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MATHEMATICAL MODELING. MATHEMATICS

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ. МАТЕМАТИКА

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MATHEMATICAL MODELING OF DYNAMICS OF RADIO-ECOLOGICAL PROCESSES AND RELIABILITY OF POLLUTANTS TRANSPORT IN A FOREST ECOSYSTEM

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***Summary**. The system of assessing reliability of forest ecosystems' components by means of construction of successive box models, the reliability theory methods and differential equations has been created and analyzed in the article. A model for determining the reliability of radionuclide / pollutants content in ecosystems has been developed. Examples of box models for forest ecosystems with consideration of coefficients / rates of radionuclide transitions between cameras have been formed. Different variants of radionuclide transport in a system have been investigated: for natural conditions, for possibility of applying protective measures, for emergency situations. The reliability assessment of radionuclide transport between cameras of investigated ecosystems is realized by means of the software MAPLE 5. It has been shown that application of mathematical modeling in the study of natural ecological systems allows determining the level of their pollution, as well as the levels of pollutants accumulation in different components of the ecosystem.*

***Key words**: mathematical modeling, box model, radionuclide transport, reliability of a forest ecosystem.*

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Statement of the problem. Constantly increasing technogenic impact on the environment results in uncontrolled changes in the natural world. In the environment there are transformations, the result of which are irreversible changes in biogeocoenoses and loss of reliability of natural biological systems.

At present there are global transformations of the biosphere including those under the influence of radiological situation: firstly, as the result of anthropogenic processes in the biosphere, accumulating amounts of artificial radionuclides not present in the biosphere previously penetrated and continue to penetrate into the biosphere; and secondly, the flows of numerous natural radionuclides migrating by abiotic and biological chains are sharply intensified in the biosphere; thirdly, in modern biosphere, zones of artificial radionuclides concentration (enterprises of complete nuclear fuel cycle, places of radioactive waste burial, etc.) have appeared and expanded developing into the sources of radionuclide dispersal, and the environment is exposed to increased radiation influence.

As the result of the accident at Chernobyl Nuclear Power Plant (ChNPP), about 3.5 million hectares of forest fell under radioactive pollution, and all the forests in Ukraine

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occupy the territory 9.9 million hectares. The largest areas of forests radioactive pollution are in Zhytomyr (60%), Kiev (52.2%) and Rivne (56.2%) regions. In Volyn, Chernihiv, Cherkassy, Vinnytsia and Sumy regions, the area of forests radioactive pollution is about 20% [1−3].

The rise of the natural background of ionizing radiation due to technological environmental impact having the become scale, and the expansion of zones with high radiation load on the natural environment put forward the task of environmental regulation of radiation influence and determination of radiation loads on living organisms. Thus, it is necessary to create the system for estimation the components reliability of environmental systems of various types, particularly, forestry ones. It is known that transport of radionuclides ^{137}Cs clearly correlates with the dynamics of transport along the ecosystem of the important potassium microelement, which practically determines the state of the ecosystem biota welfare. Therefore, observing the ^{137}Cs dynamics, which is easy to be investigate, allows us to establish the ecosystem reliability and its well-being according to the criterion of ^{137}Cs transport reliability [4].

Analysis of the available research results. The universal, comprehensive means of protecting humanity from global climate change, strong earthquakes, hurricanes, floods, and major man-made disasters really do not exist. An example of this is The nuclear accidents such as Kyshtymska in the Southern Urals (1957), Chernobyl (1986) and Fukushima-1 (2011) with global consequences, which have not yet been studied and evaluated [5] are examples of this.

The objective of the work is to develop and apply the methodology of forest ecological systems condition estimation on the basis of mathematical compartment models, reliability theory and radio frequency parameters.

Statement of the problem. On the basis of the developed mathematical compartment models of forest ecosystems, to construct the model of radionuclides transport reliability and to substantiate the application of the proposed method for the investigstion of radionuclides distribution and redistribution in the forest ecosystem while estimating the dose loads on living organisms and human beings.

The results of the investigations. Anthropogenic effects on ecosystems of different origins are the main factors that determine the biota state and the biological systems reliability.

The general algorithm for the ecosystems reliability estimation can be carried out in the following stage manner

1) detailed description of the investigated ecosystem;

2) construction of the optimal compartment model and corresponding block diagram of this ecosystem;

3) carrying out on the basis of natural and literary data estimation of velocity parameters of pollutants transition between the compartments of the investigated ecosystem;

4) description of the ecosystem compartment model using the system of differential equations, taking into account the obtained values for the transition velocities between the ecosystem compartments;

5) creation of the model for estimation the reliability of the content of radionuclides and other pollutants in ecosystems on the basis of the theory and models of the ecosystems reliability through the estimation of radiocapacity parameters. Estimation of radio frequency parameters is carried out by the formula:

$$F_j = \sum a_{ij} / \left(\sum a_{ij} + \sum a_{ji}\right), \tag{1}$$

where $\sum a_{ij}$ is the sum of pollutants transition velocities from the various ecosystem components to the specific element of the landscape or ecosystem (compartment *i*), according to the compartment models, and $\sum a_{ji}$ is the sum of the pollutants flow down velocities from the investigated compartment $-j-$ to the other ecosystem components conjugate to them.

The formula functions for a long time after reaching the equilibrium.

6) construction of the reliability model of the investigated ecosystem as system, in the form of parallel or parallel-series subsystems; and construction of the mathematical model for estimation the reliability of the radionuclides (pollutants) retension and transport in the investigated ecosystem;

7) calculation of pollutants transport reliability parameter in the investigated system on the basis of reliability model [6].

The proposed reliability estimation algorithm is applied to the forest ecosystem. The compartment model of forest ecosystem is constructed. The reliability of ^{137}Cs radionuclide transport from the forest to the river is calculated using the mathematical program MAPLE 5. Calculations according to the compartment models reflect ^{137}Cs radionuclide motion dynamics from the source [7].

The considered model consists of 3 compartments: «forest», «stream» in the forest and «river», into which the stream flows (Fig. 1). It is assumed that 100% of radioactive cesium ^{137}Cs is concentrated in the forest. Let us denote by $X(t), Y(t), Z(t)$ the content of radionuclide in the compartment «forest», «stream» and «river» respectively. The block diagram of such compartment model is presented in Fig. 1, where a_{12}, a_{23} and a_{34} are coefficients (velocities) of radionuclides transition between the compartments.

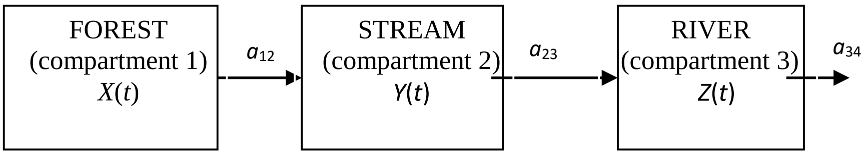

Figure 1. Block diagram of the box model of a forest

Estimation of transition coefficients between compartments is carried out. It is known that the radionuclides runoff from the plain forest is in average about 3% of the total storage, and another 3% − due to decay and fixation in the soil [8−10]. Consequently, according to the investigations, a_{12} = 0,06 (i. e., 6% of the storage). Radionuclides release from the stream into the river, according to the actual data, really does not exceed 30%, that is, a_{23} = 0,3. A certain amount of radionuclides precipitates in the bottom stream sediments. Estimated river runoff is 20% of the radionuclides storage falling into the river, i. e. a_{34} = 0,2.

Thus, the corresponding system of differential equations is as follows:

$$\begin{cases} \dfrac{dX}{dt} = -0{,}06\,X, \\ \dfrac{dY}{dt} = 0{,}06\,X - 0{,}3Y, \\ \dfrac{dZ}{dt} = 0{,}3Y - 0{,}2Z, \end{cases} \tag{2}$$

Such initial data are given: $X(0)=100, Y(0)=0, Z(0)=0.$ In the Maple 5 software environment, the system of differential equations is: sys:=diff(x(t),t)=-0.06*x(t), diff(y(t),t)=0.06*x(t)-0.3*y(t), diff(z(t),t)=0.3*y(t)- 0.2*z(t):fcns:={x(t),y(t),z(t)};

Let us set the initial data and solve the system: dsolve({sys,x(0)=100,y(0)=0,z(0)=0}, fcns, method=laplace);

$fcns := \{z(t), x(t), y(t)\}$

The following functions are its solution:

$$\{z(t) = -\frac{900}{7}\,e^{(-1/5\,t)} + 75\,e^{(-3/10\,t)} + \frac{375}{7}\,e^{(-3/50\,t)},\ y(t) = 50\,e^{(-9/50\,t)}\sinh\left(\frac{3}{25}\,t\right),$$
$$x(t) = 100\,e^{(-3/50\,t)}\}$$

Let us represent the graphs of the dynamics of finding/outflow (% from the initial storage) of radionuclides in all ecosystem compartments and analyze them (Fig. 2−4):

a) Compartment source **«forest»** (Fig. 2):

plot([100*exp(-3/50*t)],t=0..100,x=0..100);

It is evident from the graph that «forest» (*x*) over 100 years drops activity into the stream and then into the river.

b) Compartment **«stream»** (Fig. 3):

plot([50*exp(-9/50*t)*sinh(3/25*t)],t=0..100,y=0...20);

Compartment «stream» (*y*) rapidly accumulates radioactivity, accumulation peak is observed in the 8th year and is 13% from the total storage. Then follows the radionuclides discharge from «stream» to «river»;

c) Compartment **«river»** (Fig. 4):

plot([-900/7*exp(-1/5*t)+75*exp(-3/10*t)+375/7*exp(-3/50*t)], t=0..100, z=0...30);

It is evident from the graph that ^{137}Cs discharge into the «river» (*z*) is maximum in the 18th year and equals approximately 17%.

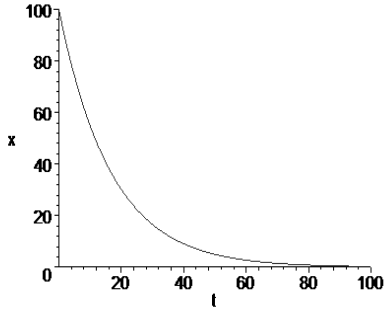

Figure 2. Dynamics of radionuclide outflow from the chamber «forest»

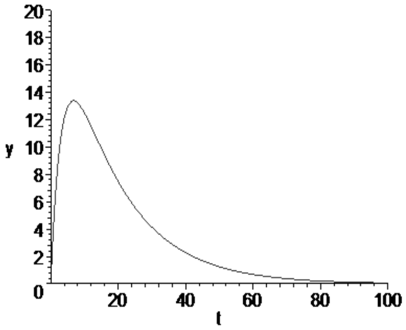

Figure 3. Dynamics of radionuclide outflow from the chamber «stream»

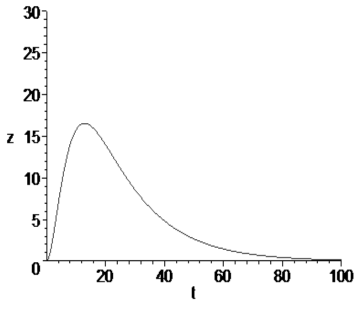

Figure 4. Dynamics of radionuclide outflow from the chamber «river»

Let us consider ^{137}Cs retension probability in the forest ecosystem, in each of the compartments (by formula (1)):

$$F_x = 1/(1+0{,}06)=0{,}94,\ F_y = 0{,}06/(0{,}3+0{,}06)=0{,}17,\ F_z = 0{,}3/(0{,}3+0{,}2)=0{,}6.$$

Since the considered system of compartments is in-series, the reliability of radionuclides transport in it is determined by the product of transport probabilities from each camera to the next.

Let us consider the probability of radionuclides discharge from the forest to the river that flows from the forest, and find reliability estimation (P) of radionuclide transport in this ecosystem ($P = 1 - F$):

$$P_x = 1- 0{,}94 =0{,}06;\ P = 1- 0{,}17 =0{,}83;\ P = 1- 0{,}6 = 0{,}4.$$

Then, $P_{\text{(transport)}} = 0{,}019$.

Therefore, only 1.9% of the total storage is able to move to the river from the forest and further to people through the water used from this ecosystem.

It is possible to limiting the radionuclides flow from the forest to other parts of the landscape by using protective measures (for example, building the cascade on the river, road along the stream, etc.). The block diagram of such compartment model will correspond to Fig. 1. The transition coefficients a_{23} and will change due to the presence of protective countermeasures preventing the radionuclides transition from one compartment to another:

$$a_{12} = a_{21} = 0{,}04;\quad a_{23} = 0{,}73;\quad a_{32} = 0{,}7;\ a_{34} = 0{,}23.$$

Relatively, the system of differential equations is asfollows:

$$\begin{cases} \dfrac{dX}{dt} = -0{,}04\,X, \\ \dfrac{dY}{dt} = 0{,}04\,X - 0{,}73Y, \\ \dfrac{dZ}{dt} = 0{,}7\,Y - 0{,}23Z, \end{cases} \tag{3}$$

In Maple 5 environment, this system of differential equations is:

```
sys:=diff(x(t),t)=-0.04*x(t),diff(y(t),t)=0.04*x(t)-0.73*y(t),
diff(z(t),t)=0.7*y(t)-0.23*z(t):fcns:={x(t),y(t),z(t)};
dsolve({sys,x(0)=100,y(0)=0,z(0)=0},fcns,method=laplace);
```

Its solution is function:

$$\{y(t) = \frac{800}{69}\,e^{\left(-\frac{77}{200}t\right)}\sinh\left(\frac{69}{200}t\right),\ z(t) = \frac{560}{69}\,e^{\left(-\frac{73}{100}t\right)} + \frac{28000}{1311}\,e^{(-1/25\,t)} - \frac{560}{19}\,e^{\left(-\frac{23}{100}t\right)},$$
$$x(t) = 100\,e^{(-1/25\,t)}\}$$

Let us represent the functions graphs and analyze the dynamics of radionuclides entry into compartments (Fig.5−7).

a) Compartment source «**forest**» (Fig. 5):

```
plot([100*exp(-1/25*t)],t=0..100,g=0...100);
```

It is evident from the graph that there is the smooth radionuclides discharge from forest (x).

b) Compartment «**stream**» (Fig. 6):

```
plot([800/69*exp(-77/200*t)*sinh(69/200*t)], t=0..100, y=0...10);
```

It is evident from the graph that in compartment «stream» the radionuclide discharge is slower and is not more than 5%, instead of 13%, as in the previous example. Due to the countermeasures applied, the radionuclides outflow decreases by 2.6 times.

c) Compartment **«river»** (Fig. 7):

```
plot([560/69*exp(-73/100*t)+28000/1311*exp(-1/25*t)-560/19*exp(-23/100*t)],t=0..100,
Z=0...20);
```

The radionuclides outflow into the river in dynamics does not exceed 12% (while in the previous version – 6%).

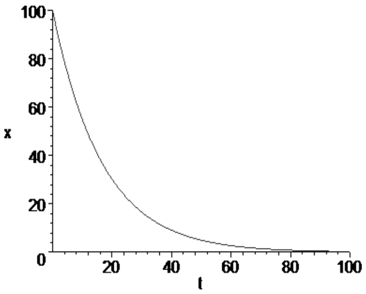

Figure 5. Dynamics of radionuclide outflow from the chamber «forest»

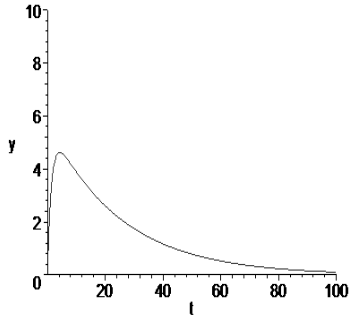

Figure 6. Dynamics of radionuclide outflow from the chamber «stream»

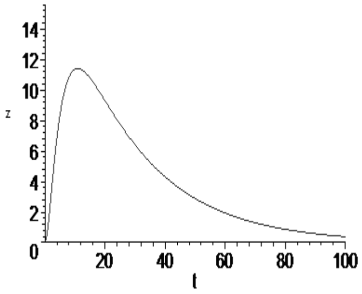

Figure 7. Dynamics of radionuclide outflow from the chamber «river»

Hence, the countermeasures significantly reduce the radionuclides outflow into the stream − in 2,6 times (5% instead of 13%).

Let us consider the probability of ^{137}Cs keeping in the forest ecosystem, as well as in each of the compartments:

$F_x = 1/(1+0{,}04)=0{,}96$; $F_y = 0{,}04/(0{,}04+0{,}73)=0{,}05$; $F_z = 0{,}7/(0{,}7+ 0{,}23)=0{,}75$.

Then the radionuclides transport reliability of such series ecosystem will be determined by the product ($P = 1 - F$):

$$P_{(general\ on\ transport)} = P_x \cdot P_y \cdot P_z = 0{,}04 \cdot 0{,}95 \cdot 0{,}25 = 0{,}0095.$$

Thus, the radionuclide transport reliability is reduced by 2 times compared with the previous version, due to protective measures, resulting in decrease in the radionuclides transport of by ecosystem compartments.

Let us consider the model with additional radionuclide discharge into the forest with time (0.05 *t* from the storage per year) while using countermeasures (for example, plowing, road). The entry additional source can be predicted by increased drainage, for example, and so on. The block diagram of such ecosystem is shown in Fig. 8

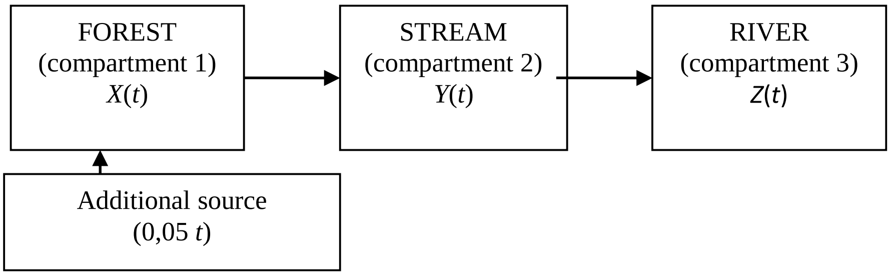

Figure 8. Block diagram of the forest box model with additional pollution source

Let us consider transition coefficients as in the previous version with additions: coefficient $a_{21} = 0{,}05t$, which corresponds to the additional discharge into the forest. The time unit is assumed 1 year, as in the previous versions.

The corresponding system of differential equations is as follows:

$$\begin{cases} \dfrac{dX}{dt} = -0{,}04\,X + 0{,}05t, \\ \dfrac{dY}{dt} = 0{,}04\,X - 0{,}73Y, \\ \dfrac{dZ}{dt} = 0{,}7\,Y - 0{,}23Z, \end{cases} \tag{4}$$

In the Maple environment the system is:

```
sys:=diff(x(t),t)=-0.04*x(t)+0.05*t,diff(y(t),t)=0.04*x(t)-0.73*y(t),
diff(z(t),t)=0.7*y(t)-0.23*z(t): fcns:={x(t),y(t),z(t)};
dsolve({sys,x(0)=100,y(0)=0,z(0)=0}, fcns, method=laplace);
```

The solution of this system is:

$$fcns := \{\mathrm{x}(t), \mathrm{y}(t), \mathrm{z}(t)\}$$

$$\{\mathrm{y}(t) = \frac{5}{73}\,t - \frac{9625}{5329} - \frac{711200}{122567}\,\mathrm{e}^{\left(-\frac{73}{100}\,t\right)} + \frac{175}{23}\,\mathrm{e}^{(-1/25\,t)},\ \mathrm{x}(t) = \frac{5}{4}\,t - \frac{125}{4} + \frac{525}{4}\,\mathrm{e}^{(-1/25\,t)},$$

$$\mathrm{z}(t) = \frac{350}{1679}\,t - \frac{18051250}{2819041} + \frac{995680}{122567}\,\mathrm{e}^{\left(-\frac{73}{100}\,t\right)} + \frac{12250}{437}\,\mathrm{e}^{(-1/25\,t)} - \frac{299040}{10051}\,\mathrm{e}^{\left(-\frac{23}{100}\,t\right)}\}$$

Let us construct the graphs of the radionuclides dynamics in compartments (Fig. 9−11).

a. Compartment **«forest + additional source»** (Fig. 9):

```
plot([5/4*t-125/4+525/4*exp(-1/25*t)], t=0..100, x=0..100);
```

It is evident from the graph that the slow discharge and then the pollution increase in compartment «forest» due to the additional external inflow of radionuclides are observed.

b. Compartment **«stream»** (Fig. 10):

```
plot([5/73*t-9625/5329711200/122567*exp(-73/100*t)+175/23*exp(-
1/25*t)],t=0..100,y=0...20);
```

Due to countermeasures, discharge into the stream (*y*) does not exceed 5% (that is, 2.6 times less than in the case without countermeasures), but the radionuclides increase due to an additional external discharge is observed.

c. Compartment **«river»** (Fig. 11):

```
plot([350/1679*t-18051250/2819041+995680/122567*exp(-73/100*t)+12250/437*exp(-
1/25*t)-299040/10051*exp(-23/100*t)],t=0..100,z=0...20);
```

It is evident from the graph that the radionuclides outflow into the river (*z*) does not exceed 12%, and in course of time gradually increases due to additional external discharge.

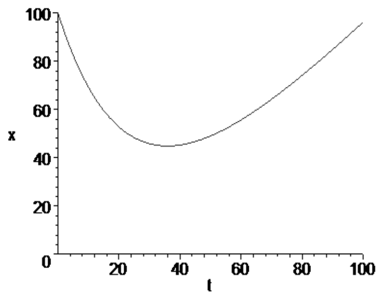

Figure 9. Dynamics of radionuclide outflow from the chamber «forest» with additional source of radionuclide

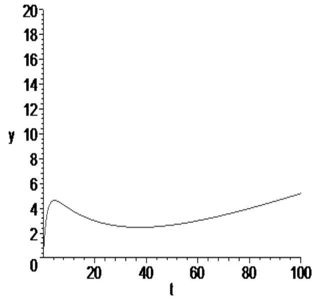

Figure 10. Dynamics of radionuclide outflow from the chamber «stream»

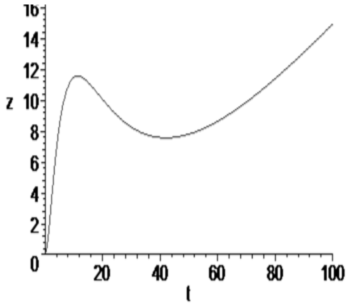

Figure 11. Dynamics of radionuclide outflow from the chamber «river»

Thus, due to the countermeasures use the radionuclides outflow from the forest into the stream sufficiently decreases in 2.6 times(5% instead of 13%), and then into the river – instead of 17%, only 12% at the beginning; in course of time, pollution increases due to additional radionuclide discharge as the result of increased discharge.

Let us consider the probability of ^{137}Cs keeping in each compartment of the ecosystem under consideration.

$$F_x=0{,}05/(0{,}05+0{,}04)=0{,}56;\ \ F_y=0{,}04/(0{,}04+0{,}73)=0{,}05;\ \ F_z=0{,}7/(0{,}7+0{,}23)=0{,}75.$$

Let us estimate the probability of radionuclides discharge from the forest further into the river that flows from the forest, that is, estimate the radionuclides transport reliability in this ecosystem (P = 1 - F):

$$P_x = 1 - 0{,}56 = 0{,}44;\ \ P_y = 1 - 0{,}05 = 0{,}95;\ \ P_z = 1 - 0{,}75 = 0{,}25.$$

Then, $P_{(transport)} = 0{,}10$.

That is, 10% of the initial total radionuclides storage in the forest are able to pass into the river, and then − through water use to people.

The considered approach is not limited by application for radionuclide transport simulation under conditions of radioactive pollution source.

Let us consider the following example. We construct the model of the ecosystem «burned forest» (for example, in case of aircraft crash or explosion on the forest territory with further fire outbrake).

While burning, the forest rapidly starts to «throw off» radioactivity, which acts as the indicator of the ecosystem condition. This is reflected in a significant increase in the coefficients of transition between the compartments «forest» and «stream» [11−12].

Block diagram of compartment model is shown in Fig. 1. The values of the transition (speed) coefficients are different: $a_{12} = a_{21} = 0{,}36$; $a_{23} = 0{,}5$; $a_{32} = 0{,}4$; $a_{34} = 0{,}2$. The compartment model is described by the system of differential equations:

$$\begin{cases} \dfrac{dX}{dt} = -0{,}36\,X, \\ \dfrac{dY}{dt} = 0{,}36\,X - 0{,}5Y, \\ \dfrac{dZ}{dt} = 0{,}4Y - 0{,}2Z, \end{cases} \qquad (5)$$

The system in Maple 5 and its solution are as follows:
sys:=diff(x(t),t)=-0.36*x(t),diff(y(t),t)=0.36*x(t)-0.5*y(t),
diff(z(t),t)=0.4*y(t)- 0.2*z(t):fcns:={x(t),y(t),z(t)};
dsolve({sys,x(0)=100,y(0)=0,z(0)=0},fcns,method=laplace);

$$\{\mathrm{z}(t)=300\,\mathbf{e}^{(-1/5\,t)}+\frac{2400}{7}\,\mathbf{e}^{(-1/2\,t)}-\frac{4500}{7}\,\mathbf{e}^{(-9/25\,t)},\ \mathrm{x}(t)=100\,\mathbf{e}^{(-9/25\,t)},$$

$$\mathrm{y}(t)=-\frac{1800}{7}\,\mathbf{e}^{(-1/2\,t)}+\frac{1800}{7}\,\mathbf{e}^{(-9/25\,t)}\}$$

Let us analyze the radionuclides dynamics in compartments (Fig. 12−14)

a. Compartment source **«forest»** (Fig. 12): plot([100*exp(-9/25*t)],t=0..100, x=0..100); It is evident from the graph that in course of 20 years “forest” dramatically discharges radioactivity into the stream.

b. Compartment **«stream»** (Fig. 13):
plot([-1800/7*exp(-1/2*t)+1800/7*exp(-9/25*t)],t=0..100,y=0...40);
The stream quickly discharges radioactivity, with maximum discharge during the 8th year and accounts 32% from the stockpile of radioactive pollution storage in this compartment.

c. Compartment **«river»** (*z*):
plot([300*exp(-1/5*t)+2400/7*exp(-1/2*t)-4500/7*exp(-9/25*t)],t=0..100, z=0...50);
Discharge into the river during the 17th year is approximately 34%.

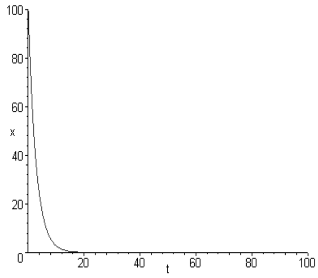

Figure 12. Dynamics of radionuclide outflow from the chamber «forest»

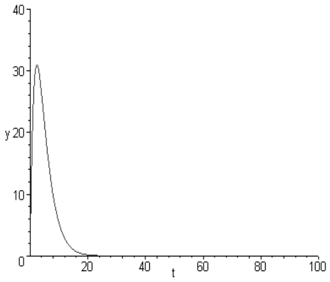

Figure 13. Dynamics of radionuclide outflow from the chamber «stream»

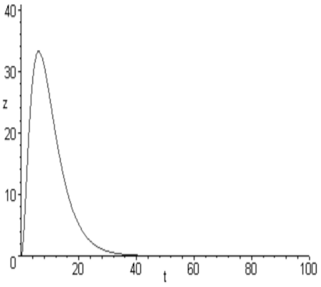

Figure 14. Dynamics of radionuclide outflow from the chamber «river»

Thus, in the considered situation, the processes of radioactivity transition from compartment to compartment are significantly accelerated.
Let us consider the probability of ^{137}Cs keeping in the forest ecosystem in each compartment.

F_x = 1/(1+0,36)=0,74; F_y = 0,36/(0,5+0,36)=0,42; F_z = 0,4/(0,4+0,2)=0,67.

Let’s estimate the probability of radionuclides discharge from the forest to the river. Such estimation of the radionuclide transport reliability in the given ecosystem (P = 1 - F):

P_x = 0,26; P_y = 0,58; P = 0,33;
$P_{(transport)}$ = 0,05.

Hence, only 5% of the total radionuclide storage in the considered ecosystem can move from the forest to the river, and then − to people through water use.

Conclusions. The principal possibility of mathematical ecosystems modeling and the countermeasures influence consideration (due to the corresponding change in the parameters of the compartment model), as well as the additional radionuclides entry (due to the introduction of additional terms in the system of differential equations) are shown in the proposed models, based on the developed method of applying the reliability theory of complex systems. The wide opportunities of such modeling and use of reliable approach to the analysis of forest ecosystems in different states are shown.

Analysis of the radioecological state of forest ecosystems by means of mathematical models and methods of reliability theory is an effective method for radiological situations estimation and modeling, as well as estimation of efficiency of real protective measures application after accidents, particularly fires. Mathematical modelling application in the forest ecosystems (and other types of ecological systems) investigation makes it possible to determine the level of pollution and accumulation of radionuclides/pollutants in them.

The influence of technogy-related factors on the environment depends on and varies from many factors: season (what season the pollutants discharges take place), the discharges intensity, the dynamics of living organisms biomass growth, their species structure, etc. Such indicators should be taken into account in mathematical modelling and carrying out investigations and calculations.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ДИНАМІКИ РАДІОЕКОЛОГІЧНИХ ПРОЦЕСІВ ТА НАДІЙНОСТІ ТРАНСПОРТУ ПОЛЮТАНТІВ У ЛІСОВІЙ ЕКОСИСТЕМІ

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Резюме. *В навколишньому середовищі під впливом техногенних чинників відбуваються перетворення, наслідком яких є необоротні зміни в біологічних угрупованнях та втрата стійкості природних систем. Створено й проаналізовано систему оцінювання надійності компонентів лісових екологічних систем за допомогою побудови послідовних камерних моделей, методів теорії надійності та диференціальних рівнянь. За допомогою алгоритму оцінювання надійності екологічних систем проведено аналіз радіоекологічного стану екосистеми лісу, розподіл та перерозподіл радіонуклідів унаслідок транспорту полютантів її компонентами. На основі теорії та моделей надійності екосистем через оцінювання параметрів радіоємності створено модель для оцінювання надійності утримання радіонуклідів/полютантів в екосистемах. Оцінювання параметрів радіоємності проведено за формулою* $F_j = \sum a_{ij} / (\sum a_{ij} + \sum a_{ji})$, *де* $\sum a_{ij}$ *– сума швидкостей переходу полютантів з різних складових екосистеми до конкретного елемента ландшафту або екосистеми (камеру i);* $\sum a_{ji}$ *– сума швидкостей відтоку полютантів з досліджуваної камери (j) до інших складових екосистеми, спряжених з ними. Сформовано приклади камерних моделей лісової екосистеми з урахуванням коефіцієнтів/швидкостей переходів радіонуклідів між камерами. Досліджено варіанти транспорту радіонуклідів у системі: за звичайних умов, при застосуванні захисних заходів (збільшення стоку радіонуклідів та застосування відповідних будівельних конструкцій), при створенні аварійної ситуації (пожежі). Надійність транспорту радіонуклідів між камерами екосистеми, що досліджується, обчислено за допомогою математичної програми MAPLE 5. Отримані розрахунки відображають динаміку міграції радіонуклідів від вихідного джерела до інших компонентів системи. Показано, що застосування математичного моделювання при вивченні стану природних екологічних систем дозволяє встановити стан їх забруднення, а також визначати рівні нагромадження полютантів у різних компонентах екосистеми.*

Ключові слова: *математичне моделювання, камерні моделі, транспорт радіонуклідів, надійність лісової екосистеми.*

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