemical, which are carried out according to a set schedule. Their results are compared with the data of the mode cards and, if the monitored indicators deviate from the recommended values, the operating personnel must restore the optimal operation of the clarifiers.

To control the dosage of reagents, control the quality and quantity of sludge in various elements and areas of the clarifier, and control the quality of clarified water at the outlet of the sludge compactor and clarifier, the latter are equipped with sampling points, the location of which is shown in Fig. 2.

Sampling point 1 is intended for sampling from the mud pit; 2 – from the reaction zone; 3 – from the cut-off pipeline; 4 – from the middle of the conical part of the clarifier; 5 – from the sludge seal, lower; 6 – from the cylindrical part of the clarifier, lower; 7 – from the sludge seal, upper; 8 – from the clarifier body at the level of the sludge intake windows; 9 – from the clarification zone; 10 – from the clarified water pipeline.



**Figure 2.** Diagram of the location of the sampling points on the clarifier:  
1-10-point numbers

During the clarifier operation, samples are taken from all sampling points every 2 hours for visual control of sludge quality and chemical control of the process and clarified water quality.

At plants with clarifiers, operational process control is provided for:

- preparation of working solutions of lime milk, coagulant, PAA and, if necessary, alkali;
- maintaining a constant temperature of the treated water of 30–40°C with fluctuations of no more than +1°C;
- maintaining the mode of possible load changes. A sharp increase in water consumption leads to sludge removal. When increasing the load, the water flow rate should be increased smoothly, without jumps of more than 10% within 10–15 minutes. Reducing the load should be done quickly;
- maintaining the normal level of clarified water in the tanks;
- strict maintenance of the established dosages of the reagents introduced into the clarifier;
- maintenance of the purge mode is carried out by means of continuous and periodic purging and changing the water cutoff to the sludge compactor;
- maintenance of the upper limit of the sludge filter at the level of the sludge intake windows;
- maintenance of the required sludge characteristics. sludge should have a curd-like structure and be homogeneous with the entire mass settling simultaneously when controlling its quality in the sample;
- during visual control of sludge quality, water with sludge is sampled in a cylinder with a diameter of 40–60 mm and a height of 250–350 mm. After a certain settling time, the height of the sludge column is measured;
- the height of the sludge filter is adjusted automatically using a sludge level indicator of the SPS or manually by changing the «cutoff» on the sludge seal and continuously blowing it.

When the sludge filter limits are exceeded, the continuous blowdown increases, while the «cut-off» is fully open. When the upper limit of the sludge filter is lowered, the continuous blowdown decreases, and the cutoff is restored. The change in continuous blowing is carried out smoothly: no more than 10% in 10–15 minutes. When the clarifier is loaded below the rated load, the level of the sludge filter may fall below the level of the sludge intake windows, which is determined by the absence of sludge in the water of the corresponding sampling point. In this case, the continuous purge is completely closed until the sludge appears at the level of the sludge inlet windows.

The scope of chemical control depends on the type of clarifier; the frequency of monitoring for various water quality indicators (COD, TOC, DO, DO, TSS, etc.) varies from 2 to 24 hours. All records of analyzes and observations of the clarifiers' operation are entered into the «Daily Statement of the Clarifiers' Operation». Based on the results of chemical analysis of the samples, the optimal operation mode of the clarifiers is established and adjusted.

#### 4. RESEARCH METHODOLOGY

1. Collect the source water in a container made of polyethylene, stainless steel or other material that does not contaminate the water with silicic acid, iron or other impurities. It is advisable to take as much water at once as will be needed for one day of work.

2. The experiments are carried out in glass cylinders with a capacity of 0.5–1.0 liters. To determine the conditions for removing unreacted silicic acid, use polyethylene or plexiglass vessels of the same capacity. Reagents are dosed in the form of solutions, measuring their estimated amounts with pipettes.

3. First, determine the optimal dose of the coagulant (1st series of experiments) by creating approximately the same pH value with insufficient alkalinity reserve by

introducing an alkali solution to the same residual alkalinity of 0.5 mg-eq / l at different doses of the coagulant. Then, the optimal pH value of the medium is determined (2nd series of experiments) by introducing the optimal dose of coagulant everywhere. Next, at the optimal coagulant dose and pH value, the flocculant dose is determined (3rd series of experiments), then the chlorine dose (4th series of experiments). In the 1st and 2nd series of experiments, neither flocculant nor chlorine is introduced; the 3rd and 4th series of experiments are conducted if it is necessary to intensify the process of flake formation or remove impurities from the water. All experiments are conducted at the same temperature and under the same mixing and filtering conditions. The temperature can be room temperature if it is 25–30°, otherwise, the experiments are carried out with a thermostatically controlled vessel with the treated water heated to a strictly defined temperature corresponding to its calculated value for an industrial plant. If necessary, an additional series of experiments is carried out to clarify the value of the optimal water temperature (other conditions are optimal, determined in the experiments of series 1–4).

4. The sequence of reagent introduction (only those reagents that are planned for each specific series of experiments are introduced) is taken as follows: acid, bleach (or chlorine water), alkali; after the introduction of each reagent, the liquid is stirred for 30 s, the last to be introduced is the coagulant. After the introduction of all the reagents used in this experiment, the water and reagents are stirred for 5 minutes, first at a speed of 3–4 rpm for 3 minutes, and then for 2 minutes at a speed of 1–2 rpm so as not to allow the flakes to settle. In experiments with flocculant, it is added after 3 min of mixing water with the rest of the reagents and then mixed for another 2 min with the flocculant. After mixing, the suspension is allowed to settle, the water is filtered through a pleated filter and analyzed. The optimal dose of the reagent in each series of experiments is the minimum dose of the reagent at which sufficiently good coagulated water quality indicators are obtained.

**Table 3**

Composition of experiments of the 1st series

Cylinder number	1	2	3	4	5	6
$D_k$ mg-eq/l	0,4	0,6	0,8	1,0	1,2	1,4

**Table 4**

Composition of experiments of the 2nd series

Cylinder number	1	2	3	4	5	6	7
Approximate pH value of coagulated water	5,4	5,8	6,2	6,6	7,0	7,5	7,8

The dose of coagulant in all experiments is optimal, determined by the results of experiments of the 1st series. The dose of alkali (or acid) is experimentally selected to ensure that the specified pH values are maintained.

**Table 5**

Composition of experiments of the 3rd series

Cylinder number	1	2	3	4	5	6
Flocculant dose, mg/l polymer	0,1	0,2	0,4	0,6	0,8	1,0

The coagulant dose and pH value in all experiments are optimal, according to the data of experiments 1 and 2 of the series.

**Table 6**

Composition of experiments of the 4th series

Cylinder number	1	2	3	4
Dose of active chlorine, mg/l	5	10	15	20

Coagulant dose and pH value in all experiments were the same as for experiments of series 3.

## 5. RESEARCH RESULTS

### 1. The performance of the clarifier under the condition of sufficient water clarification

Calculation of clarifier performance for water coagulation.

$$Q = 3.6 F_{cs} v_0 \text{ m}^3/\text{hour},$$

where  $F_{cs}$  is the cross-sectional area of the contact medium zone, m;

$2 F_{cs} = F_c - F_s \text{ m}^2$ ,  $F_c$  and  $F_s$  are the cross-sectional areas of the clarifier and sludge compactor, respectively,  $\text{m}^2$ ;  $v_0$  is the calculated upward velocity of water movement at the outlet of the clarifier contact medium zone, mm/s;

$$v_0 = (0.6 + 0.02t)(1 - 3.5C_0)v_p \text{ m/s},$$

where  $t$  is the water temperature in the range of 1–30°C;  $C_0$  is the volume concentration of sludge in the contact zone of the clarifier, ml/ml;  $v_p$  is the conditional free deposition rate of sludge particles, mm/s;

$$C_0 = \frac{1}{\alpha_c} \sqrt{\frac{Tr}{(100 + 1.65t)H_c}}, \quad Tr = (100 + 1.65t)\alpha_c^2 C_0^2 H_c \text{ sm},$$

where  $Tr$  is the cross-sectional water transparency, cm;  $\alpha_c$  is the coefficient of modeling the suspension concentration in the clarifier contact medium;  $H_c$  is the reduced height of the clarifier contact medium zone, cm.

The value of  $Tr$  and the residual turbidity of water at the outlet of the clarifier  $M_0$  are in the following approximate relationship:

$$M_0 = 880 Tr \text{ mg/l}$$

For the calculation, it is recommended to take  $Tr = 300 \text{ cm}$ ; in the least favorable short-term periods of the year, the minimum allowable value is  $Tr = 150 \text{ cm}$ .



**Table 7**

The value  $a_c$  is recommended depending on the diameter of the clarifier:

$D$ m	4	6	8	10	12	14	16
$a_c$	1,0	0,93	0,88	0,85	0,82	0,8	0,78

$$H_c = H_1 + H_2 \frac{F_2}{F_1} + \dots + H_n \frac{F_n}{F_1} \text{ cm}$$

$H_1$  – the height of the contact medium in its upper zone, having an area ( $\text{m}^2$ ) of cross section  $F_1$  sm.  $H$  and  $F$  with other indices height (cm) and area ( $\text{m}^2$ ) of the cross-section in other areas of the contact medium (with a conical shape – the average area).

When determining  $H_c$ , the upper boundary of the contact medium zone can be taken at the level of the upper edge of the upper sludge intake window, and it is also taken into account that the zone of mixing of water and reagents at the bottom of the clarifier above the water nozzles is 0.5–0.7 m in height.

Check the calculation of the clarifier performance in the following order:

- a. the initial content of the suspension is determined by analyzing the source water ( $M$  mg/l) for typical periods of the year;
- b. according to the data of experimental treatment or approximately by calculation, the value of  $D_k$  mg-eq / l is determined;
- c. calculate the values of  $\alpha_k$  for the main periods of the year and for them, determine the values of  $v_F$ ;
- d. the values of  $F_{cs}$  and  $H_c$  are determined according to the drawings of the clarifier;
- e. the value of  $C_0$  is calculated. It is necessary to comply with the desired value of  $Tr$ ;
- f. the value  $v_0$  is calculated, and  $Q$  is calculated from it.

## **2. Permissible value of water turbidity according to the conditions of sludge discharge into the sludge compactor**

The amount of sludge – sediment formed during the coagulation of water is approximately defined as:

$$K_u = M + 26 D_k \text{ mg/l.}$$

The amount of sludge that can be diverted to the sludge compactor:

$$K_u = 10(B + q_{np})\gamma_0 C_0 \text{ mg/l,}$$

where is the initial slurry content, mg/l;

$D_k$  is the coagulant dose, mg-eq/l;  $B$  is the cross-sectional area of the sludge compactor clarification zone, in % of area cross-section of the clarifier contact zone (determined according to the clarifier drawings);

$q_{np}$  – sludge seal blowdown, in % of clarifier capacity;

$\gamma_0$  – conditional volume of sludge particle weight, mg/ml).

It is required that  $K_u$  be no more than  $K_u'$ . Having determined  $K_u$ , calculate the permissible maximum value of the suspended solids content in the source water. The calculated permissible value of the suspended solids content in the feed water  $M_{\max}$  should not be less than the actual value determined from the analysis. If this condition is not met, it will be necessary to increase the clarifier blowdown. Such an increase is possible if the size of the purge does not exceed economically acceptable limits (usually up to 3%). Otherwise, it will be necessary to apply PAA dosage or reduce the calculated clarifier capacity so that the reduced upward water velocity provides the required value  $C_o$ .

The procedure for determining the reduced value  $Q$ :

- the required value of  $C'_o$ ;  $C'_o = \frac{K_u'}{10(B + q_{np})\gamma_o}$
- permissible value  $v'_o$   $v'_o = (0.6 + 0.02t)(1 + 3.5C'_o)v_p$  mm/s;
- permissible value:  $Q' = 3.6F_{cs}v'_o$  m<sup>3</sup>

### 3. Calculated size of the clarifier purge

Estimated size of the clarifier purge at a load of  $Q$ , m<sup>3</sup>/h  $q_{np} = \frac{K_u}{1000\gamma_1} Q$  m<sup>3</sup>/h, or in %  $q_{np} = \frac{K_u}{10\gamma_1}$

And from  $Q$  we can receive  $\gamma_1$  – weight content of the solid phase in the compacted sediment, g/ml.  $\gamma_1 = \frac{a p_1 \tau}{2.31 \lg \left( \frac{1 + a p_1 \tau}{\gamma_o} \right)}$  g/ml, where  $p_1 = 0.6 \frac{K_u Q}{10 F_s}$  g/cm<sup>2</sup>,  $a$  – sediment compaction coefficient, cm<sup>-1</sup> (The values of  $a$  depending on  $a_c$  are taken from Table 8);  $\tau$  – sediment accumulation time, hours

$$\tau \cong \frac{W_s}{q_{np}};$$

$F_s$  – is the cross-sectional area of the sludge compactor in the sludge compaction zone, m<sup>2</sup>;

$K_u$  is the amount of sludge water released during coagulation, g/l;

$W_s$  is the volume of the compaction zone in the sludge compactor, m<sup>3</sup> (Accepted according to the clarifier drawings). Compaction zone – the volume of the sludge compactor located 0.5–0.75 m below the bottom of the junction of the sludge outlet pipes to the sludge compactor or the bottom of the inner glass of the sludge compactor.

The value of  $\gamma_1$  is determined by selection. Given the value of  $q_{np}$ ,  $\tau$  is determined, and given the value of  $\tau$  and  $\gamma_1$  is determined. Then, according to the obtained value of  $\gamma_1$ ,  $q_{np}$  is determined. If the obtained value does not match the initially accepted value, the calculation is repeated.

The actual value of  $\gamma_1$  is often significantly lower, and  $q_{np}$  is higher than the calculated value. Usually the actual value of  $q_{np}$  is 2, sometimes 3%.

## 6. DETERMINATION OF PHYSICAL PROPERTIES

A 250 ml volumetric cylinder is used for the determinations. A scale made of millimeter paper with a height of 300 mm is glued to the cylinder. The divisions are numbered every 10 mm in the direction from bottom to top. Before taking the sample, the sampling line is purged, the flow rate is set to 300–400 ml/min and the sample is taken after 10 minutes.

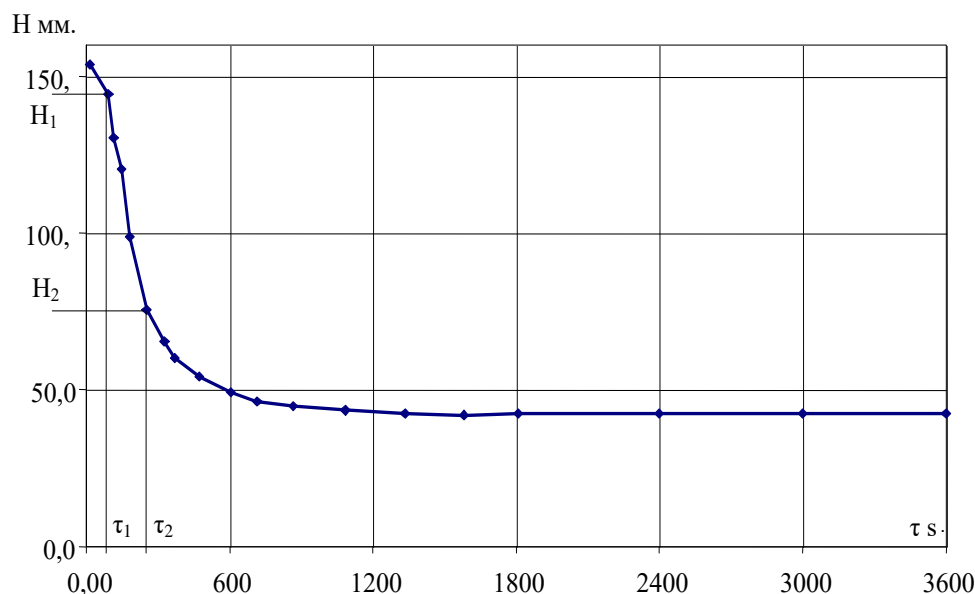
Determination of the weight concentration of the suspension in the contact medium ( $C_w$ ) and in the purge water ( $\gamma_l$ ).

Samples of liquid from the contact medium area ( $W_1$  ml) and the slurry sealer purge water ( $W_2$  ml) are taken in an amount of 100–200 ml. The sample volume must be accurately measured. Filter the samples through pre-weighed paper filters. The filters with the precipitate are brought to a constant weight and weighed. The weight of the precipitate (excluding the weight of the filters) is denoted by  $P_l$  and  $P_2$ , M, respectively.

Weight concentration of the suspension:

a) in the contact environment  $C_w = \frac{10^3 P_l}{W_1}$  g/l;

b) in purge water  $\gamma_l = \frac{10^3 P_l}{W_2}$  g/l.



**Figure 3.** Graph of sediment layer height change over time during settling

Determination of the volume concentration and conditional free sedimentation rate of sludge particles

Take a sample of liquid from the contact zone into a cylinder, note the initial height of the sludge layer ( $H_0$  mm) and then observe its deposition; during the first 10 minutes, readings are made every minute, then after 5–10 minutes, and then record the final height of the sludge layer after 60 minutes ( $\tau_k$ ) from the start of observations ( $H_k$  mm). The

temperature of the liquid during the experiment should be constant and equal to the temperature of the water in the clarifier. Draw a graph (Fig. 3) of the change in the height of the sediment layer over time and mark two points on the straight line of the graph with the coordinates  $H_1$  and  $\tau_1$  at the beginning of the deposition curve and  $H_2$  and  $\tau_2$  at the end of the straight line.

Calculate:

a) rate of constrained deposition of sludge particles:  $v_l = \frac{H_1 - H_2}{\tau_2 - \tau_1}$  mm/s;

b) volume concentration of sludge  $C_0 = \frac{W_k}{W_0} = \frac{H_k}{H_0}$  ml/ml, where  $W_0$  and  $W_k$  – respectively, the initial and final volume of sediment in the cylinder, ml.

The approximate value of the conditional free deposition rate is calculated by the equation  $v_p' = \frac{v_l}{1 - 3.5 C_0}$  mm/s.

The value  $v_p'$  refers to the temperature at which the observation was made (in the range  $t = 1 \pm 30^\circ\text{C}$ ).

Recalculation to the values of  $v_p$  at a temperature of  $20^\circ\text{C}$ , for which the dependence on  $\alpha_k$  is indicated, is carried out according to the equation  $v_p = \frac{v_p'}{0.6 + 0.02t}$  mm/s, where  $t$  is the water temperature at which the deposition of sludge particles in the cylinder was observed.

## 7. CONCLUSIONS

The study presents a mathematical model for calculating the performance of a clarifier for water coagulation, which takes into account the main physical and chemical characteristics of water and sludge. The analysis allows us to draw the following conclusions:

1. The calculation of clarifier performance is based on the geometric parameters of the structure, the upward flow rate and sludge characteristics. The resulting mathematical expressions allow us to predict the behavior of the system under various operating conditions.

2. To ensure the stability of the clarifier, it is necessary to maintain optimal temperature conditions, reagent dosage and sludge characteristics. The proposed control system ensures the efficiency of the coagulation process by timely adjusting the parameters.

3. A sequence of experimental studies has been developed to determine the optimal doses of reagents (coagulants, flocculants and chlorine) and temperature conditions that contribute to maximum water purification. This approach ensures reproducibility of results and accuracy of measurements.

4. The main indicators of clarifier performance are the clarified water turbidity, suspended solids concentration, and sludge deposition rate. The dependences of these parameters on temperature, reagent dose, and design characteristics of the clarifier were determined.

- For the efficient operation of clarifiers, it is important to ensure the stability of hydraulic modes, as sudden changes in water flow can lead to sludge removal.

- We recommend adjusting the height of the sludge filter automatically or manually, depending on the specific operating conditions.

- Adapting the operating modes of the clarifier to different types of water and seasonal conditions allows you to maintain optimal cleaning quality.

5. The proposed equations make it possible not only to determine the performance of the clarifier but also to evaluate its performance under critical conditions. This is the basis for further improvement of clarifier designs and their automation.

The results of the research can be used in the design of new coagulation water treatment plants, as well as in the modernization of existing systems to improve their efficiency.

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## **МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОДУКТИВНОСТІ ОСВІТЛЮВАЧА ДЛЯ КОАГУЛЯЦІЇ ВОДИ**

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**Резюме.** Представлено комплексний аналіз продуктивності відстійників, який наголошує на важливості геометричних параметрів, швидкості висхідного потоку й характеристик осаду у визначенні ефективності системи. Завдяки розробленні математичних виразів можна прогнозувати поведінку сепараторів за різних умов експлуатації, що полегшує про активне управління системою.

Ключовим фактором для підтримання стабільної продуктивності освітлювача є оптимізація температурних режимів, дозування реагентів і параметрів осаду. Запропонована система управління підвищує ефективність коагуляції, дозволяючи своєчасно регулювати ці параметри, забезпечуючи стабільну роботу. Розроблено експериментальну методику для визначення оптимальних дозувань реагентів, включаючи коагулянти, флокулянти та хлор, а також температурних режимів, які забезпечують максимальне очищення води. Такий підхід забезпечує відтворюваність і точність результатів.

*Ретельно досліджено такі показники роботи освітлювача, як каламутність освітленої води, концентрація завислих речовин та швидкість осідання осаду. Встановлено залежність цих показників якості від температури, дозування реагентів та конструктивних особливостей освітлювача. Підтримання стабільних гідравлічних умов має вирішальне значення, оскільки різкі коливання потоку можуть призвести до винесення осаду. Рекомендовано автоматичне або ручне регулювання висоти мулового фільтра залежно від конкретних умов експлуатації. Крім того, для підтримання оптимальної якості води необхідно регулювати режими роботи відстійників відповідно до різних типів води та сезонних коливань.*

*Отримані рівняння не тільки полегшують оцінювання продуктивності, але й оцінюють функціональність відстійника в критичних умовах, формуючи основу для майбутніх удосконалень конструкції та автоматизації відстійників.*

**Ключові слова:** *Освітлювач, коагулятор, насос-дозатор, реактиви.*

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