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THE EFFECT OF INITIAL DEFORMATIONS OF THE THICK PLATE ON ITS CONTACT INTERACTION WITH THE RING PUNCH

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Summary. *The calculation of the structural elements and mechanisms strength is one of the most important stages while their developing. Initial deformations are almost always available in the structural elements and machine parts. Stresses being initiated can cause their fracture and accelerate some phase transitions, corrosion in particular. To raise the accuracy of calculations the initial deformations must be taken into account. In the article the solution of the axesymmetric contact problem of the ring punch pressure on the preliminary stressed thick plate, being modeled by the elastic semi-space were obtained. The effect of the punch shape and the nature of the initial deformations on the distribution of the contact stresses under the punch was analysed.*

Key words: *contact stresses, initial deformation, ring punch, thick plate, semi-space.*

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Introduction. The calculation of strength of the structural elements and mechanisms is one of the most important stages while designing them. To determine the strength and durability of the contacting bodies, it is necessary to calculate the contact stresses. To minimize the error the maximum number of factors affecting their interaction must be taken into account. The residual deformations, affecting the contact stress directly being taken into account, is the factor of the paramount importance.

Analysis of the latest investigations. The problem of interaction of rigid punches and elastic bodies with residual deformations was delt with by many scientists, Ukrainian in particular. The contact problems for the bodies with the initial deformations for the certain shape of the elastic potential were studied by Arutyunyan N.Kh., Alexandrov V.M, Smetanin B.I., Filipova L.M., etc.

In general, the statement of such problems requires the envolvement of non-linear theory of elasticity complex, but the initial deformations being sufficient, its linearised option can be chosen. The fundamental results of the linearised theory of elasticity were obtained by the Ukrainian scientist, Academician of NAN of Ukraine Prof. O.M. Gooz [1 – 2]. The papers by his followers S.Y. Babich, N.B. Rudnytsky, P.P. Grygorenko, V.M. Nazarenko, Y.P. Glukhov, A.O. Ramsky, M.M. Dikhtyaruk, O.M. Panasyuk, etc., and other Ukrainian and foreign scientist, were devoted to the further development of the theory of the contact interaction of bodies with the initial stresses [3 – 7].

In spite of the continuous increase of the investigations on the contact interaction of bodies with the preliminary stress state, which has resulted from their pressing importance for both foundamental investigations on the contact interaction of bodies and for their application in many branches of industry, the problems of pressure of the parabolic and ring-parabolic punches on the preliminary stressed semi-space or sphere have not been solved yet within the linearized theory of elasticity for the stressed and non-stressed bodies in general form at the random structure of the elastic potential.

The Objective of the presented paper is to show the developed method for constructing solutions of the axesymmetric problems for finding the stress state in the preliminary stressed thick plate under its contact interaction with the rigid ring punch. To investigate the effect of the punch shape and initial deformations on the distribution of the contact stresses and vertical displacements under the contact interaction of the punch and plate.

Statement of the task. Let us analyse the axesymmetric problem of the rigid ring punch pressure on the elastic semi-space with the available residual deformations.

The punch is created while turning round the mutual axis the branches of two parabolas with the focal parameters R_1 and R_2, conjucted in the tops by the straight line section perpendicular to the rotation axis. The parabola axes, limiting the punch, are parallel to the mutual rotation axis, which coincides with the line of the force P action.

The punch is pressed into the semi-space progressively without rotation and friction with the constant force P.

Let us choose the cylindrical coordinate system $Or\theta z$ so, that the coordinate plane $rO\theta$ coincide with the boundary plane of the semi-space, and axis Oz – with the line of the force P action (Fig. 1).

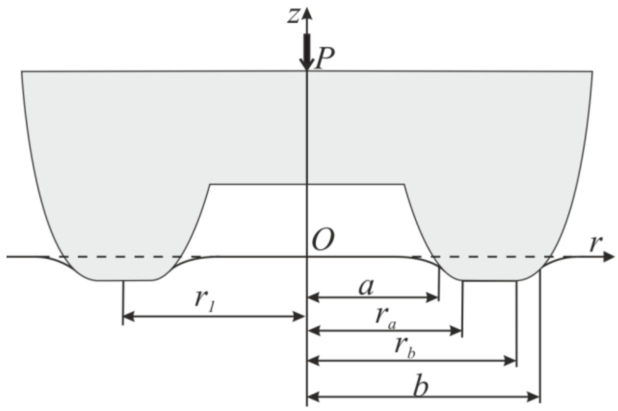

Figure 1. The scheme of the punch and semi-space contact interaction

Let us assume, that the internal radius a and the external radius b of the contact area are known, and determine the corresponding them focal parameters of the parabolas R_1 and R_2. Due to the problem statement we can describe the function, the graph rotation of which round the axis Oz the punch was created, which looks as follows:

$$W(r) = \begin{cases} \dfrac{1}{2R_1}(r - r_a)^2, \ 0 \le r \le r_a; \\ 0, \ r_a < r \le r_b; \\ \dfrac{1}{2R_2}(r - r_b)^2, \ r_b < r. \end{cases}$$

Let us consider the residual stresses, initiated in the semi-space, to be homogeneous. In this case we can use expressions for the components of stresses tensor (1) – (2) and the displacements vector (3) – (4).

$$\sigma_{rz}(r,0) = \frac{c_{44}(1+m_1)}{\sqrt{n_1}} \int_0^\infty \alpha^3 \{F_1 + s_0 F_2\} J_0(\alpha r)\, d\alpha\,; \tag{1}$$

$$\sigma_{zz}(r,0) = c_{44}(1+m_1) l_1 \int_0^\infty \alpha^3 \{F_1 + s F_2\} J_0(\alpha r)\, d\alpha\,; \tag{2}$$

$$u_r(r,0) = -\int_0^\infty \alpha^2 \{F_1 + F_2\} J_1(\alpha r)\, d\alpha\,; \tag{3}$$

$$u_z(r,0) = \frac{m_1}{\sqrt{n_1}} \int_0^\infty \alpha^2 \{F_1 + s_1 F_2\} J_0(\alpha r)\, d\alpha\,. \tag{4}$$

The limiting conditions of the stated task will look like:

$$\sigma_{rz}\left(r,0\right)=0,\ 0\leq r<\infty\,; \tag{5}$$

$$\sigma_{zz}\left(r,0\right)=0,\ 0\leq r\leq a,\ b\leq r\,; \tag{6}$$

$$u_{z}\left(r,0\right)=\omega\left(r\right),\ a\leq r\leq b\,. \tag{7}$$

Let us find the function $\omega\left(r\right)$, which describes the displacement of the boundary plane points of the elastic semi-space in the area of its contact with the rigid punch. Having used the presentation for the function $W\left(r\right)$, we will obtain

$$\omega\left(r\right)=\begin{cases}\omega\left(a\right)+\dfrac{1}{2R_{1}}\left[\left(r_{a}-r\right)^{2}-\left(r_{a}-a\right)^{2}\right],\ a\leq r<r_{a};\\ \omega\left(a\right)-\dfrac{1}{2R_{1}}\left(r_{a}-a\right)^{2},\ r_{a}\leq r<r_{1};\\ \omega\left(b\right)-\dfrac{1}{2R_{2}}\left(r_{b}-b\right)^{2},\ r_{1}\leq r<r_{b};\\ \omega\left(b\right)+\dfrac{1}{2R_{2}}\left[\left(r_{b}-r\right)^{2}-\left(r_{b}-b\right)^{2}\right],\ r_{b}\leq r\leq b;\end{cases} \tag{8}$$

where $r_{1}=\dfrac{r_{a}+r_{b}}{2}$.

In this problem the condition of the vertical displacement equality must be introduced additionally $r=r_{a}$ and $r=r_{b}$, that is, $\omega\left(r_{a}\right)=\omega\left(r_{b}\right)$.

The problem solving. Having satisfied the boundary condition (5), we will obtain the relation between the unknown function F_{1} and F_{2}

$$F_{1}=-s_{0}F_{2}\,. \tag{9}$$

Taking into account (9), the (2) and (4) will look like:

$$\sigma_{zz}\left(r,0\right)=c_{44}\left(1+m_{1}\right)\left(s-s_{0}\right)l_{1}\int_{0}^{\infty}\alpha^{3}F_{2}J_{0}\left(\alpha r\right)d\alpha\,; \tag{10}$$

$$u_{z}\left(r,0\right)=\frac{m_{1}\left(s_{1}-s_{0}\right)}{\sqrt{n_{1}}}\int_{0}^{\infty}\alpha^{2}F_{2}J_{0}\left(\alpha r\right)d\alpha\,. \tag{11}$$

Having satisfied the boundary condition (6), we will find

$$c_{44}\left(1+m_{1}\right)\left(s-s_{0}\right)l_{1}\int_{0}^{\infty}\alpha^{3}F_{2}J_{0}\left(\alpha r\right)d\alpha=0,\ 0\leq r\leq a,\ b\leq r\,. \tag{12}$$

Let us introduce the unknown function $x(r)$, $a \le r \le b$, using of which we will continue the relation (12) in the segment $0 \le r < \infty$:

$$c_{44}(1+m_1)(s-s_0)l_1\int_0^\infty \alpha^3 F_2 J_0(\alpha r)\,d\alpha = x(r)\left[\eta(r-a)-\eta(r-b)\right],\ 0 \le r < \infty . \tag{13}$$

The function $x(r)$ specifies the contact stresses distribution under the punch. Having taken into account their continuety, as well as the fact, that the contact is not available on the area boundary (at $r = a$ та $r = b$), let us present $x(r)$ as the segment of the generalized Fourier's series:

$$x(r) = \sum_{n=1}^{N} a_n L_n(r), \tag{14}$$

where $L_n(r) = J_0\left(\frac{\gamma_n}{a} r\right) Y_0(\gamma_n) - Y_0\left(\frac{\gamma_n}{a} r\right) J_0(\gamma_n)$; γ_n – positive root of the equation $J_0\left(\frac{b}{a} x\right) Y_0(x) - Y_0\left(\frac{b}{a} x\right) J_0(x) = 0$; a_n – unknown coefficient.

Having applied the reserve formula of the Hanckel integral transformation to the relation (13), we will obtain the expression:

$$\alpha^2 F_2 = \frac{1}{c_{44}(1+m_1)(s-s_0)l_1} \sum_{n=1}^{N} a_n \int_a^b r L_n(r) J_0(\alpha r)\,dr,\ 0 \le \alpha < \infty . \tag{15}$$

Having used the relations (11), (15) and the boundary condition (7) after some transformations we will obtain

$$k_1 \sum_{n=1}^{N} a_n \int_0^\infty \Phi_n(\alpha)\left[J_0(\alpha r) - J_0(\alpha a)\right] d\alpha = \omega_1^*(r),\ 0 \le r < r_1; \tag{16}$$

$$k_1 \sum_{n=1}^{N} a_n \int_0^\infty \Phi_n(\alpha)\left[J_0(\alpha r) - J_0(\alpha b)\right] d\alpha = \omega_2^*(r),\ r_1 \le r < b , \tag{17}$$

here

$$\Phi_n(\alpha) = \int_a^b r L_n(r) J_0(\alpha r)\,dr =$$

$$= \frac{\gamma_n a^2}{\gamma_n^2 - (\alpha a)^2}\left\{\frac{b}{a}\left[J_1\left(\frac{b}{a}\gamma_n\right) Y_0(\gamma_n) - Y_1\left(\frac{b}{a}\gamma_n\right) J_0(\gamma_n)\right] J_0(\alpha b) - \right.$$

$$\left. -\left[J_1(\gamma_n) Y_0(\gamma_n) - Y_1(\gamma_n) J_0(\gamma_n)\right] J_0(\alpha a)\right\}.$$

In the latter expressions the following symbols are used:

$$\omega_1^*(r)=\begin{cases}\frac{1}{2R_1}\left[(r_a-r)^2-(r_a-a)^2\right],\ a\le r<r_a;\\ -\frac{1}{2R_1}(r_a-a)^2,\ r_a\le r<r_1;\end{cases}\qquad \omega_2^*(r)=\begin{cases}-\frac{1}{2R_2}(r_b-b)^2,\ r_1\le r<r_b;\\ \frac{1}{2R_2}\left[(r_b-r)^2-(r_b-b)^2\right],\ r_b\le r\le b.\end{cases}$$

Having multiplied the relations (16) and (17) by $rL_q(r)$ and having integrated them in r from a to b, we will obtain as the result of their adding

$$\sum_{n=1}^{N}a_n\int_0^\infty \Phi_n(\alpha)\left[\Phi_q(\alpha)-K_q^{(1)}J_0(\alpha a)-K_q^{(2)}J_0(\alpha b)\right]d\alpha = \\ =\frac{1}{k_1}\{w_1+w_2\},\ q=\overline{1,N}\,, \tag{18}$$

here $K_q^{(1)}=\int_a^{r_1} rL_q(r)\,dr$, $K_q^{(2)}=\int_{r_1}^{b} rL_q(r)\,dr$; $w_1=\int_a^{r_1} r\omega_1^*(r)L_q(r)\,dr$, $w_2=\int_{r_1}^{b} r\omega_2^*(r)L_q(r)\,dr$

Having used the method of superposition and having introduced the symbols

$$a_n=\frac{1}{k_1}\left[a_n^{(1)}R_1^*+a_n^{(2)}R_2^*\right];\ R_1^*=\frac{1}{2R_1};\ R_2^*=\frac{1}{2R_2}, \tag{19}$$

from (18) we will have two independent systems of linear algebraic equations (SLAS) relatively the unknown $a_n^{(1)}$ and $a_n^{(2)}$.

The values R_i^* in the relations (19) will be found from the condition of the punch balance and equality of the vertical displacements of the boundary plane of the semi-space at $r=r_a$ and $r=r_b$

$$\begin{cases}2\pi\int_a^b r\sigma_{zz}(r,0)\,dr=-P;\\ \omega(r_a)=\omega(r_b).\end{cases} \tag{20}$$

Having introduced the symbols $\rho_i=R_i^*\frac{2\pi}{k_1P}$, $i=1,2$ from the relations (20), we will obtain the equation system relatively the unknown ρ_1 and ρ_2

$$\begin{cases}\rho_1\sum_{n=1}^{N}a_n^{(1)}M_n^{(1)}+\rho_2\sum_{n=1}^{N}a_n^{(2)}M_n^{(1)}=-1;\\ \rho_1\left[\sum_{n=1}^{N}\left[a_n^{(1)}M_n^{(2)}\right]-(r_a-a)^2\right]+\rho_2\left[\sum_{n=1}^{N}\left[a_n^{(2)}M_n^{(2)}\right]+(r_b-b)^2\right]=0,\end{cases} \tag{21}$$

де $M_n^{(1)} = \int_a^b rL_n(r)\,dr$, $M_n^{(2)} = \int_0^\infty \Phi_n(\alpha)\left(J_0(a\alpha) - J_0(b\alpha)\right)d\alpha$.

Having solved the system (21) and having found the unknown ρ_i using (19) and (14), we will obtain the function of the contact stresses distribution under the punch

$$\sigma_{zz}(r,0) = x(r) = \frac{P}{2\pi}\left[\sum_{n=1}^{N}\left(a_n^{(1)}\rho_1 + a_n^{(2)}\rho_2\right)L_n(r)\right]. \quad (22)$$

From the relations (11) and (15) the formula for finding the points vertical displacements of the semi-space boundary plane can be obtained.

$$u_z(r,0) = \frac{k_1 P}{2\pi}\sum_{n=1}^{N}\left[\left(a_n^{(1)}\rho_1 + a_n^{(2)}\rho_2\right)\int_0^\infty \Phi_n(\alpha)\,J_0(\alpha r)\,d\alpha\right]. \quad (23)$$

Analysis of regularities of the punch shape and available residual deformation in the semi-space effect on the distribution of the contact stresses and vertical displacements. Let us analyse the effect of the availability of the plane area in the punch base on the distribution of the contact stresses and the nature of the vertical displacements.

In Fig. 2, 3 the graphs of the functions σ^* and u^* are presented for the case of the ring-parabolic punch, when the residual deformations in the semi-space are not available, and the constant contact area ($a = 0.5$, $b = 1.5$) is available as well as different values of the parameters r_a and r_b. The curves 1 are constructed for $r_a = r_b = 1$, the curves 2 – for $r_a = 0.9$, $r_b = 1.1$, the curves 3 – for $r_a = 0.75$, $r_b = 1.25$.

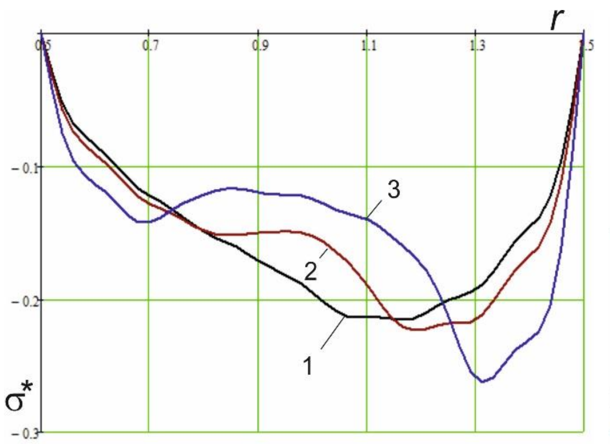

Figure 2. Distribution of the contact stresses for different r_a and r_b values

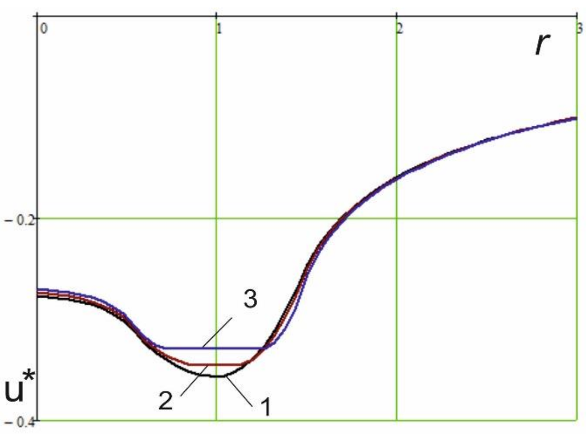

Figure 3. Vertical displacements for different r_a and r_b values

To solve the problem (5) – (7) it is necessary to provide the internal a and external b radii of the contact area correspondingly, and the radii of the parabola curvature, due to the rotation of which the punch was created, are found while solving according to the relations (19) from the system (21). But in order to study the effect of different factors, the initial stresses in particular, on the contact stresses, it is necessary to find a and b depending on P, R_1, R_2 and the features of the residual deformations field k_1. The procedure is similar to the case of the parabolic punch [3]. Let us analyse the example of the ring-parabolic punch, when $R_1 = R_2 = 0.2$, $r_a = r_b = 1$, which are pressed into the semi-space having the Bartenev – Khazanovych potential.

Let us choose 3 unit values for the parameters a and b with the step 0.1 and find values ρ_1 and ρ_2 corresponding them, which do not depend on the features of the residual deformations field. Having used the relations (19) and (21), let us find corresponding them values R_1 and R_2 for the case, when the residual deformations ($\lambda_1 = 1$) are not available in the semi-space, and when the residual tension ($\lambda_1 = 1.2$) deformations are available, as well as the case, when the residual pressing deformations ($\lambda_1 = 0.8$) are available in the semi-space. For every of the analysed cases, using the approximation of the cubic of the Mathcad medium, the functions, specifying the dependence $R_1(a,b)$ and $R_2(a,b)$, are constructed. The solutions of the equation system $R_1(a,b) = R_1$, $R_2(a,b) = R_2$ specify the parameters of the contact area corresponding to the given focal parameters of the parabolas R_1 and R_2, the force P and features of the residual deformations field k_1.

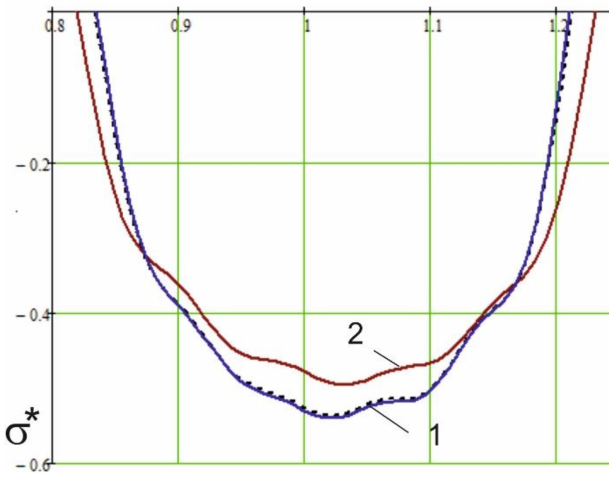

Figure 4. Contact stresses for the case of the harmony-type potential

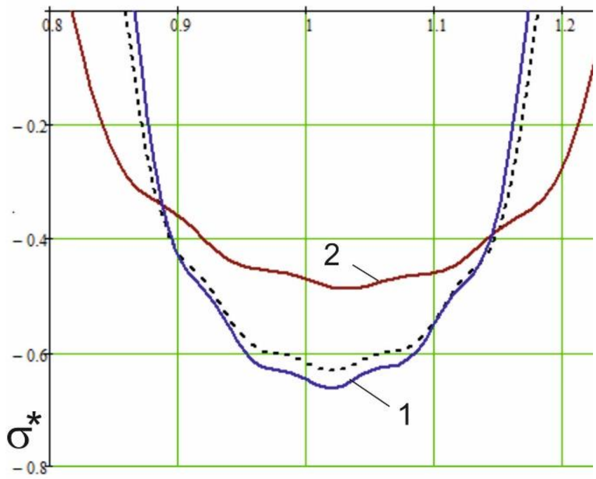

Figure 5. Contact stresses for the case of the Bartenev-Khazanovych potential

Let us analyse, for example, the case of the punch without the plane area in the base $r_a = r_b = 1$ and $R_1 = R_2 = 0.2$. Then for $\lambda_1 = 1$ we will have $a = 0.859$ and $b = 1.181$, for $\lambda_1 = 1.2$ – $a = 0.866$ and $b = 1.172$, for $\lambda_1 = 0.8$, $a = 0.817$ and $b = 1.233$. The constructed functions for the found a and b due to the formulas (22) and (23) specify the distribution of the contact stresses and vertical displacements for the fixed R_1, R_2, P and different λ_1 and make possible to analyse the effect of the residual deformations field.

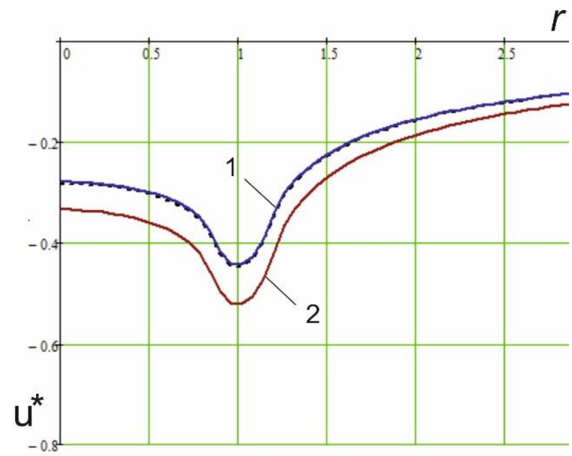

Figure 6. Vertical displacements for the case of the harmony-type potential

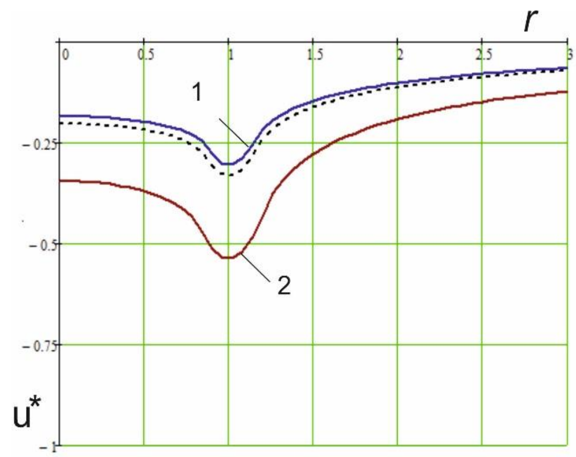

Figure 7. Vertical displacements for the case of the Bartenev – Khazanovych potential

In Fig. 4 – 7 the distribution of the contact stresses and vertical displacements, when the elastic harmony-type potential and the Bartenev – Khazanovych potential are available in the semi-space, is presented. The dotted curve corresponds to the case, when the residual deformations ($\lambda_1 = 1$) in the semi-space are not available, the curve 1 – when the tension deformations are available ($\lambda_1 = 1.2$), the curve 2 – the pressing deformations ($\lambda_1 = 0.8$).

In Fig. 8, 9 the dependence of the extreme values of the contact stresses and vertical displacements of the points of the semi-space boundary plane in the case of the ring-parabolic punch pressure is shown. The curves 1 correspond to the non-pressed semi-space, when the elastic Bartenev-Khazanovych potential is available, and curves 2 – to the compressed semi-space of the harmony-type potential. All curves are constructed for the case $P = 1$, $R_1 = R_2 = 0.2$, $r_a = r_b = 1$.

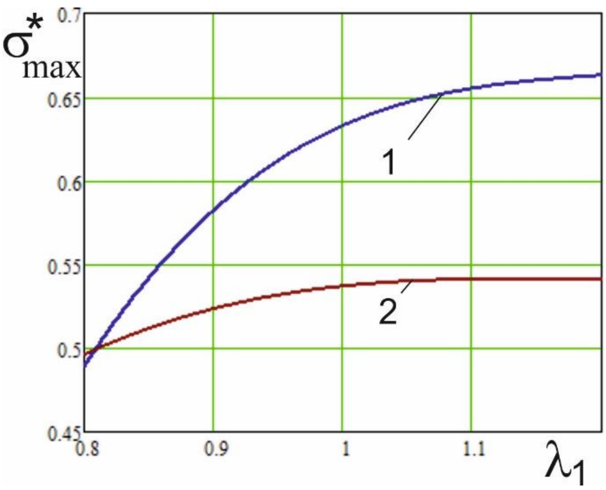

Figure 8. Extreme values of the contact stresses for the case of the ring-parabolic punch

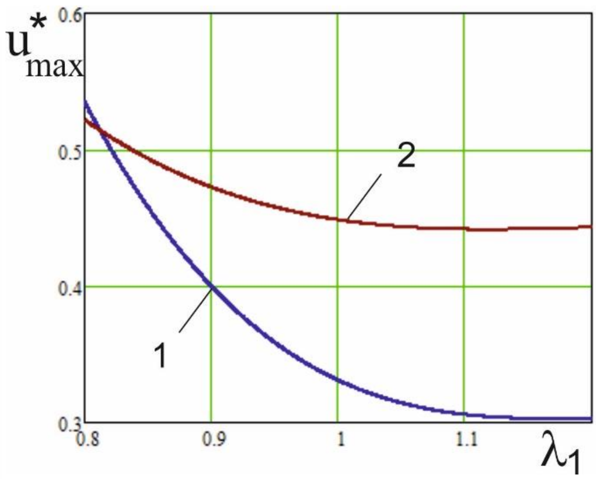

Figure 9. Extreme values of the vertical displacements for the case of the ring-parabolic punch

In Fig. 10 the dependence of the difference $d = b - a$, specifying the sizes of the contact area of the ring-parabolic punch with the preliminary stressed semi-space on the λ_1 parameter, that is, on the features of the initial deformation field, is presented. The graphs are constructed for the case $R_1 = R_2 = 0.2$, $r_a = r_b = 1$.

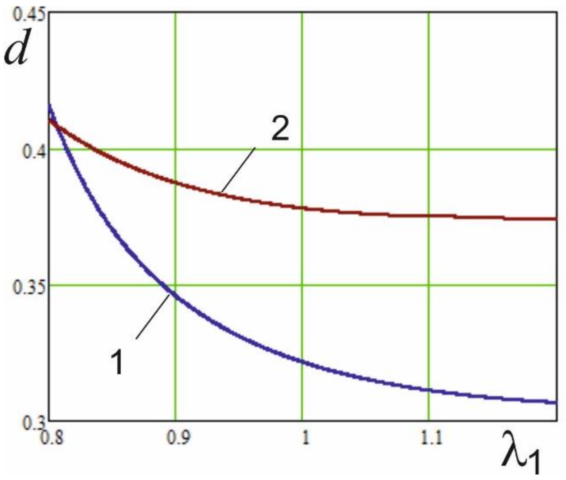

Figure 10. The contact area size for the case of the ring punch

Conclusion. As the result of the numerical experiments it has been determined, that the punch shape affects sufficiently the value and the nature of the contact stresses distribution. For the ring-parabolic punch the extreme values of the contact stresses are reached being more close to the outside boundary of the contact area, and the appearance of the flat area in the base causes the growth of the absolute value of contact stresses. The tensile residual deformations, being available in the semi-space, cause the decrease of the contact area, the increase of the absolute value of the contact forces and the decrease of the vertical displacements. The value of the changes resulted depends on the type of the elastic potential, the elastic potential of the harmony-type, for example, being available, the available 20 % of the tensile deformations causes the growth of the contact stresses only by 1 %, and in the case of the Bartenev – Khazanovych potential – by 5 %. The available pressure residual stresses in the semi-space, in its turn, results in the expansion of the contact area, the decrease of the absolute value of the contact stresses and the increase of the vertical displacements. Thus, in the case of the elastic potential of the harmony-type the 20 % pressure decreases the normal stresses by 10 % and increases the vertical displacements by 15 %. Here, in the case of the Bartenev – Khazanovych potential, the normal stresses are decreased by 25 %, the vertical displacements are increased by 40 %.

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ВПЛИВ ПОЧАТКОВИХ ДЕФОРМАЦІЙ ТОВСТОЇ ПЛИТИ НА ЇЇ КОНТАКТНУ ВЗАЄМОДІЮ З КІЛЬЦЕВИМ ШТАМПОМ

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Резюме. *Для визначення міцності та витривалості контактуючих тіл необхідним є обчислення контактних напружень. Щоб мінімізувати похибку, необхідно враховувати максимальну кількість чинників, що впливають на їх взаємодію. Урахування залишкових деформацій, які безпосередньо впливають на контактні напруження, є одним із ключових факторів. У статті продемонстровано розроблену методику побудови розв'язків осесиметричних задач визначення напруженого стану в попередньо напруженій товстій плиті при її контактній взаємодії із жорстким кільцевим штампом, а також досліджено вплив форми штампа та початкових деформацій на розподіл контактних напружень та вертикальних переміщень при контактній взаємодії штампа та плити. Співвідношення, що описують напружено-деформований стан тіл із початковими напруженнями, наведено у рамках лінеаризованої теорії пружності з використанням відлікового методу. Побудову аналітичних розв'язків контактних задач для попередньо напруженої товстої плити проводили шляхом її моделювання попередньо напруженим півпростором. Системи парних та потрійних інтегральних рівнянь, що при цьому отримали, розв'язували за допомогою представлення шуканих функцій у вигляді відрізків ряду за лінійними комбінаціями функцій Бесселя з невідомими коефіцієнтами та подальшим отриманням скінчених систем лінійних алгебраїчних рівнянь для їх відшукання. Отримано функції розподілу контактних напружень і переміщень для граничної площини півпростору. Проаналізовано вплив форми штампа та характеру початкових деформацій на розподіл контактних напружень під штампом.*

Ключові слова: *контактні напруження, початкові деформації, кільцевий штамп, товста плита, півпростір.*

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