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# CALCULATION OF THE DEFORMED STATE OF THE CELULLAR PIPELINE WITH RING SUPPORT

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***Summary.*** *The displacement and deflections of reinforced pipeline supports with cellular (hollow) walls are estimated. The investigated pipeline is constructed from a spiral tube. The pipe is divided into finite circular closed cylindrical shells of a certain length, at the ends of which elastic supports having the appropriate stiffness are located. The theory of anisotropic cylindrical shells is used. By the equations of the linear theory of anisotropic cylindrical shells, the averaged components of the displacement vector are determined. An appropriate system of differential equations is derived in order to establish displacements. The solution of these equations is presented in the form of double trigonometric Fourier series. In this case the normal ground resistance of the pipeline burial is taken into account. As a result, numerical calculations of displacements and deflections of the cellular pipeline are carried out.*

***Key words:*** *cellular (hollow) pipeline, ring support, ring stiffness; shell, stress-strain state; minimum durability; ground resistance.*

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**Introduction.** Direct application for laying and renovation of sewage drainage, water supply systems, etc. requires the calculation of the parameters of the pipelines during their operation. This particularly regards the pipelines of large diameter. Therefore, let us consider large diameter pipes that function along with the surrounding environment [3, 10], particularly with the ground. In such pipelines, the external load causes deformation greater than 3% which is considered to be boundary. One of the external load factors is the ground. It produces its own load which acts directly on the pipeline in a vertical direction. At the same time, as the result of structure interaction with the surrounding environment, the horizontal evaporation occurs greatly affecting the deformed state of the flexible pipes. Consequently, the horizontal evaporation should be taken into account when calculating and designing the structures below.

The effect of soil evaporation is mostly observed if there is a sufficiently compressed outer medium of large diameter pipelines. In such cases, it is accepted to use the empirical formula for engineering calculations [8, 10]

$$\frac{\Delta}{D} = \frac{C_1 p}{C_2 S_R + C_3 S_S} \tag{1}$$

If flexible pipes are reinforced with elastic ring supports, we can increase their ring stiffness and prevent undesirable cracking [5, 6]. This allows to optimize the design parameters, reaching the minimum mass provided with sufficient pipeline operational reliability. The investigated pipeline is constructed from a spiral tube. It is apparent that such structures with relatively small mass, while in the ground, can withstand significant external loads. Therefore, the calculation of its parameters is an important task.

**Statement of the problem.** Let us consider a long cellular (hollow) pipeline (Fig. 1. a), reinforced by circular periodic ring supports with step $l$ . The balancing force $q$ is applied to the structure, and is evenly distributed along its length (Fig. 1. b). The pipeline is made of the spiral tube with diameter ***d***, and the tube wall thickness is ***h***.

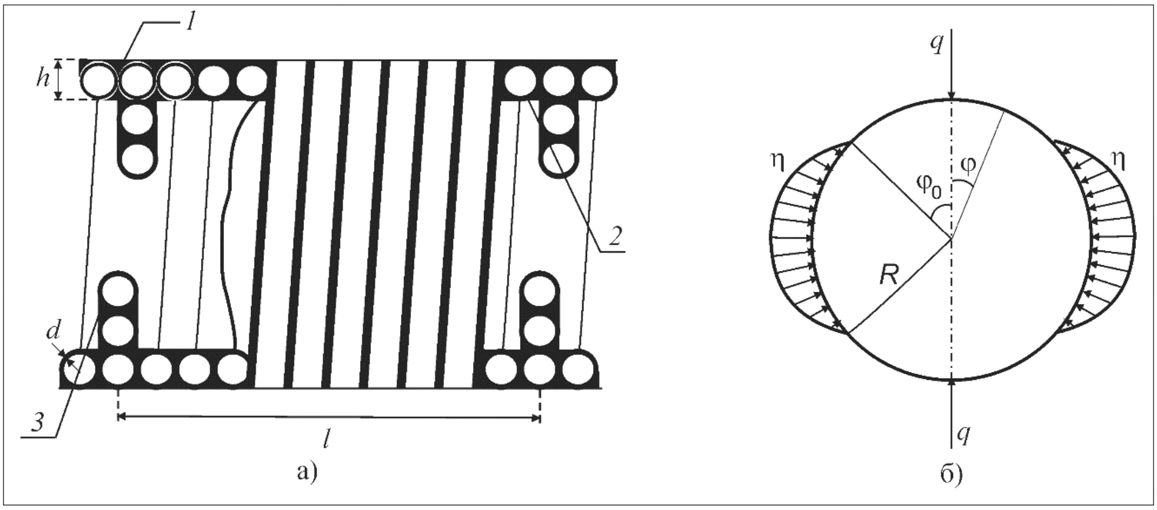


**Figure 1.** Cellular spiral single-layer pipe. 1 − welded seam; 2 − tube winding of the pipe wall; 3 − elastic ring supports

In order to increase the structure ring stiffness the pipeline is reinforced with elastic ring supports arranged periodically with step *l*.

Since we have the periodic construction, then the infinite pipe is divided into finite circular closed cylindrical shells with length *l*, at the ends of which there are elastic supports, having the corresponding stiffness.

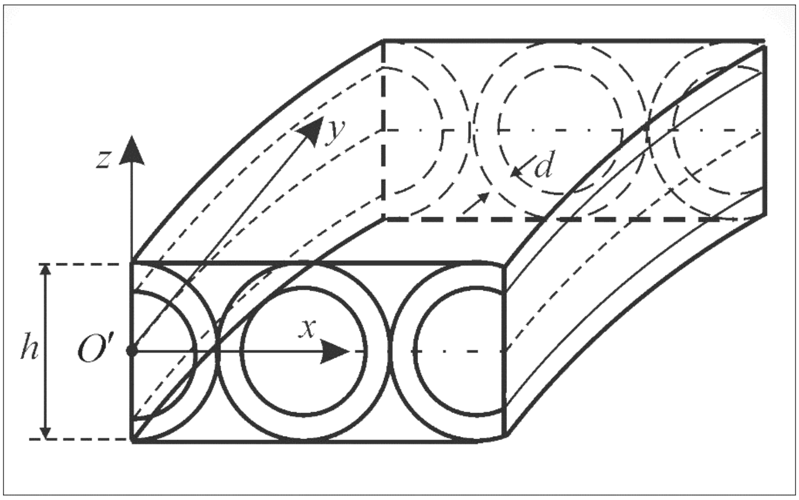


**Figure 2.** Scheme of the structure wall, mounted пустотілими трубками

Let us assume that the closed cylindrical shell consists of circular vertical hollow rings welded together.

**General method of problem solution.** Let us cut out from the shell a small element (Fig. 2), which is formed by two pairs of adjacent planes, which are normal to its median surface, and introduce mixed system of coordinates $O'xyz$ (Fig. 2). The position of the point of the shell median surface is characterized by coordinates $x$ and $y=R\varphi$, where $x$ is the distance from the point of origin from the initial equatorial section, $\varphi$ is the angle between the initial and the arbitrary meridian plane, that is, is deducted from the vertical plane.

Let us introduce the averaged displacement

$$u^*(x,y)=\Phi u(x,y),\quad \upsilon^*(x,y)=\Phi\upsilon(x,y),\quad w^*(x,y)=\Phi w(x,y). \tag{2}$$

where $u(x,y)$, $\upsilon(x,y)$ and $w(x,y)$ are are the displacement vector components of the median surface of the cylindrical shell. Here $\Phi u=\frac{1}{h}\int\limits_{x-h/2}^{x+h/2}u\,dx$ is an integral operator averaging function $u$ on the interval $[x-h/2..x+h/2]$.

Applying the theorems for the differentiation of Riemann integrals and parametric integrals [2] to expressions for deformations [1] $\varepsilon_x=\partial u/\partial x$; $\varepsilon_y=\partial\upsilon/\partial y+w/R$; $\chi_y=-\partial^2 w/\partial y^2-w/R^2$ та $\chi_x=-\partial^2 w/\partial x^2$ and proceeding from the relations (2), we obtain the averaged components of the deformation for a closed cylindrical shell

$$\varepsilon_x^*=\frac{\partial u^*(x,y)}{\partial x};\ \varepsilon_y^*=\frac{\partial\upsilon^*(x,y)}{\partial y}+\frac{w^*}{R};\ \chi_y^*=-\frac{\partial^2 w^*(x,y)}{\partial y^2}-\frac{w^*}{R^2},$$
$$\chi_x^*=-\frac{\partial^2 w^*(x,y)}{\partial x^2}. \tag{3}$$

By the expressions for the averaged displacements (2) and the averaged components of the deformations of the shell median surface (3), we derive the following relations for the averaged forces and bending moments [7]

$$N_y^*(x,y)=B_y^*\varepsilon_y^*(x,y)+\nu B_x^*\varepsilon_x^*(x,y),\quad M_y^*(x,y)=-D_y^*\chi_y^*-\nu D_x^*\chi_x^*, \tag{4}$$

where $N_y^*=\left(\Phi N'_y+\nu\Phi N'_x\right)/\left(1-\nu^2\right)$, $M_y^*=\left(\Phi M'_y+\nu\Phi M'_x\right)/\left(1-\nu^2\right)$ are relatively the averaged force and bending moment; $D_y^*$, $D_x^*$ та $B_y^*$, $B_x^*$ are cylindrical stiffness and tensile stiffness of the closed cylindrical shell, which we determine from the relations (4) in the following way:

$$B_y^*=\frac{EF}{h\left(1-\nu^2\right)},\quad B_x^*=\frac{hE}{G\left(1-\nu^2\right)};\quad D_y^*=\frac{I_xE}{h\left(1-\nu^2\right)},\quad D_x^*=\frac{hE}{J_x\left(1-\nu^2\right)}.$$

Here $F=h\Phi g(x)$ is the cross-sectional area of the tube forming the pipe wall, taking into account the weld seam; $G=h\Phi\left(1/g(x)\right)$; $I_x=h\Phi t(x)$ is the inertia moment of the tube cross-section of the construction wall set, taking into account the weld seam relatively to the axis $Ox$; $J_x=h\Phi\left(1/t(x)\right)$; $g(x)=\int\limits_{-h/2}^{h/2}f(x,z)dz$, $t(x)=\int\limits_{-h/2}^{h/2}z^2 f(x,z)dz$, де $g(x)$ та $t(x)$ are periodic functions with period $h$; $E$ is Young modulus, $\nu$ is Poisson coefficient of pipeline material; $f(x, z)=1$ when at point $(x, z)$ cylindrical shell walls is material and $f(x, z)=0$ if it is absent.

It should be noticed that other resultant forces [1, 9] $N_x(x,y)$, $N_{xy}(x,y)$, bending $M_x(x,y)$ and twisting $M_{xy}(x,y)$ moments were averaged in a similar.

Taking into account the relation (4), on the basis of [1] we obtain the equation of the linear moment theory of anisotropic cylindrical shells with solid walls for the determination of the averaged components of the displacement vector $u^*$, $\upsilon^*$ and $w^*$.

*The equation of the linear torque theory of anisotropic cylindrical shells is as follows:*

$$B_x^* \frac{\partial^2 u}{\partial x^2} + B_y^* \frac{1-\nu}{2} \frac{\partial^2 u}{\partial y^2} + B_y^* \frac{1+\nu}{2} \frac{\partial^2 \upsilon}{\partial x \partial y} + B_y^* \frac{\nu}{R} \frac{\partial w}{\partial x} = -q_x,$$

$$\frac{B_y^*(1-\nu)}{2} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 \upsilon}{\partial x^2} \right) + B_y^* \left( \frac{\partial^2 \upsilon}{\partial y^2} + \frac{\partial}{\partial y} \frac{w}{R} \right) + \nu B_x^* \frac{\partial^2 u}{\partial x \partial y} = -q_y,$$

$$\begin{gathered} D_x^* \frac{\partial^4 w}{\partial x^4} + \nu \left( D_y^* + D_x^* \right) \frac{\partial^2}{\partial x^2} \frac{w}{R^2} + D_y^* \left( \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^2}{\partial y^2} \frac{w}{R^2} \right) + \nu D_x^* \frac{\partial^4 w}{\partial x^2 \partial y^2} + \\ D_y^* (2-\nu) \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_y^*}{R^4} w + \frac{1}{R} \left[ B_y^* \left( \frac{\partial v}{\partial y} + \frac{w}{R} \right) + \nu B_x^* \frac{\partial u}{\partial x} \right] - q_z = 0. \end{gathered} \tag{5}$$

Here $q_x$, $q_x$, $q_x$ are projections of the external forces vector on the corresponding coordinate axes; $R$ is radius of the median surface of the cylindrical shell.

In the following equations (5), and below, the index «*» will be omitted understanding that we find averaged displacement for closed cellular cylindrical shell.

The horizontal ground reaction on the cylindrical shell can be represented as a radial reaction (Fig. 1. b)

$$\eta(\varphi, x) = \begin{cases} kw(\varphi, x), \text{ where } w(\varphi, x) > 0, \\ 0, \text{ where } w(\varphi, x) \le 0, \end{cases} \tag{6}$$

where $k$ is − the coefficient of resistance of the elastic medium surrounding the shell in the normal direction.

To simplify the problem we assume that $w(\varphi, x) > 0$ gets the value in straightforward area

$$\Omega = \{\varphi \in [\varphi_0, \pi - \varphi_0] \cup [\pi + \varphi_0, 2\pi - \varphi_0], \ x \in [0, l]\},$$

where angle $\varphi_0$ is shown in Fig. 1. b and its value is given below in this paper.

The solution of the system of differential equations (5) is constructed by means of the Bubnov-Galerkin method consists in the fact that the displacement vector components are presented in the form of double trigonometric Fourier series [9].

$$w = \sum_{m=0}^{\infty} \sum_{n=2}^{\infty} C_{mn} \cos\frac{m\pi x}{l} \cos n\varphi, \quad \upsilon = \sum_{m=0}^{\infty} \sum_{n=2}^{\infty} B_{mn} \cos\frac{m\pi x}{l} \sin n\varphi,$$

$$u = \sum_{m=0}^{\infty} \sum_{n=2}^{\infty} A_{mn} \sin\frac{m\pi x}{l} \cos n\varphi. \qquad (7)$$

Recorded rows do not contain members with numbers $n = 0$ and $n = 1$, since the corresponding displacements will represent the circle shift as absolute integer [9].

With such statement of the problem at the ends of the cylindrical shell, we determine the following *boundary conditions:*

$$w(\varphi) = W(\varphi), \quad \upsilon(\varphi) = V(\varphi), \text{ if } x = 0 \text{ and } x = l,$$

and

$$u = 0, \quad \frac{\partial w}{\partial x} = 0, \text{ if } x = 0 \text{ and } x = l, \qquad (8)$$

where $W(\varphi)$, $V(\varphi)$ are the displacement vector components of the frame, relatively, in the radial and tangential directions.

By direct substitution $x = 0$ and $x = l$ and in the development (7), we obtain that the components of the displacement vector of cylindrical shell satisfy the second group of boundary conditions (8).

Let us present the external distributed force $q$ through the distributed load [1, 9] in the following way:

$$q'(x, \varphi) = \frac{q}{\Delta s} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} q_{mn} \cos\frac{m\pi x}{l} \cos n\varphi, \qquad (9)$$

where $\Delta s = R\Delta\varphi$ is the arc element directed to zero;

$$q_{mn} = \begin{cases} 0, & m = 1,2,3\ldots, \ n = 1,2\ldots, \\ q(1 + \cos n\pi)/(\pi R), & m = 0, \ n = 1,2\ldots. \end{cases}$$

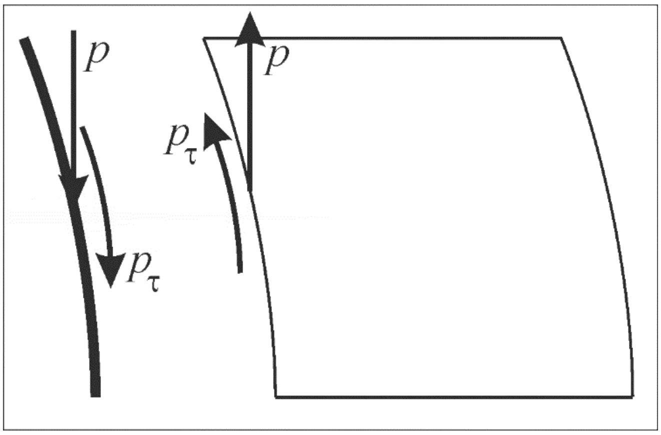


**Figure 3.** Scheme of elastic support contact with cylindrical shell

As a result of the interaction of elastic frames and cylindrical shell the unknown interacting normal $p(\varphi)$ and tangential $p_\tau(\varphi)$ forces distributed over the rim occurs on its ends. They can be represented through distributed loads [1, 9]

$$p'(x,\varphi)=\frac{p}{\Delta x}=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}p_{mn}\cos\frac{m\pi x}{l}\cos n\varphi,$$

$$p'_\tau(x,\varphi)=\frac{p_\tau}{\Delta x}=\sum_{m=0}^{\infty}\sum_{n=2}^{\infty}p_{\tau mn}\cos\frac{m\pi x}{l}\sin n\varphi, \tag{10}$$

where $\Delta x$ tends to zero; $p_{mn}=\begin{cases}2p_n\left(1+(-1)^m\right)/l, & m=1,2,3\ldots\\ 2p_n/l, & m=0,\end{cases}$,

$p_{\tau mn}=\begin{cases}2p_{\tau n}\left(1+(-1)^m\right)/l, & m=1,2,3\ldots\\ 2p_{\tau n}/l, & m=0,\end{cases}$, and $p_n$, $p_{\tau n}$ are unknown coefficients to be determined.

Let us decompose the normal evaporation of the ground $\eta(x,\varphi)$ into double trigonometric Fourier series

$$\eta(x,\varphi)=\sum_{m=0}^{\infty}\sum_{n=2}^{\infty}C'_{mn}\cos\frac{m\pi x}{l}\cos n\varphi, \tag{11}$$

where $C'_{mn}=\frac{k}{\pi}\sum_{i=2}^{\infty}C_{mi}f_{in}$, $n=2,3\ldots$, $m=0,1\ldots$ .

Here $f_{in}=\int_{\varphi_0}^{\pi-\varphi_0}\cos i\varphi\cos n\varphi d\varphi+\int_{\pi+\varphi_0}^{2\pi-\varphi_0}\cos i\varphi\cos n\varphi d\varphi$.

Substituting the development (7), (9) − (11) into the system of differential equations (5), we obtain for each value $m$ its own system of linear algebraic equations (SLAE) for the determination of the unknown coefficients $A_{mn}$, $B_{mn}$ and $C_{mn}$

$$-B_x^*\left(\frac{m\pi}{l}\right)^2A_{mn}-B_y^*\frac{(1-\nu)n^2}{2R^2}A_{mn}-B_y^*\frac{1+\nu}{2R}\frac{mn\pi}{l}B_{mn}-B_y^*\frac{\nu}{R}\frac{m\pi}{l}C_{mn}=0,$$

$$B_y^*\left(-\frac{1-\nu}{2R}\frac{mn\pi}{l}A_{mn}-\frac{1-\nu}{2}\left(\frac{m\pi}{l}\right)^2B_{mn}-\frac{n^2}{R^2}B_{mn}-\frac{n}{R^2}C_{mn}\right)-B_x^*\frac{\nu}{R}\frac{mn\pi}{l}A_{mn}=p_{\tau mn},$$

$$\left(D_x^*\left(\frac{m\pi}{l}\right)^4-\nu\left(D_x^*+D_y^*\right)\left(\frac{m\pi}{Rl}\right)^2+D_y^*\left(\frac{n^4}{R^4}-\frac{2n^2}{R^4}\right)+\nu D_x^*\left(\frac{mn\pi}{Rl}\right)^2+D_y^*(2-\nu)\left(\frac{mn\pi}{Rl}\right)^2+\frac{1}{R^4}\right)C_{mn}+$$

$$+\frac{1}{R^2}\left(B_x^*\frac{R\nu m\pi}{l}A_{mn}+B_y^*\left(nB_{mn}+C_{mn}\right)\right)=\frac{k}{\pi}\sum_{i=2}C_{mi}f_{in}-q_{mn}+p_{mn}, \tag{12}$$

where $n=2,3...$, $m=0,1,..$.

Solving the systems of linear algebraic equations (12) for each pair $m$ and $n$, we get

$$B_{mn} = p_n B'_{mn} + p_{\tau n} B''_{mn} + q_{mn} B'''_{mn},, \quad C_{mn} = p_n C'_{mn} + p_{\tau n} C''_{mn} + q_{mn} C'''_{mn}, \tag{13}$$

where $C'_{mn}$, $C''_{mn}$, $C'''_{mn}$ та $B'_{mn}$, $B''_{mn}$, $B'''_{mn}$ are constants.

The equilibrium equations of the frames are as follows [4]

$$\frac{E'I_p}{2}\left[\frac{d^3}{d\varphi^3}\left(\frac{d^2W(\varphi)}{d\varphi^2}+W(\varphi)\right)+\frac{d^2}{d\varphi^2}\left(\frac{d^2V(\varphi)}{d\varphi^2}+V(\varphi)\right)\right]+R^4\left(-p_\tau(\varphi)+\frac{dp(\varphi)}{d\varphi}\right)=0,$$

$$\frac{E'F_p}{2}\left(\frac{d^2}{d\varphi^2}+1\right)\left(\frac{dV(\varphi)}{d\varphi}-W(\varphi)\right)+R^2\left(-p(\varphi)+\frac{dp_\tau(\varphi)}{d\varphi}\right)=0, \tag{14}$$

where $I_p$ is the inertia moment of the ring support cross-section relatively to the axis of the s support symmetry, $F_p$ is the area of the ring support cross-section.

The displacement vector components of the elastic frames and the unknown interaction forces occurring on the ring supports are provided by the development [9]

$$W(\varphi)=\sum_{n=2}^{\infty}W_n\cos n\varphi, \quad V(\varphi)=\sum_{n=2}^{\infty}V_n\sin n\varphi, \quad p(\varphi)=\sum_{n=2}^{\infty}p_n\cos n\varphi,$$
$$p_\tau(\varphi)=\sum_{n=2}^{\infty}p_{\tau n}\sin n\varphi. \tag{15}$$

Substituting the development (15) into the equilibrium equation of the frame (14), we obtain the following expressions for coefficients $W_n$ and $V_n$ determination:

$$W_n=-R^2\frac{\left(F_pR^2+I_p\right)np_n+\left(F_pR^2-n^2I_p\right)p_{\tau n}}{2n\left(n^2-1\right)^2EI_pF_p},$$

$$V_n=-R^2\frac{\left(F_pR^2+n^2I_p\right)np_n+\left(F_pR^2-n^4I_p\right)p_{\tau n}}{2n^2\left(n^2-1\right)^2EI_pF_p}, \tag{16}$$

where $n=2,3,\ldots$

On the basis of the first two boundary conditions (8), taking into account (13) and (16), we obtain for each $n$ of our systems two linear algebraic equations for the determination of unknown coefficients $p_n$ and $p_{\tau n}$

$$\frac{2}{l}\left(p_nC'_{0n}+p_{\tau n}C''_{0n}+q_{0n}C'''_{0n}\right)+\sum_{m=1}^{\infty}\frac{2}{l}\left(1+(-1)^m\right)\left(p_nC'_{mn}+p_{\tau n}C''_{mn}+q_{mn}C'''_{mn}\right)=W_n$$

$$\frac{2}{l}\left(p_nB'_{0n}+p_{\tau n}B''_{0n}+q_{0n}B'''_{0n}\right)+\sum_{m=1}^{\infty}\frac{2}{l}\left(1+(-1)^m\right)\left(p_nB'_{mn}+p_{\tau n}B''_{mn}+q_{mn}B'''_{mn}\right)=V_n.$$

Here $n = 2,3...$, $m = 0,1,2...$.

*The coefficient of the ground evaporation* ***k*** *in the normal direction* depends on the type of the ground and its compression. It depends nonlinearly on the external load. In many calculations of pipelines with the surrounding elastic environment, it is assumed that

$$E'_S = kR, \tag{17}$$

where $E'_S$ [10] is ground cross-section module.

**Calculations of the pipeline deflections and displacements.** Taking into account the previous considerations, a numerical shell calculation at various input parameters was carried out. Fig. 4 − 6 represent the results of numerical analysis of the set task for the Young modulus $E = 850\ MPa$, the Poisson coefficient of the construction material $\nu = 0.25$; depth of burial $H = 1\ m$ and soil density $\rho = 1700\ kg/m^3$. Here the solid lines correspond to the pipeline with the radius of the shell median surface $R = 1\ m$, and the dash lines – with $R = 0.5m$. It is assumed that the diameter of the spiral tube equals $h = R/20$, the thickness of the tube wall is $d = h/10$.

The graphical dependences of the given maximum deflection of the structure $\Delta_{\max}/\Delta$ on the value $R/l$, where $\Delta = w(l/2,0) + w(l/2,\pi)$ is the maximum reduction of the pipeline diameter at $l \to \infty$ are shown in Fig. 4. Curves *1* and *1′* are constructed for reinforced by frames pipelines without taking into account the ground reaction, curves *2* and $2'$ are constructed at ground cross-section module $E'_s = 1MPa$, and curves 3 i $3'$ relatively when $E'_s = 2MPa$.

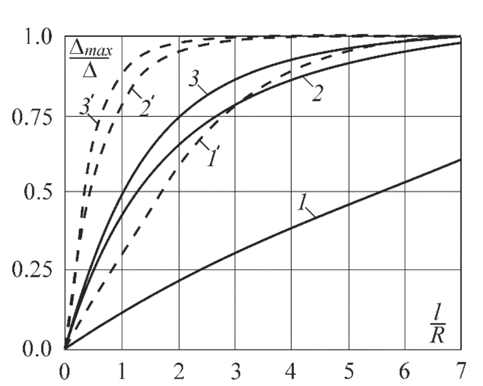


**Figure 4.** Changes of the given maximum deflection $\Delta_{max}/\Delta$ from the value *l/R*

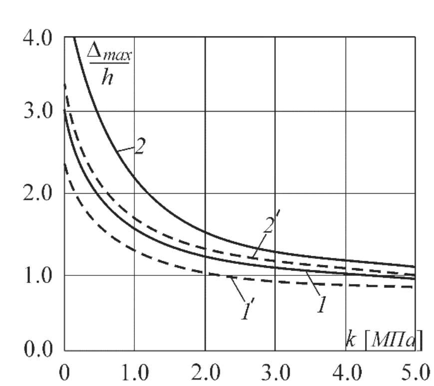


**Figure 5.** Changes in the reduced maximum pipeline deflection $\Delta_{max}/h$ from the ground reaction coefficient of filling *k*

It can be noted from Fig. 4 that at a large distance between the frames we derive the results of paper [9], which correspond to the pipe without support (with solid walls).

Figure 5 shows the graphical dependence of the given maximum deflection $\Delta_{\max}/h$ of the pipeline on the coefficient of ground evaporation $k$, where $\Delta_{\max} = w(l/2,0) + w(l/2,\pi)$ is the maximum reduction of the pipeline diameter, which is reinforced by elastic ring supports. Curves *1* and *1′* in Fig. 5 are constructed for pipelines reinforced by periodic elastic ring supports formed from five hollow rings located with step $l = 2R$; respectively curves *2* and $2'$ are constructed for structures where periodic supports are located with step $l = 3R$. The calculations assumed that the angle is $\varphi_0 \approx 0.22\pi$. On the basis of the numerical analysis, it was found that the normal displacement of the cylindrical shell $w$ changes the sign along the entire length within the range $\varphi \in [0.21\pi..0.23\pi]$.

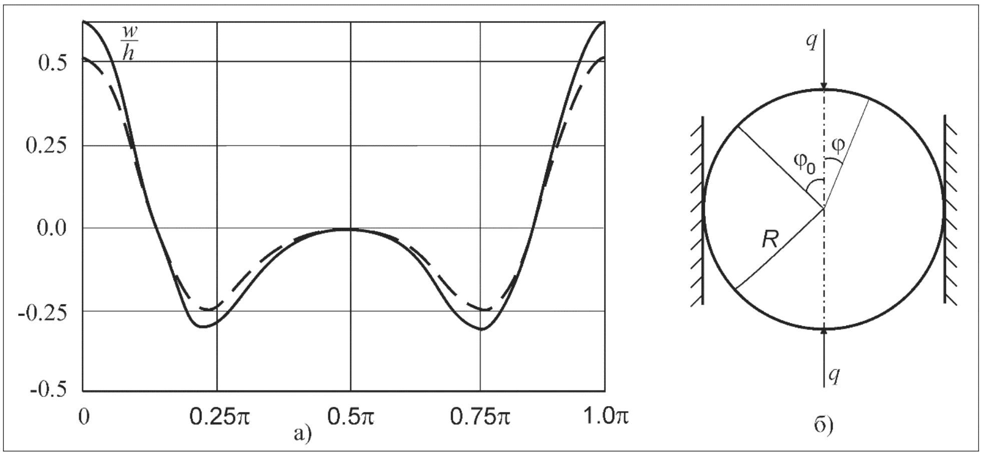


**Figure 6.** Dependence of the relative normal movement *w/h* along the pipeline rim

Accepting $\varphi_0 \to \pi/2$, and $k \to \infty$, as a partial case, averaged displacements of the reinforced cellular pipeline resting in a rigid half-space (Fig. 7a) were obtained. The relative displacement of the cellular pipeline along its rim at $x = l/2$ is shown in Fig. 7 b. Here it is assumed that $l = 3R$.

On the basis of the above mentioned results, it is possible to optimize the structure size in order to obtain its minimum mass for given external loads and if the maximum theoretical pipeline deflection is less than value $0.03D$. Application of the results given in this paper is also efficient while calculating the stress-strain state of overlaps segments with tubular inserts [7].

**Conclusion.** The calculation of displacements of the cellular pipeline reinforced by supports is carried out. The reinforcement of the flexible pipeline by the ring supports increases its ring stiffness. Increase in the distance between the supports-frames reduces the pipeline stiffness and increases its efficiency to the cell without support. It enables to optimize the structure parameters, reaching the minimum mass provided with sufficient operational reliability of the pipeline.

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# РОЗРАХУНОК ДЕФОРМОВАНОГО СТАНУ СТІЛЬНИКОВОГО ТРУБОПРОВОДУ З КІЛЬЦЕВИМИ ПІДПОРАМИ

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***Резюме.*** *Оцінено переміщення та прогини підкріпленого підпорами трубопроводу зі стільниковими (порожнистими) стінками. Розглядуваний трубопровід сконструйовано зі спіралеподібної трубки. Труба розбивається на скінченні кругові замкнені циліндричні оболонки певної довжини, на кінцях яких знаходяться пружні підпори, що мають відповідну жорсткість. Застосовано теорію анізотропних циліндричних оболонок. За рівняннями лінійної теорії анізотропних циліндричних оболонок записано усереднені компоненти вектора переміщень. Для встановлення переміщень записано відповідну систему диференціальних рівнянь. Розв'язок цих рівнянь наведено у вигляді подвійних тригонометричних рядів Фур'є. При цьому враховано нормальний відпір ґрунту засипки труби. В результаті проведено числові розрахунки переміщень та прогинів стільникового трубопроводу. На основі наведених результатів можна провести оптимізацію розмірів конструкції, щоб отримати її мінімальну масу при заданих зовнішніх навантаженнях та умові, що максимальне теоретичне прогинання трубопроводу буде менше від значення* $0.03D$*. Застосування наведених у роботі результатів є ефективним також у розрахунках напружено-деформованого стану фрагментів перекриттів із трубчастими вставками. Проведено розрахунок переміщень підкріпленого підпорами стільникового трубопроводу. Підсилення гнучкого трубопроводу кільцевими підпорами збільшує його кільцеву жорсткість. Збільшення відстані між підпорами-шпангоутами зменшує жорсткість трубопроводу та наближає його працездатність до стільника без підпор. Це дозволяє оптимізувати параметри конструкції, досягаючи мінімальної маси при забезпеченні достатньої експлуатаційної надійності трубопроводу.*

***Ключові слова:*** *стільниковий (пустотілий) трубопровід, кільцеві підпори, кільцева жорсткість, оболонка, напружено-деформований стан, мінімальна довготривала міцність, відпар ґрунту.*

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