Вісник Тернопільського національного технічного університету
https://doi.org/10.33108/visnyk_tntu
Scientific Journal of the Ternopil National Technical University
2019, № 2 (94) https://doi.org/10.33108/visnyk_tntu2019.02
ISSN 2522-4433. Web: ***visnyk.tntu.edu.ua***

UDC 620.171.3

PREDICTION TECHNIQUE FOR THIN-WALLED CYLINDRICAL TUBES BOUNDARY STATE

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Summary. *Thin-walled cylindrical tubes are used not only as structural elements but also cause great scientific-practical interest for modeling the behavior of structural elements with different geometrical shapes under complex stress state. The prediction technique for thin-walled cylindrical tubular samples of metal isotropic materials loaded by internal pressure and axial tensile strength is proposed in this paper. The investigation was carried out within momentless theory for large residual deformation areas. The material was considered to be isotropic and incompressible. Elastic deformations were neglected. The realization of Kirchhoff-Love hypothesis of thin-walled shell theory is accepted. The equilibrium boundary conditions of plastic deformation were obtained analytically. In order to derive the boundary relationships between residual relative strains and real stresses Dorn-Nadai conditions of the beginning of deformation localization process were used. The influence of stressed state and thin-walled tube geometry on the boundary real stresses and residual deformations values is observed. The analysis of the obtained conditions showed the decrease of the material strength resource when the values of primary stresses ratios approach to 0.5 and 2. It is proved analytically that with the reduction of the tensile strength and approximation of stressed state to the «internal pressure» type the strength resource of the thin-walled cylindrical tube sharply decreases.*

Key words: *large deformations, strength conditions, thin-walled cylinders, complex stress state.*

https://doi.org/10.33108/visnyk_tntu2019.02.145 *Received 10.11.2019*

Introduction. The thin-walled cylindrical tubes besides their direct use as structural elements are of great scientific-practical interest for modeling the behavior of the structural elements with different geometrical shapes under complex stress conditions. The experimental investigation of deformation mechanisms and materials fracture along with the improvement of phenomenological approaches to the estimation of the boundary states of the thin-walled cylindrical structures or their elements remains an important problem of deformable solids mechanics.

Statement of the problem. Most investigations of the boundary state of material samples under complex stress state are reduced to the construction of the strength criteria or equivalence conditions under engineering stresses. The suitability of the proposed criteria is substantiated on the basis of data bank obtained mostly from experiments on thin-walled cylindrical tubes loaded by the internal pressure and axial tensile force. Such approach makes it possible to model multioption biaxial stressed states in the structure walls predicting maximum loads and the greatest uniform deformations. At present there is a large number of such criteria quite comprehensive review of which is given in papers [1, 2, 3]. However, nowadays there is no theory or criterion that uniquely determines the boundary stress values for certain material under complex stress state. On the other hand, with the appearance of new computer technique and growth of its calculation abilities, development of new constructional materials and the need to take into account new conditions of structures operation the requirements to the accuracy and reliability of the calculated boundary values of the stress-strain state indices of the structural elements are constantly increasing. Therefore the problem of the estimation technique for structural elements boundary states still remains important for deformable solids mechanics

Analysis of the available researches and publications. Depending on the loading conditions the material can be in different mechanical states – elastic, plastic or fracture. The material stresses state where the qualitative changes in the material properties occur - the transition from one mechanical state to another is called boundary. The problem of prediction the occurrence of the boundary states of cylindrical shells under complex stress state is concerned in many experimental investigations described in the available literature [4–7]. T he basic theoretical statements and formulae for determination of stresses in the cylindrical tube wall are given in [8]. The authors Middleton J., Owen DRJ., Blachut J., Zhu L., Boyle JT., Carbonari, R. C., and others proposed the developments for optimization of thin-walled axisymmetrical shell profiles loaded by internal pressure with additional loading conditions [9–12]. However the problem of the shell geometry influence on their strength characteristics is not sufficiently investigated.

The objective of the paper is to develop the technique for obtaining the conditions of boundary state occurrence in thin-walled cylindrical tubes under the action if internal pressure and axial tension for large residual plastic deformation area; to observe the influence of the stress state and geometrical characteristics of tubular samples on the strength indices.

Statement of the problem. The paper deals with considerable uniform plastic deformations in the tube operating part to which tensile axial force *N* (in tube *z* axis direction) and internal pressure *q* are applied. The realization of Kirchhoff-Love hypothesis of thin-walled shell theory is accepted [13]. The shell material is considered to be isotropic and incompressible.

Let, R be the radius of the middle surface of cylindrical tube with thickness *h*, ε_z ε_θ i ε_r are axial, circular and radial relative residual deformations respectively.

True axial σ_z and circular σ_θ stresses are determined by relations (1), (2), as shown in [14]:

$$\sigma_z = \frac{N}{2\pi R(1+\varepsilon_\theta)h(1+\varepsilon_r)} + \frac{\left(R(1+\varepsilon_\theta) - \frac{h}{2}(1+\varepsilon_r)\right)^2 \cdot q}{2R(1+\varepsilon_\theta)h(1+\varepsilon_r)} \tag{1}$$

$$\sigma_\theta = \frac{q\left(R(1+\varepsilon_\theta) - \frac{h}{2}(1+\varepsilon_r)\right)}{h(1+\varepsilon_r)}. \tag{2}$$

For the given problem statement according to the accepted suppositions the radial stresses in the tube wall are neglected: $\sigma_r = 0$.

When the loading reaches its maximum value as the result of the cross-section area reduction and decrease of the strengthening module value of the process the uniform deformation development stops and the process of deformation localization with neck formation starts. In this regard the load starts decreasing, the process of steady uniform plastic deformation is broken. Such correspondence between the maximum loads and local deformations occurrence accepted in papers [15, 16, 17], is called as Dorn-Nadai conditions.

This approach used by the authors in papers [18, 19, 20], is analytically expressed by one of the conditions: $dq = 0$ чи $dN = 0$.

Let us consider both cases.

1) Finding the dependence between the boundary circular stresses and strains. Let us use condition

$$dq = 0\,. \tag{3}$$

From (2) we express q:

$$q = \frac{\sigma_\theta h\left(1+\varepsilon_r\right)}{R\left(1+\varepsilon_\theta\right) - \frac{h}{2}\left(1+\varepsilon_r\right)}\,. \tag{4}$$

Suppose the load is proportional i. e.

$$\frac{\sigma_z}{\sigma_\theta} = k\,. \tag{5}$$

Extending the effect of the generalizes Hooke law to the area of plastic deformations with Poisson coefficient $\mu = 0{,}5$, we get

$$\frac{\varepsilon_z}{\varepsilon_\theta} = n\,, \tag{6}$$

where

$$n = \frac{2k-1}{2-k}\,. \tag{7}$$

Let us use the material incompressibility condition for the area of large plastic deformations:

$$\left(1+\varepsilon_r\right)\left(1+\varepsilon_\theta\right)\left(1+\varepsilon_z\right) = 1\,. \tag{8}$$

Taking into account $R = const$ i $h = const$ as initial values as well as dependences (4)–(8), from (3) we derive the differential equation of the first order:

$$\frac{\partial q}{\partial \sigma_\theta} d\sigma_\theta + \frac{\partial q}{\partial \varepsilon_\theta} d\varepsilon_\theta = 0 \tag{9}$$

or, substituting the partial derivatives,

$$\frac{1}{\left(1+\varepsilon_\theta\right)^2\left(1+n\varepsilon_\theta\right) - \frac{h}{2R}} d\sigma_\theta - \sigma_\theta \frac{2\left(1+\varepsilon_\theta\right)\left(1+n\varepsilon_\theta\right) + n\left(1+\varepsilon_\theta\right)^2}{\left(\left(1+\varepsilon_\theta\right)^2\left(1+n\varepsilon_\theta\right) - \frac{h}{2R}\right)^2} d\varepsilon_\theta = 0\,. \tag{10}$$

The general equation integral (10):

$$\sigma_\theta = C_\theta \left[\left(1+\varepsilon_\theta\right)^2 \left(1+n\varepsilon_\theta\right) - \frac{h}{2R} \right]. \tag{11}$$

Condition (11) determines the values of circular stresses and circular deformations corresponding to the moment of the tube plastic deformation resistance loss from the internal pressure q action. Since under cylindrical tube loading by internal pressure, the residual circular deformations $\varepsilon_\theta > 0$, then its smallest value for $0{,}5 < k < 2$, relatively at $n \in (0;\infty)$, the boundary circular stress σ_θ will be acquired at $n \approx 0$, which is in an agreement with the accepted relation (7).
The lines in the coordinate system $\sigma_\theta : \varepsilon_\theta$ being the geometrical interpretation of condition (11), are called boundary conditions curves or boundary curves.

The integration constant is found from experiments for $k = 0{,}5$. In this case condition (11) is as follows

$$\sigma_\theta = C_\theta \left[\left(1+\varepsilon_\theta\right)^2 - \frac{h}{2R} \right]. \tag{12}$$

Let us accept the notation

$$\eta = \frac{h}{R} \tag{13}$$

We apply formula (12) for deformations localization area, defining $\eta \to 0$, and get

$$\sigma_\theta = C_\theta \left(1+\varepsilon_\theta\right)^2. \tag{14}$$

Condition (14) coincides with the condition derived for the determination of boundary circular stresses in the cylindrical tube in [20].

2) Finding the dependence between the boundary axial stresses and strains.

Let us use condition

$$dN = 0. \tag{15}$$

We use equation derived from (3) and (4):

$$N = 2\pi R h \left(1+\varepsilon_\theta\right)\left(1+\varepsilon_r\right)\sigma_z \left[1 - \frac{1}{2k}\left(1 - \frac{h\left(1+\varepsilon_r\right)}{2R\left(1+\varepsilon_\theta\right)} \right) \right]. \tag{16}$$

Let us find complete differential and apply condition (15):

$$\frac{\partial N}{\partial \sigma_z} d\sigma_z + \frac{\partial N}{\partial \varepsilon_\theta} d\varepsilon_\theta + \frac{\partial N}{\partial \varepsilon_r} d\varepsilon_r = 0. \tag{17}$$

When partial derivatives and identical transformations are found, we derive the equation

$$\frac{1-\frac{1}{2k}}{1+\varepsilon_\theta}d\varepsilon_\theta+\left[1-\frac{1}{2k}+\frac{h(1+\varepsilon_r)}{2kR(1+\varepsilon_\theta)}\right]\frac{d\varepsilon_r}{1+\varepsilon_r}+\left[1-\frac{1}{2k}\left(1-\frac{h(1+\varepsilon_r)}{2R(1+\varepsilon_\theta)}\right)\right]\frac{d\sigma_z}{\sigma_z}=0.$$

Let us use the incompressibility condition (8) and notation (6):

$$\left[1-\frac{1}{2k}\left(1-\frac{h(1+\varepsilon_r)}{2R(1+\varepsilon_\theta)}\right)\right]\frac{d\sigma_z}{\sigma_z}=\left(1-\frac{1}{2k}\right)\frac{d\varepsilon_z}{1+\varepsilon_z}-\frac{h(1+\varepsilon_r)}{2kR(1+\varepsilon_\theta)}\frac{d\varepsilon_r}{1+\varepsilon_r},$$

$$\left[1-\frac{1}{2k}\left(1-\frac{h(1+\varepsilon_r)}{2R(1+\varepsilon_\theta)}\right)\right]\frac{d\sigma_z}{\sigma_z}=\left[1-\frac{1}{2k}+\frac{h(1+\varepsilon_r)}{2kR(1+\varepsilon_\theta)}\right]\frac{d\varepsilon_z}{1+\varepsilon_z}+\frac{h(1+\varepsilon_r)}{2knR(1+\varepsilon_\theta)}\frac{d\varepsilon_z}{1+\frac{1}{n}\varepsilon_z}.$$

For uniform plastic deformation areas we consider the change of wall thinness insignificant in comparison with residual deformations in direction towards the force application axis, i. e.

$$\frac{h(1+\varepsilon_r)}{R(1+\varepsilon_\theta)}\approx\frac{h}{R}. \tag{18}$$

Finally we get:

$$\left(1-\frac{1}{2k}+\frac{h}{4kR}\right)\frac{d\sigma_z}{\sigma_z}=\left[\left(1-\frac{1}{2k}+\frac{h}{2kR}\right)\frac{1}{1+\varepsilon_z}+\frac{h}{2knR}\frac{1}{1+\frac{1}{n}\varepsilon_z}\right]d\varepsilon_z. \tag{19}$$

Taking into account notation (13), the general equation integral (19) is the following:

$$\sigma_z=C_z\left[(1+\varepsilon_z)^{2(2k-1+\eta)}\left(1+\tfrac{1}{n}\varepsilon_z\right)^{2\eta}\right]^{\frac{1}{4k-2+\eta}}. \tag{20}$$

The integration constant is found from the experiments of uniaxial tension of the cylindrical tube only by tensile strength. Under plastic-elastic conditions approximating to the uniaxial tension $(k\to\infty)$ the condition is simplified to the following form

$$\sigma_z=C_z(1+\varepsilon_z). \tag{21}$$

Condition (21) coincides with that derived for biaxial tension of the thin-walled cylinder by axial force and internal pressure which we obtained in [20]. Formula (21), obtained for the case of uniaxial strip tension and biaxial plate tension, given in the same source.

The boundary line in this case is the line with angular coefficient numerically equal to the integration constant.

Let us write the conditions in the form:

$$\begin{cases} \sigma_\theta = C_\theta \left[\left(1+\varepsilon_\theta\right)^2 \left(1+n\varepsilon_\theta\right) - \dfrac{\eta}{2} \right], \\ \sigma_z = C_z \left[\left(1+\varepsilon_z\right)^{2(2k-1+\eta)} \left(1+\frac{1}{n}\varepsilon_z\right)^{2\eta} \right]^{\frac{1}{4k-2+\eta}}. \end{cases} \qquad (22)$$

When $k=1$ we get

$$\begin{cases} \sigma_\theta = C_\theta \left[\left(1+\varepsilon_\theta\right)^3 - \dfrac{\eta}{2} \right], \\ \sigma_z = C_z \left(1+\varepsilon_z\right)^{\frac{2(1+2\eta)}{2+\eta}}. \end{cases} \qquad (23)$$

The index of thinness η affects the value of both boundary stresses. It is evident from the first formula of system (23), that the increase of index η decreases plastic-elastic resource of the tube in circular direction. For tubes with arbitrary ratio of the wall thickness to diameter $\eta \in \left(0;\frac{1}{2}\right)$ $\left[\frac{2(1+2\eta)}{2+\eta}\right]' > 0$ is realized, that is why the second formula of the system (23) analytically confirms the increase of boundary values of axial stresses with η growth.

Finding the integration constants for boundary conditions (11), (20)

Condition (11) is analytical expression for finding the boundary circular stress in thin-walled cylindrical tube for case $\sigma_\theta > \sigma_z$, condition (20) is boundary axial stress for case $\sigma_z > \sigma_\theta$. If both main stresses are $\sigma_z > 0$ та $\sigma_\theta > 0$, then conditions (11) and (20) are boundary conditions for stress states with the main stresses ratio $k = 0{,}5...2$.

In order to find integration constant C_θ in general integral (11) we consider the partial case of the occurrence of the boundary state of the plastic deformation of the cylindrical tube loaded by internal pressure q, in this case $k = 0{,}5$. The boundary circular stresses and deformation $\varepsilon_\theta{}^b$, $\sigma_\theta{}^b$ correspond to the moment of sample fracture in case with material brittle state and beginning of the deformation localization in case of material plastic state.

The results of the experiments for cylindrical thin-walled steel samples are used as the example and are given in [14, 18, 20]. The chemical composition, thermal pre-treatment of the samples and experiment conditions are also described there. The values of the integration constants are given in Table 1.

Table 1

Calculation of the integration constants values for condition (12)

Material grade	$\eta = h/R$	$\varepsilon_\theta{}^b$	$\sigma_\theta{}^b$	C_θ	Determined stresses $\sigma_\theta{}^b / C_\theta$
Steel 0,23%C	0,078	0,150	595 MPa	463,6	1,28
Steel 0,37%C	0,048	0,063	665 MPa	594,8	1,12
Steel 10GN2MFA	0,06*	0,036	705 MPa	666,2	1,06
Steel 15Kh2MFA	0,067	0,021	746 MPa	727,3	1,03
Steel 15Kh2NMFA	0,067	0,024	745 MPa	722,0	1,03

*the average value for common ratio $\eta = h/R$ for thin-walled shells with $\eta = 0,04..0,08$ is taken

Similarly on the basis of the boundary stresses and boundary residual deformations $\sigma_z = \sigma_z{}^b$, $\varepsilon_m = \varepsilon_m{}^b$ of cylindrical tubes under plastic deformation for the main stresses ratio $k = 2$ the integration steels C_z for condition (20) are determined. The calculation values are given in Table 2.

Table 2

Calculation of integration constants values for condition (20)

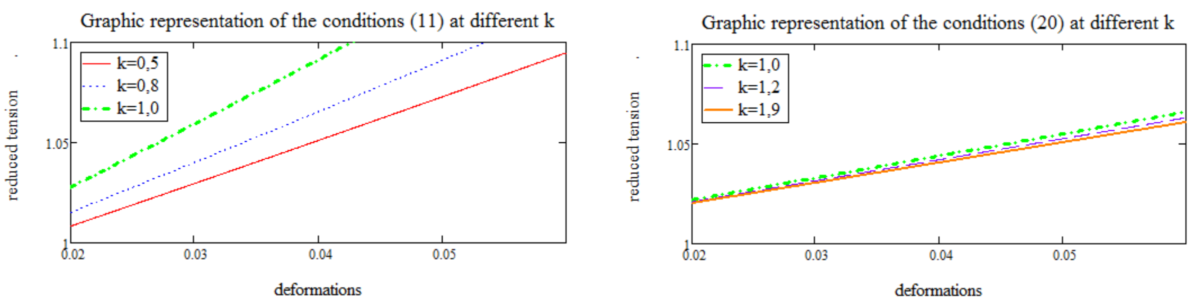
Material grade	$\eta = h/R$	$\varepsilon_z{}^b$	$\sigma_z{}^b$	C_z	Determined stresses $\sigma_z{}^b/C_z$
Steel 10GN2MFA	0,06*	0,043	722 MPa	691,9	1,04
Steel 15X2MΦA	0,067	0,042	830 MPa	796,2	1,04
Steel 15Kh2MFA	0,067	0,03	800 MPa	776,5	1,03
Steel 28 Kh 3SNMVFA	0,06*	0,016	2020 MPa	1987,9	1,02

*the average value for the common common ratio $\eta = h/R$ for thin-walled shells with $\eta = 0,04..0,08$ is taken

Graphic investigation of conditions (11) and (20)

The graphs of the strength loss conditions (11) of cylindrical tube for the case $\sigma_\theta > \sigma_z$ were constructed for three values of the main stresses ratio in the range 0,5…1 and thinness index $\eta = 0,067$ (Fig. 1, a). The amplitude of the values of relative conditional residual deformations $\varepsilon_\theta{}^b$ and determined stresses $\sigma_\theta{}^b/C_\theta$ were chosen according to Table 1.

The graphs of the strength loss conditions (20) for the case $\sigma_z > \sigma_\theta$, constructed for the values of the normal stresses ratio in the range 1…2 are shown in Fig. 1, b. The amplitude of the values of relative residual deformations $\varepsilon_z{}^b$ and determined stresses $\sigma_z{}^b/C_z$ and value $\eta = 0,067$ were chosen according to Table 2.



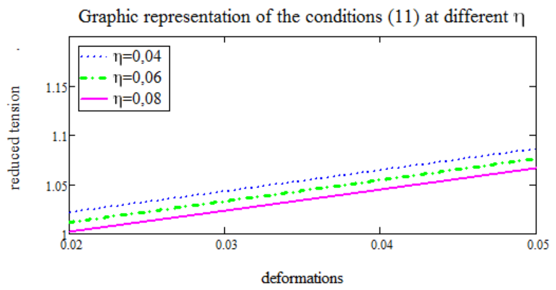
a) b)

Figure 1. Graphic representation of the strength loss conditions at different k : a) condition (11) for the case $\sigma_\theta > \sigma_z$ b) condition (20) for the case $\sigma_z > \sigma_\theta$

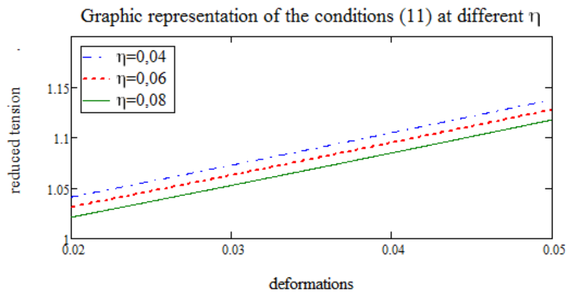
Analysis of the graphical images of conditions (11) and (20), performed for the cases $\sigma_\theta > \sigma_z > 0$ (Fig. 1, a) and $\sigma_z > \sigma_\theta > 0$ (Fig. 1, b) respectively, showed the decrease of material strength resource while approximating the main stresses ratio values to 0,5 and 2, increasing at $k \to 1$. It should be noted that when the impact of tensile strength decreases and stress state approximates to the «internal pressure» form the strength resource of the thin-walled cylindrical tube sharply decreases.

In order to observe the influence of the thinness index η of the cylindrical tube on the values of boundary residual deformations and true stresses a series of graphs were constructed where the index of stress state k and specified parameter were recorded.

The boundary curves described by condition (11) at main stresses ratios $k = 0,5$ та $k = 1$ are shown in Figure 2. Parameter η was given the values 0,04; 0,06 and 0,08.



a) $k = 0,5$

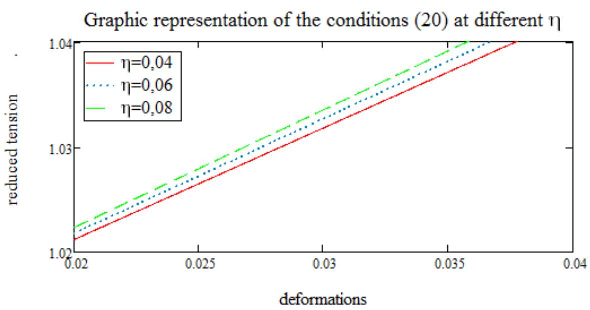


b) $k = 1$

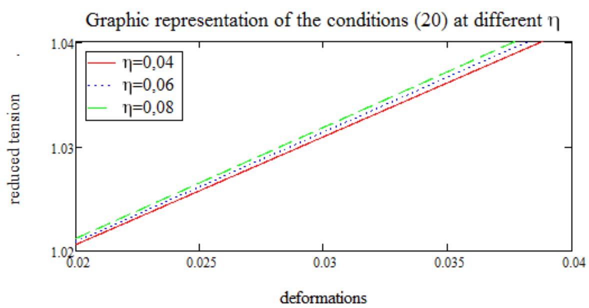
Figure 2. Graphic representation of the dependence (11) of boundary circular stresses σ_θ and residual deformations ε_θ on parameter η

Graphs analysis (Fig. 2) showed the decrease of the calculation strength resource of the cylindrical shell with the increase of values η in both cases. The boundary stress is achieved at lower level of residual plastic deformations if the ratio of the initial wall thickness of the tube to its initial radius is larger. I t should be also noted that at $k = 0,5$ a certain level of the relative residual deformations is reached by lower stress level than at $k = 1$, that is for the same thinness index η the tube resource is decreased while approaching the value $k = \sigma_m / \sigma_t$ до 0,5.

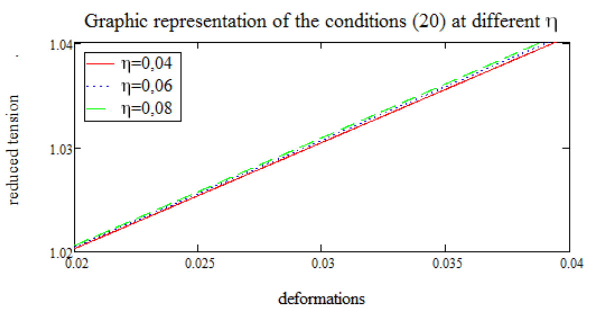
Boundary curves described by condition (20) for main stresses ratios $k = 1,0; 1,2; 1,5; 1,9$ are shown in Fig. 3. Parameter η is given values 0,04; 0,06 and 0,08.



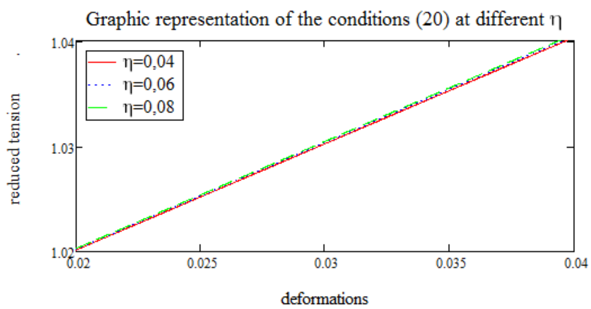
a) $k = 1$



b) $k = 1,2$



c) $k = 1,5$



d) $k = 1,9$

Figure 3. Graphic representation oft he dependence (20) of boundary axial stresses σ_z and residual deformations ε_z on parameter η at different k

Analysis of Fig. 3 showed that strength resource of the cylindrical tube under conditions of loading by internal pressure and axial tensile force is slightly higher while increasing the initial ratio $\eta = h / R$ for values k, close to 1 (Fig. 3, a, b), and are practically independent on ratio h / R in case when stresses σ_z are determining and precede the cylindrical sample neck formation (Fig. 3, c, d).

Conclusions and prospects. The developed technique makes it possible to predict the behavior of the thin-walled cylindrical tubular samples made of metal isotropic materials loaded by internal pressure and axial tensile force taking into account the results of two experiments – loading by internal pressure *P* and uniaxial tension by force *N;* the boundary conditions of the plastic deformation boundary equilibrium are obtained.

Analysis of the investigation results showed that the level of boundary stresses and residual plastic deformations depends not only on the values of boundary axial and circular stresses and strains but on the type of stress state, by represented in the paper coefficient k and tube thinness index η. Generalization of the proposed technique for axisymmetric thin-walled shells of the form makes it possible to apply it for the wide range of structural elements made of isotropic plastic-elastic materials operating under complex stress conditions.

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УДК 620.171.3

МЕТОДИКА ПРОГНОЗУВАННЯ ГРАНИЧНИХ СТАНІВ ТОНКОСТІННИХ ЦИЛІНДРИЧНИХ ТРУБОК

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Резюме. *Переважна більшість конструктивних елементів та апаратів, що застосовуються в авіабудуванні, суднобудуванні, харчовій, хімічній та інших галузях промисловості – це циліндричні оболонки, що відрізняються матеріалом, співвідношенням товщини стінки до діаметру, конструкцією конструкції та призначенням. Осесиметричні тонкостінні оболонки обертання – це частини конструкцій, що характеризуються високою несучою здатністю і використовуються в різних галузях техніки. Тонкостінні циліндричні трубки, окрім використання безпосередньо як елементів конструкцій, становлять науково-практичний інтерес для моделювання поведінки елементів конструкцій інших геометричних форм в умовах складного напруженого стану. В роботі запропоновано методику прогнозування поведінки тонкостінних циліндричних трубчастих зразків металевих ізотропних матеріалів, навантажених внутрішнім тиском та осьовим розтягуючим зусиллям. Дослідження проведено в рамках безмоментної теорії для ділянки великих залишкових деформацій. Матеріал вважався ізотропним та нестисливим. Пружними деформаціями було знехтувано. Прийнято виконання гіпотез Кірхгофа-Лява теорії тонкостінних оболонок. Аналітично отримано умови граничної рівноваги пластичного деформування. Для виведення граничних співвідношень між залишковими відносними деформаціями та істинними напруженнями використано умови Дорна-Надаі початку процесу локалізації деформацій. Прослідковано вплив виду напруженого стану та показників геометрії тонкостінної трубки на величину граничних істинних напружень та залишкових деформацій. Аналіз отриманих умов показав зменшення ресурсу міцності матеріалу при наближенні значень співвідношень головних напружень до 0,5 та 2. Аналітично доведено, що при зменшенні впливу розтягуючого зусилля та наближенні напруженого стану до виду «внутрішній тиск» ресурс міцності тонкостінної циліндричної трубки стрімко спадає. Запропонована у статті методика прогнозування критичних значень напружень у стінках тонкостінних циліндрів удосконалює теоретичний та інженерний апарат для оцінки та запобігання небезпечних станів у конструкціях типу котлів, реакторів та трубопроводів.*

Ключові слова: *великі деформації, умови міцності, тонкостінні циліндри, складний напружений стан.*

https://doi.org/10.33108/visnyk_tntu2019.02.145 *Отримано 10.11.2019*