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THERMOMAGNETOELECTROELASTICITY OF ANISOTROPIC SOLIDS WITH SPATIAL NON-FLAT THIN INCLUSIONS

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***Summary**. Based on the application of coupling principle for continua of different dimension the mathematical models of thin deformable inclusions for thermomagnetoelectroelastic solids are proposed. Corresponding integral equations are derived and the boundary element method for their solution is developed. The key features of the latter are the usage of discontinuous boundary elements, special shape functions, nonlinear mappings for smoothing the sub-integral at the element's boundary and the modified Kutt's quadrature for numerical evaluation of singular integrals. All these made possible to develop efficient numerical approach for the solution of the stated problem class. Numerical example is considered, which studies thin inhomogeneity of paraboloidal shape.*
***Key words**: thermomagnetoelectroelasticity, thin inclusion, integral equation, boundary element method.*

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Statement of the problem. Thermomagnetoelectroelastic materials are widely used now for the production of different devices by modern advanced high-tech manufactures, fine mechanic devices in particular. These materials are intellectual composites created on the basis of the mechanical combination (stochastic or ordered) of the pyroelectric (ferroelectric) and magnetoelectric (piezomagnetic) materials, which will make possible to transform the fields of different physical nature, that is, it is the method for creating the sensors, position fine sensors in particular. In its turn, the science is challenged by the production to deal with the tasks and problems to build integral mathematic models and analysis methods of certain bodies, which have both structure defects and specially introduced thin layers, which change the operation macro-and micro-properties of these bodies.

Analysis of the available investigations. Nowadays the methods of analysis of the mechanical, electric and magnetic fields interaction in the anisotropic intellectual materials are developed efficiently, the boundary element method in particular. For example, Rungamornrat and Mear [1], as well as Rungamornrat et al. [2] have obtained the symmetric Galerkin boundary element method for the investigation of the spatial cracks in the piezoelectric bodies. Zhao et al. [3] have proposed the method of boundary integral equations for extended jumps of physical-mechanic fields for the study of the vertical crack systems in the magnetoelectroelastic medium. Muñoz-Reja et al. [4] have developed three-dimensional boundary element method for the study of the mechanic problems of the anisotropic magnetoelectroelastic materials fracture.

On the contrary to the tasks, where the thermal effects are taken into account, there are only some available papers, which deal with the flat defects, in the papers [5 – 7] the tasks on thermoelectroelasticity being analysed for the transversal-isotropic bodies with one or two concentric disk cracks.

Only recently it was managed to obtain the integral equations of the three-dimensional thermomagnetoelectroelasticity of the anisotropic bodies of arbitrary shape with the hole system or internal sections [8, 9], which made possible to study the interaction of fields with different physical nature in the anisotropic bodies with the spatial cracks [9]. These integral

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relations make possible to analyse more thouroughly the class of tasks, bodies with thin inclusions in particular.

The objective of the paper. To derive the integral equations and highly precise and effective scheme of the boundary elements method for the numerical analysis of the anisotropic thermomagnetoelectroelastic bodies with the thin non-flat inclusions, non-smooth in particular.

Statement of the task. According to [8 – 1] in the fixed rectangular coordinate system $Ox_1x_2x_3$ the balance equation, the Maxwell equation (Gauss theorem for electric and magnetic fields) and balance relations of the heat conduction in the stationary case will look like:

$$\sigma_{ij,j} + f_i = 0,\ \ D_{i,i} - q = 0,\ \ B_{i,i} + b_m = 0,\ \ h_{i,i} - f_h = 0 \quad (i, j = 1,2,3). \tag{1}$$

Here σ_{ij} – stress tensor components; h_i – heat flow density vector components; D_i – electric displacement; B_i – magnetic field induction; f_i – volume forces; q – free charges density; f_h – density of distributed heat (discharge) sources; b_m – DC volume density, which equals zero for the dielectric. In the formulas the Einstein summarising rule due to the repetitive index is assumed. Comma in the indexes is treated as differentiation according to the coordinate, the index of which follows the comma, that is, $u_{i,j} \equiv \partial u_i / \partial x_j$.

Constitutive relations of the linear thermomagnetoelectroelasticity and heat conductivity according to [10] look like:

$$\begin{aligned} &\sigma_{ij} = C_{ijkm}u_{k,m} - e_{pij}E_p - h_{pij}H_p - \beta_{ij}\theta, \\ &D_i = e_{ikm}u_{k,m} + \kappa_{ip}E_p + \gamma_{ip}H_p + \chi_i\theta, \\ &B_i = h_{ikm}u_{k,m} + \gamma_{ip}E_p + \mu_{ip}H_p - \nu_i\theta, \\ &h_i = -k_{ij}\theta_{,j}, \end{aligned} \tag{2}$$

where u_i – body points displacement; ϕ – electric potential; ψ – stationary magnetic field potential; θ – temperature change compared with the initial; C_{ijkm} – elastic constants; k_{ij} – heat conductivity coefficients; β_{ij} – thermal expansion moduli (thermal stress coefficients); e_{ijk} – piezoelectric constants; κ_{ij} – material dielectric constants; h_{ijk} – piezomagnetic constants; μ_{ij}, γ_{ij} – material magnetic and electromagnetic permeability; χ_i – pyroelectric coefficients; ν_i – pyromagnetic coefficients. The tensors with components C_{ijkm}, k_{ij}, κ_{ij}, μ_{ij}, γ_{ij} and β_{ij} are considered to be symmetric.

The equations (1) and (2) are easily to be unificated and presented as follows:

$$\tilde{\sigma}_{Ij,j} + \tilde{f}_I = 0,\ \ h_{i,i} - f_h = 0; \tag{3}$$

$$\tilde{\sigma}_{Ij} = \tilde{C}_{IjKm}\tilde{u}_{K,m} - \tilde{\beta}_{Ij}\theta,\ \ h_i = -k_{ij}\theta_{,j}, \tag{4}$$

where

$$\begin{aligned}
&\tilde{u}_i = u_i,\ \tilde{u}_4 = \phi,\ \tilde{u}_5 = \psi\ ;\ \tilde{f}_i = f_i,\ \tilde{f}_4 = -q,\ \tilde{f}_5 = b_m;\\
&\tilde{\sigma}_{ij} = \sigma_{ij},\ \tilde{\sigma}_{4j} = D_j,\ \tilde{\sigma}_{5j} = B_j;\\
&\tilde{C}_{ijkm} = C_{ijkm},\ \tilde{C}_{ij4m} = e_{mij},\ \tilde{C}_{4jkm} = e_{jkm},\ \tilde{C}_{4j4m} = -\kappa_{jm},\\
&\tilde{C}_{ij5m} = h_{mij},\ \tilde{C}_{5jkm} = h_{jkm},\ \tilde{C}_{5j5m} = -\mu_{jm},\\
&\tilde{C}_{4j5m} = -\gamma_{jm},\ \tilde{C}_{5j4m} = -\gamma_{jm};\\
&\tilde{\beta}_{ij} = \beta_{ij},\ \tilde{\beta}_{4j} = -\chi_j,\ \tilde{\beta}_{5j} = \nu_j.
\end{aligned} \tag{5}$$

Here and below the indexes marked by capital letters change from 1 to 5, and those by small ones – from 1 to 3, that is, $I = 1, 2, ..., 5$, $i = 1, 2, 3$.

According to [9] boundary value problems for the differential equations in the partial derivatives (3), (4) in the case of the body with the discontinuous surfaces of the physical-mechanical fields deal with the solving of such systems of hypersingular integral equations:

- heat conduction

$$\begin{aligned}
&\frac{1}{2}\Sigma\theta(\mathbf{x}_0) = \iint_S \Theta^*(\mathbf{x},\mathbf{x}_0)\Sigma h_n(\mathbf{x})\,dS(\mathbf{x}) - \mathrm{CPV}\iint_S H^*(\mathbf{x},\mathbf{x}_0)\Delta\theta(\mathbf{x})\,dS(\mathbf{x}),\\
&\frac{1}{2}\Delta h_n(\mathbf{x}_0) = n_i(\mathbf{x}_0)\Bigg[\mathrm{CPV}\iint_S \Theta_i^{**}(\mathbf{x},\mathbf{x}_0)\Sigma h_n(\mathbf{x})\,dS(\mathbf{x}) -\\
&\qquad - \mathrm{HFP}\iint_S H_i^{**}(\mathbf{x},\mathbf{x}_0)\Delta\theta(\mathbf{x})\,dS(\mathbf{x})\Bigg],
\end{aligned} \tag{6}$$

- thermomagnetoelectroelasticity

$$\begin{aligned}
\frac{1}{2}\Delta\tilde{t}_I(\mathbf{x}_0) = n_j\Bigg[&\mathrm{CPV}\iint_S D_{IjK}(\mathbf{x},\mathbf{x}_0)\Sigma\tilde{t}_K(\mathbf{x})\,dS(\mathbf{x}) - \mathrm{HFP}\iint_S S_{IjK}(\mathbf{x},\mathbf{x}_0)\Delta\tilde{u}_K(\mathbf{x})\,dS(\mathbf{x})\\
&+ \mathrm{CPV}\iint_S Q_{Ij}(\mathbf{x},\mathbf{x}_0)\Delta\theta(\mathbf{x})\,dS(\mathbf{x}) + \iint_S W_{Ij}(\mathbf{x},\mathbf{x}_0)\Sigma h_n(\mathbf{x})\,dS(\mathbf{x})\Bigg],
\end{aligned} \tag{7}$$

where S – discontinuity surface of the physical-mechanical fields with the shores S^+ and S^- correspondingly; $\Sigma f = f^+ + f^-$; $\Delta f = f^+ - f^-$; n_p – components of the normal unit vector to the surface S $\left(S^+\right)$; $\tilde{t}_I = \tilde{\sigma}_{Ij} n_j$ – components of the extend stress vector; $h_n = h_i n_i$ – heat flow through the surface; CPV – Cauchy Principal Value; HFP – Hadamard Finite Part. Nucleus of these integral dependencies are presented in [9].

Modelling of thin inclusion. While modelling the bodies with thin inhomogeneity the principle of coupling the continua of different dimension [11] is often used. The essence of it is in the displacement of the thin inclusion of some volume by some discontinuous surface of the stress fields, displacements, temperature, electric or magnetic potentials, etc. (Fig. 1). The most often this surface is chosen as the medium surface of this inhomogeneity. The inclusion is excluded from the analysis as the geometric unit and it is assumed, that its effect is treated as formation of some surface in the body (in two-dimension tasks) of the physical-mechanical fields discontinuity line. Here according to the function jump method [11] the study of the body stress state can be associated with the unknown jump functions and analysed without real properties of the inclusion, material being taken into account. It is clear, that it depends on the jump function, properties of the body material, geometric configuration of the task, contact conditions of the thin inhomogeneity with the medium and external loading.

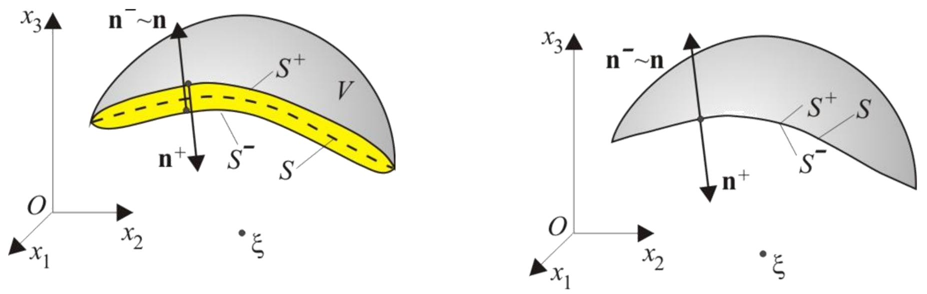

Figure 1. Sketch for modeling of thin inclusion based on the coupling principle

On the other hand, because of small thickness of inclusions stress and displacement vectors, temperature and the heat flow, electrical potentials and displacements on its opposite shores must be somehow connected. Corresponding dependencies, which contain physical-mechanical properties of the inclusion and its thickness, are the mathematic model of the inclusion, which do not depend on the properties of the main material and can be treated as some internal task. The mathematic model of the thin inclusion must meet only three main requirements [11]: 1) The number of equations must be equal to the number of unknown values on the shores, of the mathematic section in the outside task (number of the jump function); 2) the model must be simple enough in order the resultant equation system to be solved easily or at least possible; 3) the model must be adequate enough to demonstrate specific characteristics of the inclusion deformation and other investigated processes.

Using the conditions of the body and inclusion contact in the mathematic model makes possible to obtain the so-called conditions of the inclusion and body interrelation, which can be interpreted as the special conditions of the non-ideal contact between the body surfaces, which are adjoining to take opposite shores of the inclusion. Because of it the specific characteristics of the physical-mechanical properties of the inclusion and its contact with the medium are in the interrelation condition itself. If basing on the outside task the stress and displacement vectors, thermal flow and temperature, electrical potentials and displacement, magnetic potential and induction on the opposite shores of the inclusion are found and presented in the interrelation conditions, the equations of the unknown functions of the jump will be obtained.

As the solutions of the outside and inside tasks in the method of jump function are absolutely independent, the change of the interrelation conditions under the same solution of the certain outside task makes possible to analyse the tasks for the same body with the different models of the inclusion. The specified interrelation conditions, moreover, the model or the type of inclusion, can be associated with the different solutions of the outside tasks.

In the paper in question the principle of different dimension continua coupling is used. Thus, the outside tasks relatively the inclusion is described by the relations (6), (7). The inclusion model is obtained by averaging the initial equations (3), (4) under the small inclusion thickness, having assumed there being softly and insufficient electrical, magnetic and thermal permeability (as compared with those of the body properties) of its material. The equations are written as follows:

$$\Sigma h_n(\mathbf{x}_0)=0;\quad \Delta h_n(\mathbf{x}_0)=\frac{k_{11}^{\prime i}(\mathbf{x}_0)}{h(\mathbf{x}_0)}\Delta\theta(\mathbf{x}_0)+2h_0(\mathbf{x}_0);$$

$$\Sigma\tilde{t}_I=0;\quad \mathbf{\Delta}\tilde{\mathbf{t}}(\mathbf{x}_0)=-\frac{\mathbf{V}(\mathbf{x}_0)}{h(\mathbf{x}_0)}\mathbf{\Delta}\tilde{\mathbf{u}}(\mathbf{x}_0)-\mathbf{v}(\mathbf{x}_0)\Sigma\theta(\mathbf{x}_0)+2\tilde{\mathbf{t}}_0(\mathbf{x}_0);\tag{8}$$

$$\mathbf{V}=\mathbf{\Omega}^{\mathrm{T}}\mathbf{C}_{33}^{\prime i}\mathbf{\Omega};\quad \mathbf{v}=\mathbf{\Omega}^{\mathrm{T}}\tilde{\boldsymbol{\beta}}_3^{\prime i}.$$

Here $\mathbf{\Omega}$ – rotation matrix to the coordinate system, the axis Ox_3' of which is directed along the normal $\mathbf{n}$; $2h$ – inclusion thickness; h_0, $\tilde{\mathbf{t}}_0$ – outside thermal and magneto-electromechanical loading applied to the inclusion.

The relations (7), (8) form the system of integral equations relatively the unknown functions of the temperature jump $\Delta\theta$ and the extended displacement vector $\mathbf{\Delta}\tilde{\mathbf{u}}$ on the shores of the medium surface S of the model thin inclusion. The solution of these integral equations, especially in the case of non-canonic shape surfaces, is easily made numerically taking advantage of the boundary elements method.

The boundary element method for non-flat thin inclusions. The scheme of the boundary element method proposed in the paper [9] is the basis for our work. According to it the surface S is divided into quadrilateral square discontinuous boundary elements. It means, that collocation nodes are exclusively on the element, and not on its boundary. In the case of non-flat surfaces the application of such boundary elements makes possible not to calculate the boundary transitions and gradients at the collocation point threshold, as the latter always is on the smooth surface.

The curvilinear coordinate system $O\xi\eta$ is connected with every boundary element and the element itself is mapped onto the square $-1\le\xi\le1$, $-1\le\eta\le1$, here the interpolation nodes of the element geometry being in 9 points, for which the curvilinear coordinates are equal correspondingly –1; 0; 1, and the collocation nodes are in nine points, for which $\xi=(-2/3;0;2/3)$; $\eta=(-2/3;0;2/3)$.

Boundary conditions with the unknown boundary functions and the jump functions are interpolated with the collocation points on every boundary element Γ_N as follows:

$$\mathbf{b}_N(\xi,\eta)=\sum_{i=1}^{3}\sum_{j=1}^{3}\mathbf{b}_N^{i,j}\phi_i(\xi)\phi_j(\eta),\tag{9}$$

where $\mathbf{b}=(\theta,\Delta\theta,\Sigma\theta,h_n,\Sigma h_n,\Delta h_n,\tilde{u}_I,\Delta\tilde{u}_I,\Sigma\tilde{u}_I,\tilde{t}_I,\Sigma\tilde{t}_I,\Delta\tilde{t}_I)^{\mathrm{T}}$ and the shape discontinuous functions are presented like

$$\phi_1(\xi)=\xi\left(\frac{9}{8}\xi-\frac{3}{4}\right),\quad \phi_2(\xi)=\left(1-\frac{3}{2}\xi\right)\left(1+\frac{3}{2}\xi\right),\quad \phi_3(\xi)=\xi\left(\frac{9}{8}\xi+\frac{3}{4}\right).\tag{10}$$

Beside the shape function (10), the other ones can be used in the equation (9), that is, while modelling the front line of the thin inclusion or the crack in order to take into account the root characteristics of the thermal flow fields and expended stresses the shape functions for the temperature jumps and extended displacements are chosen as follows [9]

$$\phi_i^{\Delta}(\xi)=\sqrt{1\pm\xi}\left(\Phi_{i1}^{\Delta}+\sum_{j=2}^{3}\Phi_{ij}^{\Delta}(1\pm\xi)^{j-1}\right), \tag{11}$$

where Φ_{ij}^{Δ} constants are found from the equations system $\phi_i(\xi_j)=\delta_{ij}$ at $\xi_j=(-2/3;0;2/3)$.

These shape functions make possible to calculate very precisely the generalised intensity factors of the physical-mechanical fields on the front of the thin inhomogeneity [9].

Besides, in the case of the inflexion line on the surface S, the shape functions must be chosen taking into account the peculiarities appearing on this line. That is why while finding integrals (6), (7) on the boundary elements tangential to the inflexion line or to the inclusion line, non-linear reflections were proposed to be used

$$\xi=\frac{1}{2}\left(3-\xi_1^2\right)\xi_1,\ \ \eta=\frac{1}{2}\left(3-\eta_1^2\right)\eta_1,\ \ d\xi d\eta=\frac{9}{4}\left(1-\xi_1^2\right)\left(1-\eta_1^2\right)d\xi_1 d\eta_1, \tag{12}$$

smoothing the sub-integral expression on the boundary element, as the variables substitution jacobian there equals zero.

Such mappings make possible to increase sufficiently the accuracy of the numerical realisation of the method, which, in its turn, contributes to the efficiency of calculations as the result of smaller number of the division elements.

While calculating singular and hyper-singular integrals the transition to the polar coordinate system has been given advantage of and further application of the modified Kutt's quadrature [9], which will make possible to find easily the main value and the finite Adamar's part of the special surface integral.

Numerical example. Let us analyse the transversal-isotropic pyroelectric tyntanatum barium medium possessing such properties [12]:

- modulus of elasticity (MPa): $C_{11}=C_{22}=150$; $C_{33}=146$; $C_{12}=C_{13}=C_{23}=66$; $C_{44}=C_{55}=44$; $C_{66}=\left(C_{11}-C_{12}\right)/2=42$;
- piezoelectric constants (C/m^2): $e_{31}=e_{32}=-4.35$; $e_{33}=17.5$; $e_{15}=e_{24}=11.4$;
- dielectric steels (nF/m): $\kappa_{11}=\kappa_{22}=9.86775$; $\kappa_{33}=11.151$;
- heat conductivity coefficients (W/(m·K)): $k_{11}=k_{22}=k_{33}=2.5$;
- heat expansion coefficients (K^{-1}): $\alpha_{11}=\alpha_{22}=8.53\cdot10^{-6}$; $\alpha_{33}=1.99\cdot10^{-6}$;
- pyroelectric constants (GV/(m·K)): $\lambda_3=13.3\cdot10^{-6}$.

The rest of the mentioned above coefficients are zero. Here the Voigt sign being used (10), according to which the index pairs in (3) are substituted by one index according to the rule $11\leftrightarrow1$; $22\leftrightarrow2$; $33\leftrightarrow3$; $23,32\leftrightarrow4$; $13,31\leftrightarrow5$; $12,21\leftrightarrow6$.

Let us analyse thin inhomogeneity, the medium surface of which forms the section of the elliptic paraboloid of rotation:

$$x_3=\rho\left(x_1^2+x_2^2\right),\ \ x_1^2+x_2^2\le R^2. \tag{13}$$

The inclusion is considered to be very yielding (non-permeable crack), because in this case the intensity factor of the physical-mechanical fields is the greatest. Besides, let us assume, that the inclusion is not affected by the heat expansion.

Let the given self-balanced heat loading $h_0 = \mathrm{const}$ be on the inclusion surface. Additional mechanical loading is not available: $\tilde{\mathbf{t}}_0 \equiv 0$. Any other loading is not applied to the thermomagnetoelectroelastic medium with the inclusion.

Let us divide the medium inhomogeneity surface into 12 boundary elements so, as it is shown in Fig. 2 (the view along the Ox_3 axis).

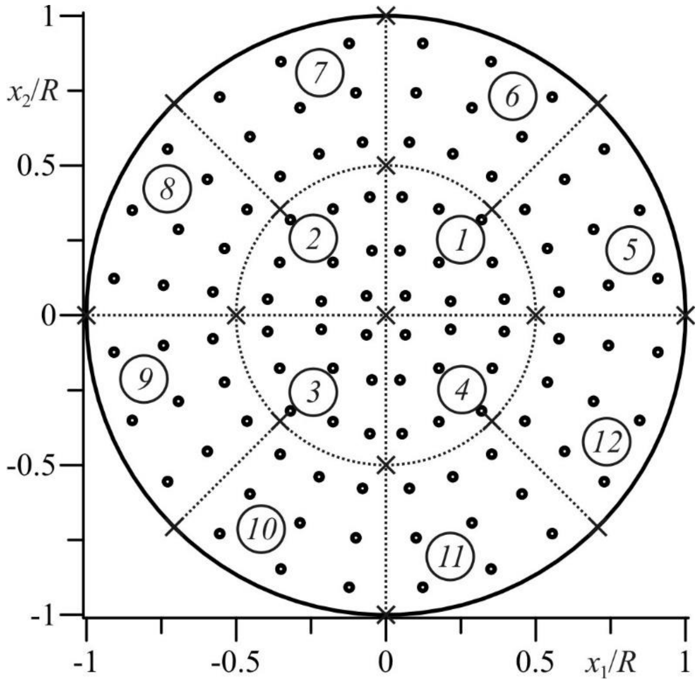

Figure 2. Boundary element mesh

Let us analyse the effect of the ρ parameter of the inclusion medium surface shape (crack) on the intensity factors of physical-mechanical fields on inhomogeneity line, here the rating factors being the values $K_\sigma = h_0 \beta_{11} R \sqrt{\pi R} / k_{11}$, $K_\chi = h_0 \chi_3 R \sqrt{\pi R} / k_{11}$, $K_{h0} = -2h_0 \sqrt{R/\pi}$. Calculated for the fixed values ρ (0,2; 0,4; 0,6 and 0,8) the values of the intensity rating factors of the physical-mechanical fields are shown in Fig. 3.

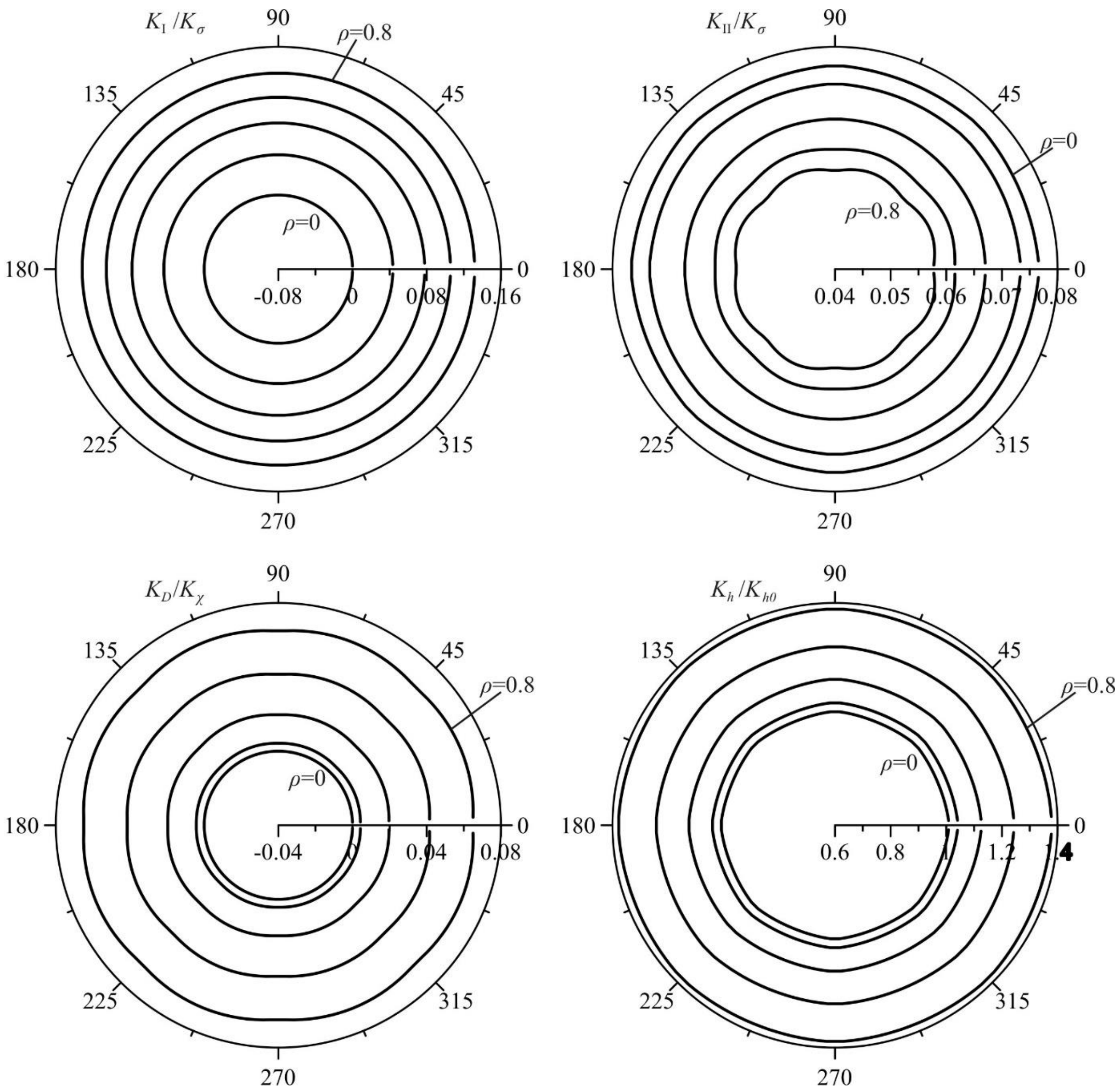

Figure 3. Field intensity factors at inclusion's front line

It is seen, that intensity factors of the physical-mechanical fields are, in fact, constant along the front. Insufficient oscillations are caused by the approximation of the real circular threshold surface by the square boundary elements. It can be noticed, that these vibrations are symmetric and similar at every of the elements. But they are within only 0,7%. Constant values of the intensity factors along the front is caused by the fact, that it is in plane of the medium material isotropy, and the inclusion medium surface itself (crack) is the rotation surface around the polarization axis Ox_3.

In the case $\rho = 0$ of the flat disk-like crack the calculation results coincide with those known [9], which verifies the developed approach. When the ρ parameter increases, which specifies the "non-flat" thin inhomogeneity, stress intensity factors of the Mode I increase, and those of the Mode II – decrease. Stress intensity factors of the Mode III equal zero. The heat flow intensity factors increase to, because the crack surface size increases, when ρ is greater. For the non-flat defects, on the contrary to those flat ones, the electric displacement intensity factors K_D becomes sufficiently.

Conclusions. The mathematic model of the thermomagnetoelectroelastic body with thin inclusions, as well as the boundary element method, which will make possible to solve certain spatial tasks more efficiently (highly precise and quick), have been developed. The characteristic of the proposed boundary-element approach is taking advantage of the coupling principle for continua of different dimension for modeling of thin inclusions, as well as application of the discontinuous boundary elements, non-linear mapping and modified quadratures for the solving of the obtained on its basis the principle of the integral equation system. Besides, using special shape functions it is possible to take into account both the characteristics on the inhomogeneity front and those corresponding on the fracture lines or in the angle points. All these make possible to solve precisely the thermomagnetoelectroelasticity tasks for the bodies with the non-flat thin inclusions or cracks, which could not be done before using conventional numerical approaches, the boundary or finite elements methods in particular.

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УДК 539.3

ТЕРМОМАГНІТОЕЛЕКТРОПРУЖНІСТЬ АНІЗОТРОПНИХ ТІЛ ІЗ ПРОСТОРОВИМИ НЕПЛОСКИМИ ТОНКИМИ ВКЛЮЧЕННЯМИ

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Резюме. *На основі застосування принципу спряження континуумів різної вимірності запропоновано математичні моделі тонких деформівних включень у термомагнітоелектропружних тілах. Побудовано інтегральні рівняння відповідної задачі та метод граничних елементів для розв'язування. Ключовими особливостями останнього є використання розривних елементів, спеціальних функцій форми, нелінійних відображень для згладжування підінтегральних виразів на межах елементів та модифікованих квадратур Кутта для обчислення особливих інтегралів. Усе це дало можливість створити високоефективний числовий підхід розв'язування сформульованого класу задач. Наведено числовий приклад із вивчення тонкої неоднорідності у формі параболоїда.*

Ключові слова: *термомагнітоелектропружніть, тонке включення, інтегральні рівняння, метод граничних елементів.*

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