Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
*2016, № 3 (83)*
*ISSN 1727-7108. Web:* ***visnyk.tntu.edu.ua***

**UDC 539.3**

# PRESSING OF THE PRELIMINARY-STRESSED PLATE BY TWO CO-AXIAL PUNCHES OF COMPLEX CONFIGURATION

**Oleh Panchuk**

*Ternopil Ivan Puluj National Technical University, Ternopil, Ukraine*

***Summary***. *Solution of the contact task on the interrelation of the ring-parabolic and parabolic punch with the preliminary-stressed plate has been presented in the article. Development of analytical solutions for the plate is performed by its modeling by the finale thickness preliminary-stressed layer. The systems of double and triple integral equations obtained here are solved by presenting necessary functions as finale set sums according to the Bessel's functions with the unknown coefficients and further obtaining finale systems of the linear algebraic equations for their finding. Basing on the obtained solution the effect of the initial deformations field nature on the stressed state of thick plate has been analyzed.*

***Key words****: linearized elasticity theory, contact interrelation, contact stresses, ring-parabolic punch, parabolic punch, plate, layer, initial deformations, preliminary stresses.*

*Received 10.09.2016*

**Problem setting.** Determination of contact stresses and deformations in the interaction of hard punches with the elastic plate is an important objective in the design of machine parts and elements of buildings. In particular, when assessing the strength of reinforced concrete beams, monolithic foundation plates in construction, road coatings, etc. To minimize the error of calculations, it is necessary to consider the maximum number of factors that affect the contact interaction. The initial deformations, on which contact stresses and movement depend directly, are one of the key factors.

**Analysis of the known research results.** A lot of scientists, including domestic, dealt with the interaction of bodies with existing residual deformations. In general, the statement of such problems requires the use of a linear system of elasticity, but at fairly large initial deformations it is possible to be restricted by its linearized version [1].

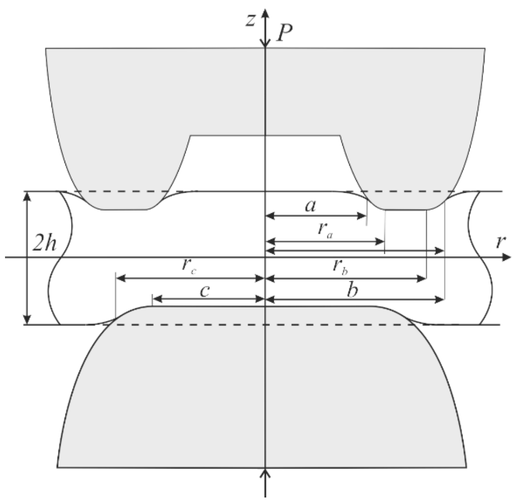


**Figure 1.** Scheme of the contact interrelation of the parabolic punch and layer

Despite the increasing number of researches, dedicated to contact interaction of bodies of preliminary-stressed state [2-4], the task about the compression of the preliminary-stressed plate by the parabolic and the ring-parabolic punch is still unresolved for compressible and incompressible bodies in general, according to the arbitrary structure of the elastic potential.

**Problem statement.** We will conduct the formulation and problem solving within the theory of linearized elasticity. We will model thick plate by means of isotropic preliminary-stressed layer.

Let us consider preliminary-stressed layer with the thickness of $2h$, which is compressed by the parabolic and ring-parabolic punches.

Let us choose a cylindrical coordinate system $(O, r, \theta, z)$ in such a way that coordinate plane $(r, O, \theta)$ coincides with the middle plane layer, and the axis $Oz$ – coincides with a common axis of the punches (fig.1).

We will assume parameters as the known areas of contact on both boundary planes and will define the corresponding configuration of punches. The function, by means of the rotation of the graph of which round the axis $Oz$ the ring-parabolic punch is made, we will show as

$$W_1(r) = \begin{cases} \frac{1}{2R_1}(r - r_a)^2, \ 0 \le r \le r_a; \\ 0, \ r_a < r \le r_b; \\ \frac{1}{2R_2}(r - r_b)^2, \ r_b < r. \end{cases}$$

Similarly, for a parabolic punch we will have

$$W_2(r) = \begin{cases} 0, \ 0 \le r \le r_c; \\ -\frac{1}{2R_3}(r - r_c)^2, \ r_c < r. \end{cases}$$

The boundary conditions of the problem have the following look

$$\sigma_{zz}(r,h) = 0, \ r \le a, b \le r < \infty; \quad (1)$$

$$\sigma_{rz}(r,h) = 0, \ 0 \le r < \infty; \quad (2)$$

$$u_z(r,h) = \omega_1(r), \ a \le r \le b; \quad (3)$$

$$\sigma_{zz}(r,-h) = 0, \ c \le r < \infty; \quad (4)$$

$$\sigma_{rz}(r,-h) = 0, \ 0 \le r < \infty; \quad (5)$$

$$u_z(r,-h) = \omega_2(r), \ 0 \le r \le c. \quad (6)$$

Functions $\omega_1(r)$ and $\omega_2(r)$ describe the movement of the points respectively the upper and lower boundary plane of the stressed layer at the site of his contact with hard punches. Therefore, based on the appearance of functions, $W_1(r)$ and $W_2(r)$, we can write down:

$$\omega_1(r) = \begin{cases} \omega_1(a) + \frac{1}{2R_1}\left[(r_a - r)^2 - (r_a - a)^2\right], \ a \le r < r_a; \\ \omega_1(a) - \frac{1}{2R_1}(r_a - a)^2, \ r_a \le r < r_1; \\ \omega_1(b) - \frac{1}{2R_2}(r_b - b)^2, \ r_1 \le r < r_b; \\ \omega_1(b) + \frac{1}{2R_2}\left[(r_b - r)^2 - (r_b - b)^2\right], \ r_b \le r \le b; \end{cases} \qquad r_1 = \frac{r_a + r_b}{2}. \quad (7)$$

$$\omega_2(r) = \begin{cases} \omega_2(c) + \frac{1}{2R_3}(c - r_c)^2, \ 0 \le r < r_c; \\ \omega_2(c) + \frac{1}{2R_3}\left[(c - r_c)^2 - (r - r_c)^2\right], \ r_c \le r \le c. \end{cases} \quad (8)$$

**The solution of the problem.** We will assume residual stresses, available in a layer, as homogenous. So we can use the following expressions for the components of the stress tensor and displacement vector [1]

$$\sigma_{zz}(r,z) = c_{33}\int_0^\infty \alpha^3 \left\{A_1 ch(\alpha z) + A_2\left[sch(\alpha z) + \alpha z sh(\alpha z)\right] + \right.$$

$$\left. + B_1 sh(\alpha z) + B_2\left[ssh(\alpha z) + \alpha z ch(\alpha z)\right]\right\} J_0(\alpha r)\, d\alpha\,; \tag{9}$$

$$\sigma_{rz}(r,z) = -c_{31}\int_0^\infty \alpha^3 \left\{A_1 sh(\alpha z) + A_2\left[s_0 sh(\alpha z) + \alpha z ch(\alpha z)\right] + \right.$$

$$\left. + B_1 ch(\alpha z) + B_2\left[s_0\, ch(\alpha z) + \alpha z\, sh(\alpha z)\right]\right\} J_1(\alpha r)\, d\alpha\,; \tag{10}$$

$$u_z(r,z) = m\int_0^\infty \alpha^2 \left\{A_1 sh(\alpha z) + A_2\left[s_1 sh(\alpha z) + \alpha z\, ch(\alpha z)\right] + \right. \tag{11}$$

$$\left. + B_1 ch(\alpha z) + B_2\left[s_1 ch(\alpha z) + \alpha z\, sh(\alpha z)\right]\right\} J_0(\alpha r)\, d\alpha\,.$$

Constants $c_{31}$, $c_{33}$, $m$, $s$, $s_0$, $s_1$ depend on the nature of elastic potential and are selected in each case separately [1]. Unknown functions $A_1$, $B_1$, $A_2$, $B_2$ are defined by the boundary conditions of the problem.

Demanding the fulfillment of the boundary conditions (2) and (5) out of the ratio (10), we will have:

$$A_1 sh(\alpha h) + A_2\left[s_0 sh(\alpha h) + \alpha h ch(\alpha h)\right] + B_1 ch(\alpha h) + B_2\left[s_0\, ch(\alpha h) + \alpha h\, sh(\alpha h)\right] = 0\,;$$

$$-A_1 sh(\alpha h) - A_2\left[s_0 sh(\alpha h) + \alpha h ch(\alpha h)\right] + B_1 ch(\alpha h) + B_2\left[s_0\, ch(\alpha h) + \alpha h\, sh(\alpha h)\right] = 0\,.$$

From the last equations we get

$$A_1 = -\frac{A_2\left[s_0 sh(\alpha h) + \alpha h ch(\alpha h)\right]}{sh(\alpha h)}; \quad B_1 = -\frac{B_2\left[s_0\, ch(\alpha h) + \alpha h\, sh(\alpha h)\right]}{ch(\alpha h)}\,. \tag{12}$$

Demanding the fulfillment of the boundary conditions (1) and (4) out of the ratio (9), we will have:

$$c_{33}\int_0^\infty \alpha^3 \left\{A_1 ch(\alpha h) + A_2\left[sch(\alpha h) + \alpha h sh(\alpha h)\right] + \right.$$

$$\left. + B_1 sh(\alpha h) + B_2\left[s\, sh(\alpha h) + \alpha h ch(\alpha h)\right]\right\} J_0(\alpha r)\, d\alpha = 0,\ r \le a, b \le r < \infty; \tag{13}$$

$$c_{33}\int_0^\infty \alpha^3 \left\{A_1 ch(\alpha h) + A_2\left[sch(\alpha h) + \alpha h sh(\alpha h)\right] - \right.$$

$$\left. - B_1 sh(\alpha h) - B_2\left[s\, sh(\alpha h) + \alpha h ch(\alpha h)\right]\right\} J_0(\alpha r)\, d\alpha = 0,\ c \le r < \infty. \tag{14}$$

Having introduced two unknown functions $x(r)$ and $y(r)$, sets at intervals $[a,b]$ and $[0,c]$ respectively, we will continue the ratio (13) and (14) on the whole additional semi-axis $[0,\infty)$

$$c_{33}\int_0^\infty \alpha^3 \left\{A_1\, ch(\alpha h) + A_2\left[s\, ch(\alpha h) + \alpha h\, sh(\alpha h)\right] + \right.$$

$$+B_1 sh(\alpha h)+B_2\left[s\, sh(\alpha h)+\alpha h\, ch(\alpha h)\right]\Big\} J_0(\alpha r)d\alpha = x(r)\eta(a-r),\ \ 0\le r<\infty; \qquad (15)$$

$$c_{33}\int_0^\infty \alpha^3\left\{A_1 ch(\alpha h)+A_2\left[s\, ch(\alpha h)+\alpha h\, sh(\alpha h)\right]-\right.$$

$$\left.-B_1 sh(\alpha h)-B_2\left[s\, sh(\alpha h)+\alpha h\, ch(\alpha h)\right]\right\} J_0(\alpha r)d\alpha = y(r)\eta(c-r),\ \ 0\le r<\infty. \qquad (16)$$

where $\eta(r)$ – is Heaviside function.

Applying the formula of the inverse Hankel transformation to the equations (15) and (16), using the ratios (12) we will get the system of relatively unknown $A_2$, $B_2$. Solving it we will have

$$\alpha^2 A_2 = \frac{sh(\alpha h)}{2c_{33}\left[(s-s_0)ch(\alpha h)sh(\alpha h)-\alpha h\right]}\left[I_1(\alpha)+I_2(\alpha)\right]; \qquad (17)$$

$$\alpha^2 B_2 = \frac{ch(\alpha h)}{2c_{33}\left[(s-s_0)ch(\alpha h)sh(\alpha h)+\alpha h\right]}\left[I_1(\alpha)-I_2(\alpha)\right], \qquad (18)$$

where

$$I_1(\alpha)=\int_a^b r x(r) J_0(r\alpha)dr\,; \quad I_2(\alpha)=\int_0^c r y(r) J_0(r\alpha)dr\,.$$

From the boundary conditions (3) and (6), based on the ratios (7), (8), (11), (12), (17) and (18), we will have:

$$\frac{k_1}{2}\int_0^\infty\left[\varphi_1(\alpha)I_1(\alpha)+\varphi_2(\alpha)I_2(\alpha)\right]\left(J_0(\alpha r)-J_0(\alpha a)\right)d\alpha =$$

$$=-\frac{1}{2R_1}(r_a-a)^2\left\{\eta(r-a)-\eta(r-r_1)\right\}+\frac{1}{2R_1}(r_a-r)^2\left\{\eta(r-r_a)-\eta(r-r_1)\right\}; \qquad (19)$$

$$\frac{k_1}{2}\int_0^\infty\left[\varphi_1(\alpha)I_1(\alpha)+\varphi_2(\alpha)I_2(\alpha)\right]\left(J_0(\alpha r)-J_0(\alpha b)\right)d\alpha =$$

$$=-\frac{1}{2R_2}(r_b-b)^2\left\{\eta(r-r_1)-\eta(r-b)\right\}+\frac{1}{2R_2}(r_b-r)^2\left\{\eta(r-r_1)-\eta(r-r_b)\right\}; \qquad (20)$$

$$\frac{k_1}{2}\int_0^\infty\left[\psi_1(\alpha)I_1(\alpha)+\psi_2(\alpha)I_2(\alpha)\right]\left(J_0(\alpha r)-J_0(\alpha c)\right)d\alpha =$$

$$=\frac{1}{2R_3}(c-r_c)^2\eta(c-r)-\frac{1}{2R_3}(r-r_c)^2\left\{\eta(r-r_c)-\eta(r-c)\right\}. \qquad (21)$$

In the latter ratios the following notations have been introduced:

$$\varphi_1(\alpha)=\Delta_1(\alpha)+\Delta_2(\alpha);\quad \varphi_2(\alpha)=\Delta_1(\alpha)-\Delta_2(\alpha);$$

$$\psi_1(\alpha)=-\varphi_2(\alpha);\quad \psi_2(\alpha)=-\varphi_1(\alpha);$$

$$\Delta_1(\alpha)=\frac{(s-s_0)sh^2(\alpha h)}{(s-s_0)ch(\alpha h)sh(\alpha h)-\alpha h};\quad \Delta_2(\alpha)=\frac{(s-s_0)ch^2(\alpha h)}{(s-s_0)ch(\alpha h)sh(\alpha h)+\alpha h};$$

$$k_1=\frac{m(s_1-s_0)}{c_{33}(s-s_0)}.$$

Unknown functions $x(r)$ and $y(r)$ determine the distribution of contact stresses under the punches. Having taken into account their continuity, and the lack of contact at the border area (provided $r=a$ , $r=b$ and $r=c$ ), we will introduce $x(r)$ and $y(r)$ in the form of partial sums of series according to the linear combinations of Bessel functions

$$x(r)=\sigma_{zz}(r,h)=\sum_{n=1}^{N}a_nL_n(r),\ a\le r\le b; \tag{22}$$

$$y(r)=\sigma_{zz}(r,-h)=\sum_{n=1}^{N}b_nJ_0\left(\frac{r}{c}\beta_n\right), 0\le r\le c, \tag{23}$$

where $L_n(r)=J_0\left(\frac{\gamma_n}{a}r\right)Y_0(\gamma_n)-Y_0\left(\frac{\gamma_n}{a}r\right)J_0(\gamma_n),\ a\le r\le b$;

$\beta_n$ and $\gamma_n$ – additional roots of equations $J_0(x)=0$ and $J_0\left(\frac{b}{a}x\right)Y_0(x)-Y_0\left(\frac{b}{a}x\right)J_0(x)=0$ .

Having substituted the ratios (22) and (23) into equalities (19) – (21), we will get

$$\frac{k_1}{2}\left\{\sum_{n=1}^{N}a_n\int_0^\infty\left[\varphi_1(\alpha)I_n^{(1)}(\alpha)\left(J_0(\alpha r)-J_0(\alpha a)\right)d\alpha+\right.\right.$$
$$\left.+\sum_{n=1}^{N}b_n\int_0^\infty\left[\varphi_2(\alpha)I_n^{(2)}(\alpha)\left(J_0(\alpha r)-J_0(\alpha a)\right)d\alpha\right\}=$$
$$=-\frac{1}{2R_1}(r_a-a)^2\left\{\eta(r-a)-\eta(r-r_1)\right\}+\frac{1}{2R_1}(r_a-r)^2\left\{\eta(r-r_a)-\eta(r-r_1)\right\}; \tag{24}$$

$$\frac{k_1}{2}\left\{\sum_{n=1}^{N}a_n\int_0^\infty\left[\varphi_1(\alpha)I_n^{(1)}(\alpha)\left(J_0(\alpha r)-J_0(\alpha b)\right)d\alpha+\right.\right.$$
$$\left.+\sum_{n=1}^{N}b_n\int_0^\infty\left[\varphi_2(\alpha)I_n^{(2)}(\alpha)\left(J_0(\alpha r)-J_0(\alpha b)\right)d\alpha\right\}=$$
$$=-\frac{1}{2R_2}(r_b-b)^2\left\{\eta(r-r_1)-\eta(r-b)\right\}+\frac{1}{2R_2}(r_b-r)^2\left\{\eta(r-r_1)-\eta(r-r_b)\right\}; \tag{25}$$

$$\frac{k_1}{2}\left\{\sum_{n=1}^{N}a_n\int_0^\infty\left[\psi_1(\alpha)I_n^{(1)}(\alpha)\left(J_0(\alpha r)-J_0(\alpha c)\right)d\alpha+\right.\right.$$
$$\left.+\sum_{n=1}^{N}b_n\int_0^\infty\left[\psi_2(\alpha)I_n^{(2)}(\alpha)\left(J_0(\alpha r)-J_0(\alpha c)\right)d\alpha\right\}=$$
$$=\frac{1}{2R_3}(c-r_c)^2\eta(c-r)-\frac{1}{2R_3}(r-r_c)^2\left\{\eta(r-r_c)-\eta(r-c)\right\}. \tag{26}$$

$$I_n^{(1)}(\alpha)=\int_a^b rL_n(r)J_0(r\alpha)dr;\quad I_n^{(2)}(\alpha)=\int_0^c rJ_0\left(\frac{r}{c}\beta_n\right)J_0(r\alpha)dr.$$

Having multiplied both sides of ratios (24) and (25) by $r\,L_q(r)$ and the ratio (26) by $r\,J_0\left(\frac{r}{c}\beta_q\right)$ by $q=\overline{1,N}$ , and having integrated the obtained expressions in $r$ we will have

$$\begin{cases}\sum_{n=1}^{N}a_nK_{qn}^{(11)}(\alpha)+\sum_{n=1}^{N}b_nK_{qn}^{(12)}(\alpha)=B_q^{(1)};\\ \sum_{n=1}^{N}a_nK_{qn}^{(21)}(\alpha)+\sum_{n=1}^{N}b_nK_{qn}^{(22)}(\alpha)=B_q^{(2)};\\ \sum_{n=1}^{N}a_nK_{qn}^{(31)}(\alpha)+\sum_{n=1}^{N}b_nK_{qn}^{(32)}(\alpha)=B_q^{(3)}.\end{cases} \tag{27}$$

$$K_{qn}^{(11)}(\alpha)=\int_0^\infty\left[\varphi_1(\alpha)I_n^{(1)}(\alpha)\left[I_q^{(1)}(\alpha)-J_0(a\alpha)F_q^{(1)}\right]d\alpha\right.;$$

$$K_{qn}^{(12)}(\alpha)=\int_0^\infty\left[\varphi_2(\alpha)I_n^{(2)}(\alpha)\left[I_q^{(1)}(\alpha)-J_0(a\alpha)F_q^{(1)}\right]d\alpha\right.;$$

$$K_{qn}^{(21)}(\alpha)=\int_0^\infty\left[\varphi_1(\alpha)I_n^{(1)}(\alpha)\left[I_q^{(1)}(\alpha)-J_0(b\alpha)F_q^{(1)}\right]d\alpha\right.;$$

$$K_{qn}^{(22)}(\alpha)=\int_0^\infty\left[\varphi_2(\alpha)I_n^{(2)}(\alpha)\left[I_q^{(1)}(\alpha)-J_0(b\alpha)F_q^{(1)}\right]d\alpha\right.;$$

$$K_{qn}^{(31)}(\alpha)=\int_0^\infty\left[\psi_1(\alpha)I_n^{(1)}(\alpha)\left[I_q^{(2)}(\alpha)-J_0(c\alpha)F_q^{(2)}\right]d\alpha\right.;$$

$$K_{qn}^{(32)}(\alpha)=\int_0^\infty\left[\psi_2(\alpha)I_n^{(2)}(\alpha)\left[I_q^{(2)}(\alpha)-J_0(c\alpha)F_q^{(2)}\right]d\alpha\right.;$$

$$B_q^{(1)}=\frac{1}{k_1R_1}\left[\int_{r_a}^{r_1}r(r_a-r)^2L_q(r)dr-(r_a-a)^2\int_a^{r_1}rL_q(r)dr\right];$$

$$B_q^{(2)}=\frac{1}{k_1R_2}\left[\int_{r_1}^{r_b}r(r_b-r)^2L_q(r)dr-(r_b-b)^2\int_{r_1}^{b}rL_q(r)dr\right];$$

$$B_q^{(3)}=\frac{1}{k_1R_3}\left[(c-r_c)^2\int_0^c rJ_0\left(\frac{r}{c}\beta_q\right)dr-\int_{r_c}^{c}r(r-r_c)^2J_0\left(\frac{r}{c}\beta_q\right)dr\right];$$

$$F_q^{(1)}=\int_a^b rL_q(r)dr;\quad F_q^{(2)}=\int_0^c rJ_0\left(\frac{r}{c}\beta_q\right)dr.$$

Having used the method of superposition and entering the designation

$$a_n=\frac{1}{k_1}\left[\frac{1}{R_1}a_n^{(1)}+\frac{1}{R_2}a_n^{(2)}+\frac{1}{R_3}a_n^{(3)}\right];\ b_n=\frac{1}{k_1}\left[\frac{1}{R_1}b_n^{(1)}+\frac{1}{R_2}b_n^{(2)}+\frac{1}{R_3}b_n^{(3)}\right],\tag{28}$$

from (27) we will obtain the systems of equations relatively to the unknown $a_n^{(1)}$, $a_n^{(2)}$, $a_n^{(3)}$, $b_n^{(1)}$, $b_n^{(2)}$ та $b_n^{(3)}$, $n=\overline{1,N}$.

The values $R_1$, $R_2$ та $R_3$ in the ratios (28) we determine from the conditions of balance of the punches

$$2\pi\int_a^b r\sigma_{zz}(r,h)dr=-P;\qquad 2\pi\int_0^c r\sigma_{zz}(r,-h)dr=-P,$$

and the condition of equality of the vertical displacements of points of the upper boundary plane of the layer provided $r=r_a$ and $r=r_b$.

Finally, from the ratios (22), (23) and (28) we will get the formulas for finding the function of distribution of contact stresses under punches

$$\begin{aligned}\sigma_{zz}(r,h)&=\frac{1}{k_1}\sum_{n=1}^{N}\left[\frac{1}{R_1}a_n^{(1)}+\frac{1}{R_2}a_n^{(2)}+\frac{1}{R_3}a_n^{(3)}\right]L_n(r);\\ \sigma_{zz}(r,-h)&=\frac{1}{k_1}\sum_{n=1}^{N}\left[\frac{1}{R_1}b_n^{(1)}+\frac{1}{R_2}b_n^{(2)}+\frac{1}{R_3}b_n^{(3)}\right]J_0\left(\frac{r}{c}\beta_n\right).\end{aligned}\tag{29}$$

**Numerical example.** On the fig. 2 and 3 the graphs of functions are introduced $\sigma^* = \dfrac{\sigma_{zz}(r,0)}{P}$, that characterize the distribution of contact stresses (30) under the upper and lower punches according to the case of existence in the plane of elastic potential of Bartenev-Khazanovych [1] and on the fig. 4 and 5 – according to the case of the potential of harmonic type. The dashed curve on figures corresponds to the case of the absence of the residual deformations ($\lambda_1 = 1$), the curve 1 – corresponds to the case of available deformations of the stretching ($\lambda_1 = 1.2$), and the curve 2 – corresponds to the deformation of the compression ($\lambda_1 = 0.8$).

As can be seen from the figures, the nature of the field of initial deformations influences the distribution of contact stresses significantly.

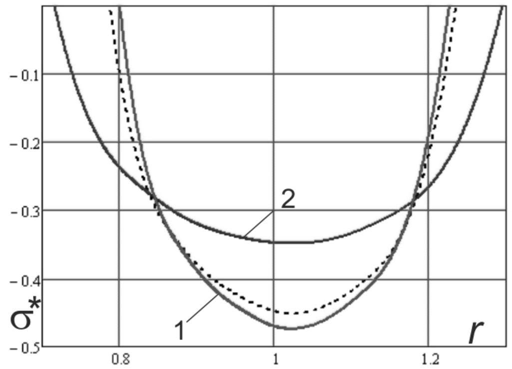


**Figure 2.** Distribution of contact stresses under the upper punch in the case of the Bartenev-Khazanovych potential

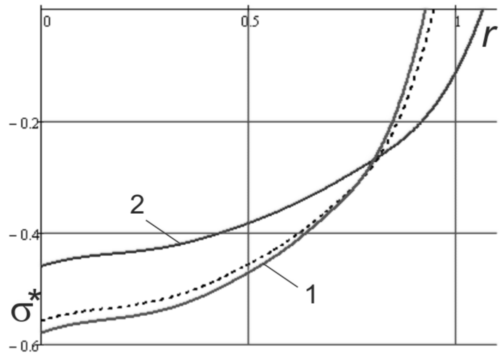


**Figure 3.** Distribution of contact stresses under the lower punch in the case of the Bartenev-Khazanovych potential

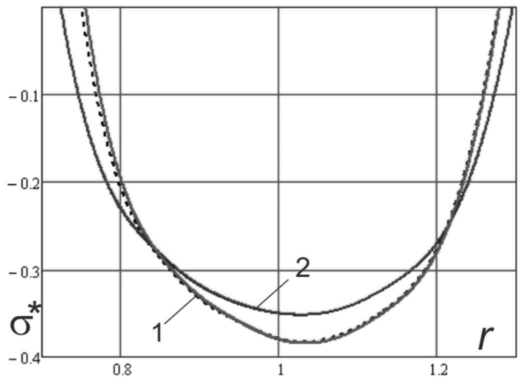


**Figure 4.** Distribution of contact stresses under the upper punch in the case of the harmonic-type potential

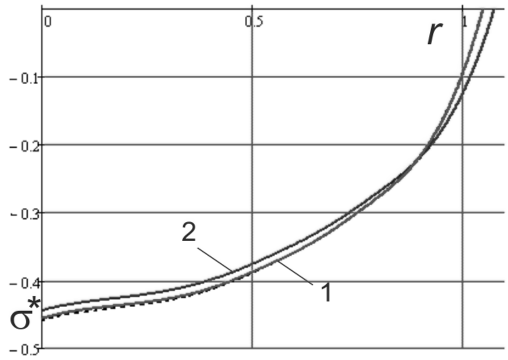


**Figure 5.** Distribution of the contact stresses under the lower punch in the case of harmonic type potential

**Conclusions.** Within the framework of linearized elasticity the formulation and the solution of axisymmetric contact task has been presented about the compression of the ring-parabolic and parabolic punches of the preliminary-stressed plate. The influence of the initial deformations on the distribution of contact stresses has been analyzed. The patterns have been revealed that are traced also in the works of other authors [2-4]:

a) the presence of residual deformations of stretching in the plate cause narrowing of the contact area and increasing of the absolute value of the contact efforts. The value of the caused changes depends on the type of elastic potential. In case of availability of the elastic potential of harmonic type, availability of 20% of stretching deformations leads to increased contact stresses by only 1% and in the case of potential Bartenev-Khazanovych – by 5%;

б) the presence of residual deformations of compression in the plate in turn entails the contact area and reduce the absolute value of contact stresses. Particularly in the case of elastic potential of harmonic type 20% compression reduces to 10% of normal stresses. Thus in cases of potential of Bartenev-Khazanovych normal stresses are reduced by 25%.

**References**

1. Huz O.M. Kontaktna vzaiemodiia pruzhnykh til z pochatkovymy napruzhenniamy: Navch. posibnyk, Huz O.M., Babych S.Yu., Rudnytskyi V.B. K: Vyshcha shkola, 1995. – P. 304 [in Ukrainian].
2. Habrusieva I.Yu. Contact interaction of a circular punch with a preliminarily stressed isotropic layer, B.G. Shelestovs`kyi, I.Yu. Habrusieva, Journal of mathematical sciences. 2012. Vol. 186, No. 1. P. 48 – 60.
3. Habrusieva I.Yu. Kontaktna zadacha dlia parabolichnoho shtampa ta poperedno napruzhenoho pivprostoru, I.Yu. Habrusieva, Mizhvuzivskyi zbirnyk "Naukovi notatky". 2015. Vol. 51. P. 231 – 236 [in Ukrainian].
4. Shelestovskyi B.H. Vzaiemodiia kiltsevoho shtampa iz poperedno napruzhenym sharom u vypadku potentsialu Bartenieva-Khazanovycha, B.H. Shelestovskyi, I.Yu. Habrusieva, Visnyk TNTU. 2010. Vol. 15, No. 3. P. 14 – 22 [in Ukrainian].

**Список використаної літератури**

1. Гузь, О.М. Контактна взаємодія пружних тіл з початковими напруженнями: навч. посібник [Текст] / О.М. Гузь, С.Ю. Бабич, В.Б. Рудницький. – К: Вища школа, 1995. – 304 с.
2. Gabrusseva, I.Yu. Contact interaction of a circular punch with a preliminarily stressed isotropic layer [Text] / B.G. Shelestovs`kyi, I.Yu. Gabrusseva // Journal of mathematical sciences. – 2012. – Vol. 186, No. 1. – P. 48 – 60.
3. Габрусєва, І.Ю. Контактна задача для параболічного штампа та попередньо напруженого півпростору [Текст] / І.Ю. Габрусєва // Міжвузівський збірник «Наукові нотатки». – 2015. – Вип. 51. – С. 231 – 236.
4. Шелестовський, Б.Г. Взаємодія кільцевого штампа із попередньо напруженим шаром у випадку потенціалу Бартенєва-Хазановича [Текст] / Б.Г. Шелестовський, І.Ю. Габрусєва // Вісник ТНТУ. – 2010. – Т. 15, № 3. – С. 14 – 22.

**УДК 539.3**

## СТИСНЕННЯ ПОПЕРЕДНЬО НАПРУЖЕНОЇ ПЛИТИ ДВОМА СПІВВІСНИМИ ШТАМПАМИ СКЛАДНОЇ КОНФІГУРАЦІЇ

**Олег Панчук**

*Тернопільський національний технічний університет імені Івана Пулюя, Тернопіль, Україна*

***Резюме.*** *Наведено розв'язок контактної задачі про взаємодію кільцево-параболічного та параболічного штампів із попередньо напруженою плитою. Побудову аналітичних розв'язків для плити проведено шляхом її моделювання попередньо напруженим шаром скінченної товщини. Системи парних та потрійних інтегральних рівнянь, які при цьому отримано, розв'язано за допомогою подання шуканих функцій у вигляді скінченних сум ряду за функціями Бесселя з невідомими коефіцієнтами та подальшим отриманням скінчених систем лінійних алгебраїчних рівнянь для їх знаходження. На основі отриманого розв'язку проаналізовано вплив характеру поля початкових деформацій на напружений стан товстої плити.*

***Ключові слова:*** *лінеаризована теорія пружності, контактна взаємодія, контактні напруження, кільцево-параболічний штамп, параболічний штамп, плита, шар, початкові деформації, попередні напруження.*

*Отримано 10.09.2016*