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EVALUATION OF FIRE RESISTANCE OF STRUCTURAL ELEMENTS CONSIDERING NONLINEARITY OF DEFORMATION PROCESSES DURING FIRE

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Summary. *A mathematical model for quantitative description of thermomechanical processes in structural elements under fire was proposed, with taking into account an elastic-plastic deformation and temperature dependence of material properties. The model is grounded on the equations of heat transfer theory and nonlinear thermo mechanics and is focused on the numerical methods of research. A method for numerical simulation of deformation processes in element structures subjected to intensive thermal- power loads was built on the bases of finite element method. The temperature dependent deformation curves, physical and mechanical characteristics are approximated by interpolation splines reconstructed by the experimental points of well known dependencies capturing the mechanical behavior of materials in wide temperature range. As an example, the computer simulations of thermo mechanical behavior of a steel structure subjected to fire conditions was carried out. Its fire resistance has been estimated.*

Key words: *thermo-mechanics, finite element method, fire resistance, the principle of virtual displacements.*

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Problem setting. One of the most important characteristics of building structures is their fire resistance or property to withstand high temperatures during the fire, without losing its load capacity. The majority of works devoted to the study of the properties of structures, has an experimental character. The establishment of modern research programs of fire resistance of structural elements from fires is mainly based on some experiments under conditions of full-scale fires, or a large number of probations and tests in special furnaces, which can withstand ambient temperature according to introduced typical standards [1, 2]. However, opportunities for the study of the behavior of buildings in full-scale fire are rather limited and extremely expensive, and during the experiments in furnaces, as a rule, only some parameters of individual structural elements are taken into consideration often referring to unrealistic size samples, load conditions and destruction. Such experiments do not reflect the behavior of the design as a whole even at low temperatures, as many aspects of the behavior of complex mechanical structures resulting from the interaction of various components cannot be predicted or traced in experiments with isolated elements. Therefore, the actual stresses in real structures during a fire are often substantially lower than anticipated and are based on partial experiments with elements of designs in standard fire scenarios (due to their structural continuity and providing alternative ways of loading individual items).

Due to huge losses during the full-scale experiments and partial nature of the results of experiments in furnaces with individual structural elements a practical need to develop mathematical models with quantitative description and evaluation of fire resistance of structures under different deployment of fire scenarios arises. There is also the necessity to create appropriate software and on this basis to conduct computer modeling of deformation processes of structures under conditions of intense thermal loadings, that meets the requirements of fire.

Analysis of recent researches and publications. In recent years, researches which are dedicated to building of mathematical models predicting fire resistance of elements of building structures, particularly concrete and reinforced concrete buildings have been intensified.

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Some authors believe that exact model for concrete under conditions of high temperatures should take into account connectivity process of thermal conductivity, the flow of viscous liquid, vapor diffusion, capillary effects, and to highlight hidden heat of water phases variability. Thus series of complex hydro-thermo mechanical models have been proposed (look, e.g. [3, 4] and other), where moving, temperature, gas pressure, capillary pressure, are considered to be related. However, due to the complexity of such models, numerical researches on its basis for the elements for concrete building structures under fire conditions as a rule are not provided. The basis of simple models are unbound heat equations and the ratio of thermo-mechanics on the basis of which it is much easier nowadays to conduct computational experiments and to get adequate results. Transient temperature distributions in structural elements structures that were defined during the first stage of solving complex tasks are considered as the input for the second phase of the problem – analysis of stress-strain state of the construction. There is wide debate concerning the necessity and the methods of consideration of different factors of influence on the level of development of such models (look e.g., [5, 6, 7 – 9]).

The purpose of the work is the development of mathematical model, methodology and software for modeling of deformation processes of structural elements under conditions of intensive thermal power taking into account loadings, temperature dependence of material properties and elastic-plastic character of deformation. Availability of reliable software allows analyzing the behavior of structures during fire, in the process of extinguishing and during exploitation after fire, to get expert evaluation of fire resistance, stock of resource in the individual structural elements during the fire, the level of residual stresses after the fire and so on. This provision provides opportunities for rational choice of geometrical sizes of elements of constructions, properties of structural and insulating materials with the purpose to continue the duration of the elements of construction in the midst of the fire with the saving of bearing capacity. This dramatically reduces the number of field experiments, as only rational projects are experimentally tested that are obtained through computer simulation.

The formulation of the problem and methods of its solution.

Let's consider a solid deformable body, which covers area V with the continuous surface S according to Lipchitz. The body is influenced by the volumetric f_i^B and surface f_i^S forces, given in accordance in the area V and in the part S_f of the surface S, of the movements u_i^S, given on the part S_u of the part S ($S_f \bigcup S_u = S,\ S_f \bigcap S_u = \varnothing$), and also due to the action of thermal factors caused by the fire. The problem is to determine the stress state of the body, caused by these influences. In the first phase we form a non-linear non-stationary problem of thermal conductivity of the body. Environmental temperature change is given in accordance with the time-temperature curve, which practically defines the scenario of fire [1, 2].

The temperature field $T(\mathbf{r}, t)$ in the body describes the equation

$$c\rho \frac{\partial T}{\partial t} = \vec{\nabla} \cdot \left(\lambda \vec{\nabla} T\right) \qquad \mathbf{r} \in V,\ t \in [0, \tau_*] \tag{1}$$

under initial

$$T(\mathbf{r}, 0) = T_0(\mathbf{r}) \tag{2}$$

and boundary

$$-\lambda\vec{\nabla}T\mathbf{n}=\beta\left(T-T_S\right) \quad \mathbf{r}\in S \qquad (3)$$

conditions, where $\mathbf{r}$ – radius vector of point; $c=c(\mathbf{r},T)$ – specific heat; $\rho=\rho(\mathbf{r},T)$ – density; $\lambda=\lambda(\mathbf{r},T)$ – thermal conductivity; $\vec{\nabla}$ – operator of Hamilton, ($\vec{\nabla}\cdot$) means the operation of divergence; $\mathbf{n}$ – vector of external single unit to the surface S of the body V ; $\beta=\beta(\mathbf{r},T)$ – heat transfer coefficient; T_S – environmental temperature. As we can see, thermal characteristics depend on temperature and on the point which makes it possible to consider heterogeneous, piecewise and thermo-sensitive bodies.

Since the fire conditions the body is at high temperatures, it can be assumed that the heat radiation will significantly affect the temperature distribution in the body.

Assuming the generalized heat transfer coefficient as

$$\beta'\left(\grave{O},T_S\right)=\beta\left(\grave{O}\right)+\upsilon\chi\varepsilon\left(T^3+T^2T_S+TT_S^2+T_S^3\right), \qquad (4)$$

we can consider that heat transfer between flame and the surface of the body by radiation within the boundary conditions (3), where υ – is the index of type of radiation, χ – constant of Stefan Boltsman, T та T_S – absolute temperatures according to the surface of the body and the environment (temperature of «pseudo-fire», which rises in accordance with the standards imposed by fire scenarios), $\varepsilon=\left(1/\varepsilon_f+1/\varepsilon_S-1\right)^{-1}$ (ε_f – emissivity of the flame; ε_S – absorption capacity of the body surface) [5]. The methodology of solving the problem of thermal conductivity for thermo-sensitive, piecewise bodies by means of the method of finite elements of bodies has been proposed and tested in work [10], which presents the results of solving specific practical problems and the comparative analysis of the numerical solutions with the results of other authors and relevant experiments has been done.

Unsteady temperature fields that have been defined are the input for the problem of the second stage which is the determination of the stress-strain state of the body using Lagrange's approach, and considering body movement in a fixed Cartesian coordinate system step by step, when the approximate solution of basic equations, describing equilibrium and compatibility conditions of the body, is got for discrete moments in time $t_{i+1}=t_i+\Delta t_i$, $i=0,1,...$

Let's consider the random next step loading [t, $t+\Delta t$] (solutions for all previous discrete moments in time till t including are known).

According to the principle of virtual displacements the body is in equilibrium at the moment of time $t+\Delta t$, if [11]

$$\int_{{}^0V} {}^{t+\Delta t}_{\ \ 0}\sigma_{ij}\,\delta\,{}^{t+\Delta t}_{\ \ 0}\in_{ij}\ d\,{}^0V={}^{t+\Delta t}R\,, \qquad (5)$$

where ${}^{t+\Delta t}_{\ \ 0}\sigma_{ij}$, ${}^{t+\Delta t}_{\ \ 0}\in_{ij}$ – are components of the stress tensor of Piol-Kirchhoff of 2nd type and deformations of Green-Lagrange that refer to the virtual movements δu_i imposed on configuration of the body in the moment of time $t=t+\Delta t$, which are the functions of Cartesian coordinates ${}^{t+\Delta t}x_j$ of material point in the moment of time $t=t+\Delta t$ (${}^{t+\Delta t}x_j={}^0x_j+{}^{t+\Delta t}u_j$);

$${}^{t+\Delta t}R=\int_{{}^{t+\Delta t}V} {}^{t+\Delta t}f_i^B\,\delta u_i\,d\,{}^{t+\Delta t}V+\int_{{}^{t+\Delta t}S_f} {}^{t+\Delta t}f_i^S\,\delta u_i^S\,d\,{}^{t+\Delta t}S\,; \qquad (6)$$

${}^{0}V,\ {}^{t+\Delta t}V$ – volume of body according to the moments of time $t=0$ i $t=t+\Delta t$. Deformations are determined through movement by using correlation

$$ {}_{0}^{t}\in_{ij}=\frac{1}{2}\cdot\left(\frac{\partial_{0}^{t}u_{i}}{\partial^{0}x_{j}}+\frac{\partial_{0}^{t}u_{j}}{\partial^{0}x_{i}}+\frac{\partial_{0}^{t}u_{k}}{\partial^{0}x_{i}}\frac{\partial_{0}^{t}u_{k}}{\partial^{0}x_{j}}\right). \tag{7}$$

It should be noted that the relationship between the components of Piol-Kirchhoff's tensors ${}_{0}^{t}\sigma_{ij}$ and Koch's tensors ${}^{t}\tau_{nm}$ (when the forces are related to the unit of the area of the deformed body) is:

$$ {}_{0}^{t}\sigma_{ij}=\frac{{}^{0}\rho}{{}^{t}\rho}\frac{\partial^{0}x_{i}}{\partial^{t}x_{m}}\frac{\partial^{0}x_{j}}{\partial^{t}x_{n}}\cdot{}^{t}\tau_{nm}, \tag{8}$$

where ${}^{0}\rho,\ {}^{t}\rho$ – the density of the body in two configurations (when $t=0$ and at the moment of time t).

All basic quantities in correlation with (5) are referred to one and the same (initial) configuration of the body ${}^{0}V$. It gives the opportunity to define the searching movements, deformations and tenses at the moment of time $t=t+\Delta t$ through the known definitions ${}_{0}^{t}u_{i}$, ${}_{0}^{t}\in_{ij}$, ${}_{0}^{t}\sigma_{ij}$ at the beginning of the proposed step of the loading and the corresponding increases ${}_{0}u_{i}$, ${}_{0}\in_{ij}$, ${}_{0}\sigma_{ij}$ on this stage, that is:

$$ {}_{0}^{t+\Delta t}\sigma_{ij}={}_{0}^{t}\sigma_{ij}+{}_{0}\sigma_{ij};\quad {}_{0}^{t+\Delta t}\in_{ij}={}_{0}^{t}\in_{ij}+{}_{0}\in_{ij};\quad {}^{t+\Delta t}u_{i}={}^{t}u_{i}+{}_{0}u_{i}. \tag{9}$$

Taking into consideration the trivial correlation ${}_{0}\in_{ij}={}_{0}^{t+\Delta t}\in_{ij}-{}_{0}^{t}\in_{ij}$ and presentation ${}_{0}^{t+\Delta t}\in_{ij}$ and ${}_{0}^{t}\in_{ij}$ through the movement we get schedule for components

$$ {}_{0}\in_{ij}={}_{0}e_{ij}+{}_{0}\eta_{ij}, \tag{10}$$

where linear (${}_{0}e_{ij}$) and non-linear (${}_{0}\eta_{ij}$) terms look like:

$$ {}_{0}e_{ij}=\frac{1}{2}\cdot\left(\frac{\partial_{0}u_{i}}{\partial^{0}x_{j}}+\frac{\partial_{0}u_{j}}{\partial^{0}x_{i}}+\frac{\partial_{0}^{t}u_{k}}{\partial^{0}x_{i}}\frac{\partial_{0}u_{k}}{\partial^{0}x_{j}}+\frac{\partial_{0}^{t}u_{k}}{\partial^{0}x_{j}}\frac{\partial_{0}u_{k}}{\partial^{0}x_{i}}\right) \tag{11}$$

$$ {}_{0}\eta_{ij}=\frac{1}{2}\cdot\frac{\partial_{0}u_{k}}{\partial^{0}x_{i}}\frac{\partial_{0}u_{k}}{\partial^{0}x_{j}}, \tag{12}$$

Since variation is taken relatively to configuration, corresponding the moment of time $t=t+\Delta t$, then $\delta_{0}^{t}\in_{ij}=0$ i $\delta^{t+\Delta t}_{0}\in_{ij}=\delta_{0}\in_{ij}$ the original equation of principle of virtual work (1) taking into account the correlations (9) and (10) looks like:

$$ \int_{{}^{0}V}{}_{0}\sigma_{ij}\delta_{0}\in_{ij}d^{0}V+\int_{{}^{0}V}{}_{0}^{t}\sigma_{ij}\delta_{0}\eta_{ij}d^{0}V={}^{t+\Delta t}R-\int_{{}^{0}V}{}_{0}^{t}\sigma_{ij}\delta_{0}e_{ij}d^{0}V. \tag{13}$$

Let's linearize the resulting equation (13), as we still have a lot of non-linear expressions in the left part.

Let's consider approximation to deformation variations ($\delta_0 \in_{ij} = \delta_0 e_{ij}$) and to relationships between the stresses and deformations:

$$ {}_0\sigma_{ij} = {}_0C_{ijrs}\,({}_0e_{rs} - {}_0e^T_{rs}), \tag{14} $$

where ${}_0C_{ijrs} = \partial^t_0\sigma_{ij} / \partial^t_0 \in_{rs}$ (the process of constructing a matrix ${}_0C_{ijrs}$ in the case of non-isothermal thermo-elastic-plasticity which is described in detail in the monograph [12]); ${}_0e^T_{rs}$ – increases of the temperature deformations in a step of loading. Then:

$$ \int_{{}^0V} {}_0C_{ijrs}\,{}_0e_{rs}\delta_0 e_{ij}\,d\,{}^0V + \int_{{}^0V} {}^t_0\sigma_{ij}\delta_0\eta_{ij}\,d\,{}^0V = {}^{t+\Delta t}R - \int_{{}^0V} {}^t_0\sigma_{ij}\delta_0 e_{ij}\,d\,{}^0V + \int_{{}^0V} {}_0C_{ijrs}\,{}_0e^T_{rs}\delta_0 e_{ij}\,d\,{}^0V . \tag{15} $$

We are going to write this equation in a convenient matrix-vector form:

$$ \begin{gathered} \int_{{}^0V} \{\delta_0\mathbf{e}\}^T[{}_0\mathbf{C}]\{{}_0\mathbf{e}\}\,d\,{}^0V + \int_{{}^0V} \{\delta_0\mathbf{g}\}^T[{}^t_0\mathbf{S}]\{{}_0\mathbf{g}\}\,d\,{}^0V = \\ {}^{t+\Delta t}R - \int_{{}^0V} \{\delta_0\mathbf{e}\}^T\{{}^t_0\mathbf{S}\}\,d\,{}^0V + \int_{{}^0V} \{\delta_0\mathbf{e}_T\}^T[{}_0\mathbf{C}]\{{}_0\mathbf{e}\}\,d\,{}^0V . \end{gathered} \tag{16} $$

Here tensor components of deformations and stresses are collected in vectors:

$\{\mathbf{e}\} = (e_{11}, e_{22}, e_{33}, 2e_{12}, 2e_{13}, 2e_{32})^T$; $\{\mathbf{g}\} = (g_{11}, g_{22}, g_{33}, 2g_{12}, 2g_{13}, 2g_{32})^T$;
$\{{}^t_0\mathbf{S}\} = ({}^t_0\sigma_{11}, {}^t_0\sigma_{22}, {}^t_0\sigma_{33}, {}^t_0\sigma_{12}, {}^t_0\sigma_{13}, {}^t_0\sigma_{23})^T$.

After substituting the kinematic equations (11) and (12) in (16) we get the linear approximation for the equations of the motion:

$$ \begin{gathered} \int_{{}^0V} \{\delta_0\mathbf{u}\}^T\left[{}^t_0\mathbf{B}_L\right]^T[{}_0\mathbf{C}]\left[{}^t_0\mathbf{B}_L\right]\{{}_0\mathbf{u}\}\,d\,{}^0V + \int_{{}^0V} \{\delta_0\mathbf{u}\}^T\left[{}^t_0\mathbf{B}_{NL}\right]^T[{}^t_0\mathbf{S}]\left[{}^t_0\mathbf{B}_{NL}\right]\{{}_0\mathbf{u}\}\,d\,{}^0V = \\ = {}^{t+\Delta t}R - \int_{{}^0V} \{\delta_0\mathbf{u}\}^T\left[{}^t_0\mathbf{B}_L\right]^T\{{}^t_0\mathbf{S}\}\,d\,{}^0V + \int_{{}^0V} \{\delta_0\mathbf{u}\}^T\left[{}^t_0\mathbf{B}_L\right]^T[{}_0\mathbf{C}]\{{}_0e^T\}\,d\,{}^0V , \end{gathered} \tag{17} $$

where $\{\delta_0\mathbf{u}\} = ({}_0u_1, {}_0u_2, {}_0u_3)^T$ – is the vector of increases of movements on the proposed step of loading, $\left[{}^t_0\mathbf{B}_L\right]$ and $\left[{}^t_0\mathbf{B}_{NL}\right]$ – matrix of differential operators of kinematic equations (11) and (12) in accordance,

$$ {}^{t+\Delta t}R = \int_{{}^0V} \{\delta_0\mathbf{u}\}^T\mathbf{f}^B\,d\,{}^0V + \int_{{}^0S} \{\delta_0\mathbf{u}\}^T\mathbf{f}^S\,d\,{}^0V . \tag{18} $$

We use the correlation of the principle of virtual works (17) for the definition of the increases of the movements on the proposed step of loading. It should be noted that its structure is universal and remains unchanged for all partial cases, in particular, for axisymmetric or plane problems, and also for the problems which were got due to the usage of assumptions of the theory of beams and cores. As a result, during the numerical implementation there will be the only general structure of solution and only the blocks will be changed, that are responsible for calculating the specific characteristics of the matrix-vector $\left[{}_0^t\mathbf{B}_L\right]$, $\left[{}_0^t\mathbf{B}_{NL}\right]$, $[{}_0\mathbf{C}]$, $[{}_0^t\mathbf{S}]$ etc. For example, in the case of three-dimensional space problem

$$[{}_0^t\mathbf{B}_L]=\begin{bmatrix} \frac{\partial_0}{\partial^0x_1} & 0 & 0 & \frac{\partial_0}{\partial^0x_2} & \frac{\partial_0}{\partial^0x_1} & 0 \\ 0 & \frac{\partial_0}{\partial^0x_2} & 0 & \frac{\partial_0}{\partial^0x_3} & 0 & \frac{\partial_0}{\partial^0x_1} \\ 0 & 0 & \frac{\partial_0}{\partial^0x_3} & 0 & \frac{\partial_0}{\partial^0x_3} & \frac{\partial_0}{\partial^0x_2} \end{bmatrix}^T +$$

$$+\begin{bmatrix} \frac{\partial_0^tu_1}{\partial^0x_1}\frac{\partial_0}{\partial^0x_1} & \frac{\partial_0^tu_2}{\partial^0x_1}\frac{\partial_0}{\partial^0x_1} & \frac{\partial_0^tu_3}{\partial^0x_1}\frac{\partial_0}{\partial^0x_1} \\ \frac{\partial_0^tu_1}{\partial^0x_2}\frac{\partial_0}{\partial^0x_2} & \frac{\partial_0^tu_2}{\partial^0x_2}\frac{\partial_0}{\partial^0x_2} & \frac{\partial_0^tu_3}{\partial^0x_2}\frac{\partial_0}{\partial^0x_2} \\ \frac{\partial_0^tu_1}{\partial^0x_3}\frac{\partial_0}{\partial^0x_3} & \frac{\partial_0^tu_2}{\partial^0x_3}\frac{\partial_0}{\partial^0x_3} & \frac{\partial_0^tu_3}{\partial^0x_3}\frac{\partial_0}{\partial^0x_3} \\ \frac{\partial_0^tu_1}{\partial^0x_1}\frac{\partial_0}{\partial^0x_2}+\frac{\partial_0^tu_1}{\partial^0x_2}\frac{\partial_0}{\partial^0x_1} & \frac{\partial_0^tu_2}{\partial^0x_1}\frac{\partial_0}{\partial^0x_2}+\frac{\partial_0^tu_2}{\partial^0x_2}\frac{\partial_0}{\partial^0x_1} & \frac{\partial_0^tu_3}{\partial^0x_1}\frac{\partial_0}{\partial^0x_2}+\frac{\partial_0^tu_3}{\partial^0x_2}\frac{\partial_0}{\partial^0x_1} \\ \frac{\partial_0^tu_1}{\partial^0x_1}\frac{\partial_0}{\partial^0x_3}+\frac{\partial_0^tu_1}{\partial^0x_3}\frac{\partial_0}{\partial^0x_1} & \frac{\partial_0^tu_2}{\partial^0x_1}\frac{\partial_0}{\partial^0x_3}+\frac{\partial_0^tu_2}{\partial^0x_3}\frac{\partial_0}{\partial^0x_1} & \frac{\partial_0^tu_3}{\partial^0x_1}\frac{\partial_0}{\partial^0x_3}+\frac{\partial_0^tu_3}{\partial^0x_3}\frac{\partial_0}{\partial^0x_1} \\ \frac{\partial_0^tu_1}{\partial^0x_2}\frac{\partial_0}{\partial^0x_3}+\frac{\partial_0^tu_1}{\partial^0x_3}\frac{\partial_0}{\partial^0x_2} & \frac{\partial_0^tu_2}{\partial^0x_2}\frac{\partial_0}{\partial^0x_3}+\frac{\partial_0^tu_2}{\partial^0x_3}\frac{\partial_0}{\partial^0x_2} & \frac{\partial_0^tu_3}{\partial^0x_2}\frac{\partial_0}{\partial^0x_3}+\frac{\partial_0^tu_3}{\partial^0x_3}\frac{\partial_0}{\partial^0x_2} \end{bmatrix};$$

$$[{}_0^t\mathbf{B}_{NL}]=\begin{bmatrix} \frac{\partial_0}{\partial^0x_1} & \frac{\partial_0}{\partial^0x_2} & \frac{\partial_0}{\partial^0x_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial_0}{\partial^0x_1} & \frac{\partial_0}{\partial^0x_2} & \frac{\partial_0}{\partial^0x_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial_0}{\partial^0x_1} & \frac{\partial_0}{\partial^0x_2} & \frac{\partial_0}{\partial^0x_3} \end{bmatrix}^T;$$

$$[{}_0^t\mathbf{S}]=\begin{bmatrix} {}_0^t\mathbf{S}_* & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & {}_0^t\mathbf{S}_* & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & {}_0^t\mathbf{S}_* \end{bmatrix},\ {}_0^t\mathbf{S}_*=\begin{bmatrix} {}_0^t\sigma_{11} & {}_0^t\sigma_{12} & {}_0^t\sigma_{13} \\ {}_0^t\sigma_{21} & {}_0^t\sigma_{22} & {}_0^t\sigma_{23} \\ {}_0^t\sigma_{31} & {}_0^t\sigma_{32} & {}_0^t\sigma_{33} \end{bmatrix},\ \mathbf{0}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

For two-dimensional beam of Euler-Bernoulli, which has been got by means of the introduction of the hypothesis of flat sections [11], whereby movement u_1, u_2 of beam points define longitudinal movements v_1 and hogging v_2 of the points of its axis in relations:

$$u_1({}^0x_1,\ {}^0x_2)=v_1({}^0x_1)-{}^0x_2\frac{d^2v_2({}^0x_1)}{d\,{}^0x_1^{\,2}},\ u_2({}^0x_1,\ {}^0x_2)=v_2({}^0x_1)$$

(x_1 – coordinate along the beam axis), corresponding matrix characteristics have much simpler form:

$$\left[{}_0^t\mathbf{B}_L\right]=\left[\frac{d}{d\,{}^0x_1}\left(1+\frac{d\,{}_0^tu_1}{d\,{}^0x_1}\right)\quad \frac{d\,{}_0^tu_2}{d\,{}^0x_1}\frac{d}{d\,{}^0x_1}-{}^0x_2\frac{d^2}{d\,{}^0x_1{}^2}\right];\ \left[{}_0^t\mathbf{B}_{NL}\right]=\begin{bmatrix}\frac{d}{d\,{}^0x_1} & 0\\ 0 & \frac{d}{d\,{}^0x_1}\end{bmatrix};$$

$[{}_0^t\mathbf{S}]=\{{}_0^t\mathbf{S}\}={}_0^t\sigma_{11}$.

All integrals in equation are simplified (17), because

$$\int_{{}^0V} d\,{}^0V=\int_{{}^0A} d\,{}^0A\int d\,{}^0x_1\,, \tag{19}$$

where ${}^0A$ – is a cross-section of the beam.

Because of standard finite-element of sampling area ${}^0V$ and a certain approximation of desired increases of nodes movements of every separate element by means of form function [11] key correlation (17) takes the form

$$([\mathbf{K}_L]+[\mathbf{K}_{NL}])\{\mathbf{q}\}={}^{t+\Delta t}\{\mathbf{P}\}-\{\mathbf{F}\}\,, \tag{20}$$

where $\{\mathbf{q}\}$ – is the global vector of the unknown nodes, that allow to determine the increases movements of body.

We get matrix-vector properties in correlation (20) by means of summing certain characteristics of specific finite elements: ${}^0V^e$:

$$[\mathbf{K}_L]^e=\int_{{}^0V^e}[\mathbf{N}]^T[{}_0^t\mathbf{B}_L]^T[{}_0\mathbf{C}][{}_0^t\mathbf{B}_L][\mathbf{N}]dv\,;$$
$$[\mathbf{K}_{NL}]^e=\int_{{}^0V^e}[\mathbf{N}]^T[{}_0^t\mathbf{B}_{NL}]^T[{}_0^t\mathbf{S}][{}_0^t\mathbf{B}_{NL}][\mathbf{N}]\,dv\,;$$
$${}^{t+\Delta t}\{\mathbf{P}\}^e=\int_{{}^0S_\sigma^e}[\mathbf{N}]^T\ {}^{t+\Delta t}\mathbf{f}^S\,ds+\int_{{}^0V^l}[\mathbf{N}]^{\mathrm{T}}\ {}^{t+\Delta t}\mathbf{f}^B\,dv\,;$$
$$\{\mathbf{F}\}^e=\int_{{}^0V^e}[\mathbf{N}]^T[{}_0^t\mathbf{B}_L](\{{}_0^t\mathbf{S}\}-[{}_0\mathbf{C}]\{{}_0e^T\})dv\,. \tag{21}$$

here $[\mathbf{N}]$ – is functions matrix of the form [11], that provides deformation compatibility through transition from one finite element to another.

Iterative process of the building of nonlinear equations system solving (20) using the method Newton-Raphson [11] can be represented in three steps.

Step 1. At i – iteration (at the beginning of calculations $i=1$) we have initial approach: for movements ${}^{t+\Delta t}\mathbf{u}^{(i-1)}$, which are defined from finite-element representation ${}^{t+\Delta t}\mathbf{u}^{(i-1)}=[\mathbf{N}]^t\{\mathbf{q}\}^{(i-1)}$ ($\{\mathbf{q}\}^{(0)}={}^t\mathbf{u}$), and deformations ${}^{t+\Delta t}\in^{(i-1)}$.

Step 2. According to these approximations we calculate ${}^{t+\Delta t}\sigma^{(i-1)}$ and then we calculate tangential matrix of state $[{}_0\mathbf{C}]^{(i-1)}$. If there is elastic on the certain step of deformation, then deformations ${}^{t+\Delta t}\in^{(i-1)}$ give the opportunity to receive voltage directly ${}^{t+\Delta t}\sigma^{(i-1)}$ and matrix $[{}_0\mathbf{C}]^{(i-1)}$. According to elastic-plastic deformation

$${}^{t+\Delta t}\sigma^{(i-1)}={}^t\sigma+\int_t^{t+\Delta t^{(i-1)}}d\sigma\,.$$

According to received approach of stresses ${}^{t+\Delta t}\sigma^{(i-1)}$ (at the end of $(i-1)$ - iteration) we calculate matrix $[{}_0\mathbf{C}]^{(i-1)}$.

Step 3. We form matrix equation:

$$({}^{t+\Delta t}[\mathbf{K}_L]+{}^{t+\Delta t}[\mathbf{K}_{NL}])\{\mathbf{q}\}^{(i)}={}^{t+\Delta t}\{\mathbf{P}\}-{}^{t+\Delta t}\{\mathbf{F}\}^{(i-1)}$$

out of which we can define another approach of increases of nodal movements $\{\mathbf{q}\}^{(i)}$ on a certain step of loading, next – approach for the moving of nodes ${}^{t+\Delta t}\{\mathbf{q}\}^{(i)}={}^{t+\Delta t}\{\mathbf{q}\}^{(i-1)}+\{\mathbf{q}\}^{(i)}$, movements ${}^{t+\Delta t}\mathbf{u}^{(i)}$ and deformations ${}^{t+\Delta t}\in^{(i)}$, after which we go to step 1 of iteration process (when $i=i+1$). Then we continue the calculation on the steps 1 – 3 on the step of loading $[t,\ t+\Delta t]$ to achieve convergence.

On the basis of the mathematical model and methodology of solving the formulated problem of thermal conductivity and thermal elastic-plasticity appropriate software has been developed.

Methodology of its development is described in detail in the works [12 – 14]. Using software development thermo-mechanical behavior of a number of steel, concrete and reinforced concrete structures has been investigated.

As an example of the proposed method we are going to simulate the thermomechanical behavior of the П-construction, made of heat-sensitive steel C30 under conditions of fire.

The results of computer modeling and analysis.

The dimensions of the construction, terms of its consolidation and power of loading are shown at the Figure 1.

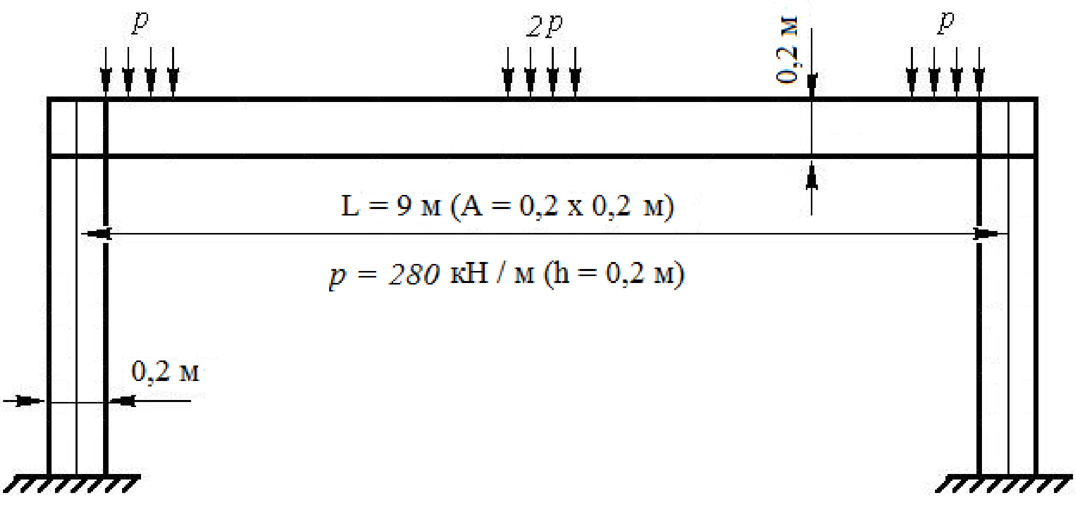

Figure 1. Steel П-Construction

We consider that out of the inner surface the construction is under the influence of «pseudo-fire», variable in temperature over time is considered as

$$T_s = T_0 + 345\log_{10}(8\cdot t+1) \qquad (21)$$

according to the international standard ISO 834 [2] (this temperature is taken into account in the boundary condition) (3) as the temperature of the environment). Other surfaces of the constructions are isolated.

The dependence of the thermal and mechanical properties of steel from the temperature is given in the monograph [14]. For approximation of the temperature-dependent deformation of the curves and physical and mechanical properties of materials the interpolation splines have been used, which are built according to the points of already known experimental curves that describes thermo-mechanical behavior of materials in a wide temperature range.

Thermal conductivity problem is solved by means of using isoperimetric quadratic finite elements [10]. In order to solve thermal elastic-plasticity problem cubic beam finite elements have been applied [11]. Convergence of the solutions has been investigated by means of comparison of the numerical solutions on different, according to the density, finite-element divisions of the construction at various steps of sampling process of thermal conductivity according to time (when the solutions, that have been got on the certain steps coincide within 1% with the solutions on twice less steps and this convergence was considered to be achieved).

The temperature division in the middle section of the beam cross-bar at the moment of time, when the construction exhausts its carrying capacity, is shown at Fig. 2.

At Fig. 3 temperature changes of «pseudo-fire» over time are shown T_S (curve 1), and also the temperature on the upper (curve *2*) and the lower (curve *3*) surfaces of the bar in its median section of beam which have been in the midst of the fire for two hours.

Analysis of the results shows that during the fire in the beam cross-bar at the beginning of the sixth minute (t = 320 c) first plastic deformations appear. Ability to elastic-plastic deformation and further strengthening steel beam is exhausted after 19 minutes. Axial stresses in the beam at this time are shown at Fig. 4.

Obviously, increasing of the fire resistance of the constructions is connected with the using of insulation materials from the surface of possible ignition and also with the manufacturing of the constructions from the materials physical and mechanical properties of which counteract rapid heating and growth of temperature difference in the construction.

The approach will be used in future for the investigation of thermomechanical behavior of structures during fire extinguishing and determination of residual stresses in them after a sharp cooling on the stage of the fire extinguishing that have been launched until the loss of structure bearing capacity, and also in order to receive expertise review of opportunities for further exploitation after the fire extinguishing.

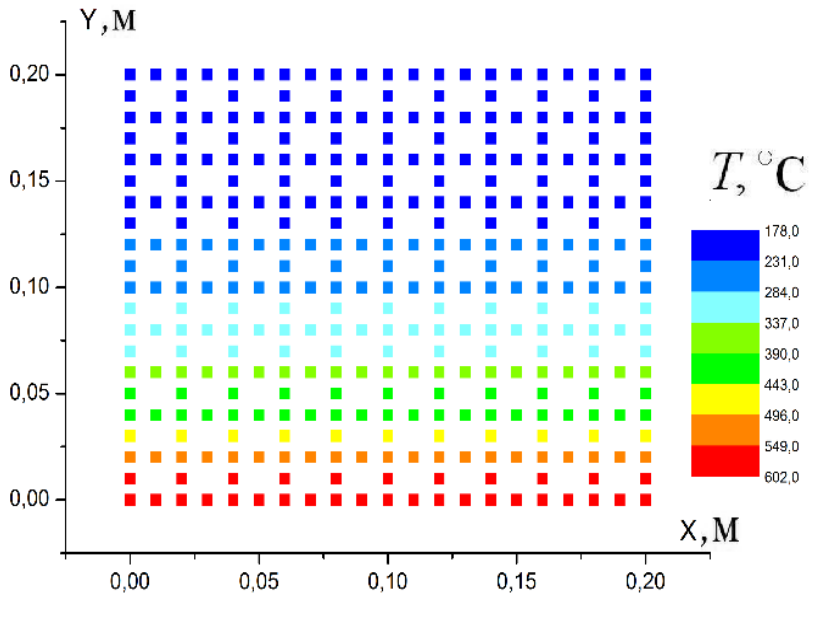

Figure 2. Temperature distribution in nodes of the bar middle section at t =19 min.

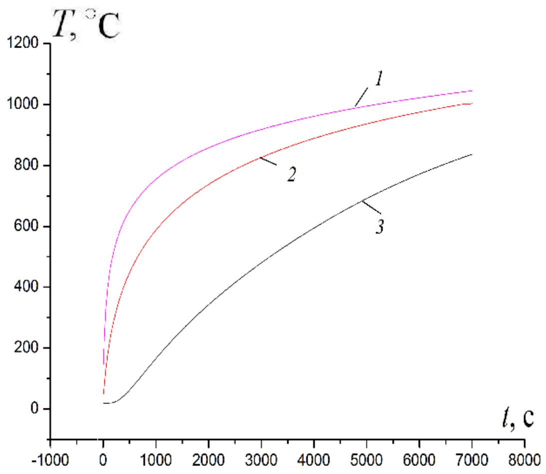

Figure 3. Temperature changes at the bottom (curve 1) and the top (curve 2) surfaces of the bar, and of a «pseudo-fire» (curve 3).

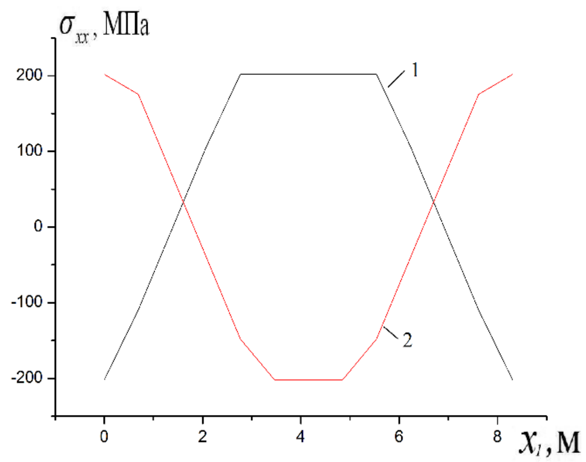

Figure 4. Stresses at the bottom (curve 1) and the top (curve 2) surfaces of the bar at moment t = 19 min.

Conclusions. The process of elastic-plastic deformation and temperature dependence of material properties should be taken into account when modeling thermomechanical processes in structural elements under the fire. We get the significant deviations in the distributions of received parameters characterizing stress-strain state structures from actual ones if we do not consider these factors. It is also important to consider the stage of extinguishing the fire, on which the residual stresses are actually formed, and evaluate the strength of structures at the stage of their operation after the fire.

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УДК 539.3

ОЦІНЮВАННЯ ВОГНЕТРИВКОСТІ ЕЛЕМЕНТІВ КОНСТРУКЦІЙ З УРАХУВАННЯМ НЕЛІНІЙНОСТІ ПРОЦЕСІВ ЇХ ДЕФОРМУВАННЯ

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Резюме. *Запропоновано орієнтовану на використання числових методів дослідження математичну модель кількісного опису термомеханічних процесів в елементах конструкцій під час пожежі з урахуванням пружно-пластичного характеру деформування та температурної залежності фізико-механічних характеристик матеріалів. Модель базується на залежностях теорії теплопровідності та нелінійної термомеханіки. Побудовано з використанням методу скінченних елементів методику чисельного моделювання процесів деформування конструкцій за інтенсивних температурних та силових навантажень. Для апроксимації температурно-залежних кривих деформування та фізико-механічних характеристик матеріалів використано інтерполяційні сплайни, побудовані за точками відомих експериментальних кривих. Як приклад, виконано комп'ютерне моделювання термомеханічної поведінки сталевої конструкції за умов пожежі. Отримано оцінку її вогнетривкості.*

Ключові слова*: термомеханіка, метод скінченних елементів, вогнетривкість, принцип віртуальних переміщень.*

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