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INVESTIGATIONS OF A THERMOMECHANICAL PROCESSES IN PRODUCTS SUBJECTED TO ANNEALING AND OPERATING CONDITIONS AFTER ANNEALING: THEORY, CALCULATIONS

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Summary *An approach to investigation of the thermomechanical processes in solids during cooling under high-temperature annealing and following after annealing thermomechanical conditions is proposed. The annealing is oriented on the relaxation of the residual stresses. This approach is based on the non-stationary thermal conductivity problem, on the problem about a thermoelastoplastic stress and strain state of the thermal sensitive hardenable solids in the deforming process and also on the software based on the finite element method (FEM). As an example the evolution of the residual stress state in disk subjected to annealing and following influence of the static loading applied to the disk hole is analyzed. The hole in disk is coaxial and circular. The residual stresses are specific after coupling by butt welding of two ring disks.*

Key words*: annealing, cooling, thermomechanical conditions, residual stresses, relaxation, plastic deforming, thermosensitivity, hardening, finite element method, disk.*

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Statement of the problem. Residual stresses in solids, products, structural elements are acquired and usually undesirable consequence of a number of technological processes and operating conditions, as they reduce the strength of products and their durability [1–3]. In particular, residual stresses occur as a result of welding [4, 5], casting, quenching, pressing, heat treatment, etc. To increase the strength properties of such elements, in engineering, high-temperature annealing with successively implemented stages namely heating to high temperature, holding at this temperature for a long time and cooling to natural temperature is widely used [1, 2, 6] (Figure 1).

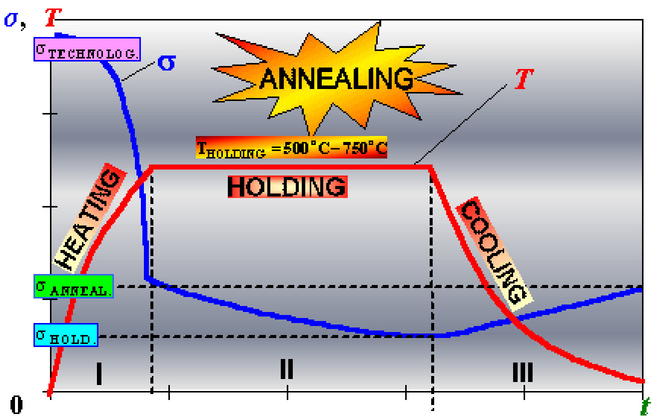

Figure 1. The stages of the high-temperature annealing and scheme of temperature and stresses change under annealing

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In Figure 1 stress, temperature and time are indicated with $\boldsymbol{\sigma}$, $\boldsymbol{T}$ and $\boldsymbol{t}$ respectively.

Estimation of the stresses relaxed at the end of annealing is important for predicting the operation of structural elements with residual stresses in operating conditions. High-temperature annealing is a costly technological process. For welded structures, the price of heat treatment is 25%–28% of the total cost of welding [7]. Therefore, the use of annealing heat treatment is not expedient if the products with residual stresses are reliably operated without previously application of annealing. For products with residual stresses, the following tasks are in demand in practice: a) evaluation of stresses relaxed as a result of annealing; b) evaluation of stresses in annealed products operating in thermal conditions. The problems of mathematical physics, which are formulated for both presented cases, are characterized by initial stresses. These initial stresses may not be described by functional dependencies and make it difficult to solve the corresponding problems.

Analysis of available investigations. Approximate methods, and especially the finite element method (FEM), make it possible to solve a wide range of problems in mechanics and thermomechanics [8, 9]. However, the presented above tasks in regard to stress estimation are not sufficiently considered. An important reason for this is that the known FEM software systems are not focused on solving problems for solids with initial stresses. The developed set of FEM programs [10] is an extension of algorithms [11, 12] and generalization of computational schemes in case of initial stresses.

The problem of annealing, namely the estimation of stresses in annealed products, which are operated under certain technological conditions related to the action of forces or temperature, is considered less than mechanical phenomena in the annealing process. The operational capabilities of annealed products are associated with the study of microstructure and mechanical properties [13, 14], as well as the improvement of design properties [15], etc.

Theoretical studies on the estimation of residual stresses are based on the relationships of thermomechanics and allow to obtain quantitative distributions of thermomechanical fields as a result of welding, heat treatment, etc. technological processes [1–3, 16]. There is a number of difficulties associated with taking into account the specific features of high-temperature or mechanical treatment, such as the presence of domains of plastic deformation, hardenability of materials, thermal sensitivity of material properties in the range of realized temperatures, etc., which are significant factors influencing current and residual stress and strain states (SSS). As a rule, the study of the mechanical state is restricted to theoretical beliefs based on certain simplifications and does not comprehensively take into account these significant physically observable phenomena. Experimental studies require expensive equipment to perform a series of well-engineered experiments that can often be destructive [1]. Estimation of annealing stresses is also difficult due to the phenomena of creep and phase transformations, which are significantly manifested at the holding stage and studied more experimentally than theoretically [1]. Experiments carried out about certain materials do not allow making more general predictions about other materials used in practice. After this type of heat treatment, the products are operated with the acquired stresses, which are often not estimated quantitatively. In this regard, the theoretically made and physically justified prediction of residual stresses is an important technological, applied and scientific problem. Based on the known stress distributions at the end of holding, neglecting the insignificant influence of phase transformations and the phenomenon of creep at the cooling stage, it is possible to theoretically predict the final stresses at the end of annealing (which become residual during subsequent operation of the product) and stresses in result of realized operating conditions after annealing.

Work purpose. The aim of the work is to develop a theoretical approach for studying of thermomechanical processes in plastically deformable products and structural elements of thermosensitive in the general case piece-homogeneous materials hardenable in the deformation process. These thermomechanical processes are caused by cooling conditions during high-temperature annealing and the simulated operational thermal and (or) static mechanical loads following after annealing. The theoretical approach includes the formulation of the corresponding problem of mathematical physics, FEM for its solving and software development. Based on this approach, as an example, investigate the stress in a thin round disk with a coaxial concentric hole under cooling conditions during annealing and under the action of a pressure of a given value applied to the boundary of the hole of the annealed disk.

Formulation of a mathematical problem. Basic regulations and relationships of the model. In this paper, the formulation of problems of mathematical physics, which take into account the possibility of plastic deformation, thermosensitivity and hardenability of materials, is carried out both to describe thermomechanical processes during cooling annealing, and under subsequent thermoforce influences as well. It is essential to estimate the residual and acquired stresses at the holding end. Presented in Figure 1, schematic redistribution of stresses during high-temperature annealing, taken from [2] and specified by experimental data on the investigated material [1], illustrates the following regularities regarding the behavior of residual stresses during annealing. The main stress relaxation occurs during the heating stage due to the decrease in the Young's modulus E and the yield limit σ_Y at higher temperatures. Holding at high temperatures causes further and less significant stress reduction. At the cooling stage at certain thermal conditions, an increase in stresses due to an increase in the Young's modulus E and yield limit σ_Y with decreasing temperature can be observed [2].

For an isotropic solid subjected to annealing heat treatment in order to relax known residual technologically acquired stresses $\{\sigma_{techno\,log\,.}\}$ (Figure 1) to an unknown desired level $\{\sigma_{anneal\,.}\}$ (Figure 1) and its subsequent operation under given thermomechanical conditions, a range of problems is formulated based on the same relationships of thermal conductivity and thermoplasticity with different initial conditions for temperature and stress distribution and different boundary cooling conditions due to convective heat transfer and the nature of the applied static mechanical forces. The cooling stage during annealing is considered, for which the initial temperature is known $T_0 = T_{hold.}$ (Figure 1), and on the basis of the known before annealing stress distribution $\{\sigma_{techno\,log\,.}\}$ and experimental and theoretical data [1] on stress reduction during heating and holding, the distribution of self-balanced stresses $\{\sigma^{(0)}\} = \{\sigma_{hold.}\}$ (Figure 1) relaxed before cooling is substantiated. The quasistaticity of deformation processes is assumed. The possibility of plastic deformation, thermal sensitivity and hardenability of the material are taken into account. The joint consideration of the listed phenomena is practically not realized in the known approaches to solving applied and theoretical problems in different domains of mechanics of deformable rigid body, including the ones in this work.

Thermomechanical fields are investigated in the initial undeformed domain Ω_0 with the boundary Γ_0 occupied by the solid in the orthogonal Cartesian coordinate system (x_1, x_2, x_3). In the following relations, the components of the tensor quantities for stresses are arranged similarly as in the vector $\{\sigma\} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}\}'$, the components of the tensor quantities for strains are arranged as in the vector $\{\varepsilon\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{12}, 2\varepsilon_{13}, 2\varepsilon_{23}\}'$, where the symbol «□» means the transposition operation.

The corresponding problems of mathematical physics both for the cooling stage during high-temperature annealing and for thermomechanical factors of influence in a product that has undergone annealing heat treatment are formulated as follows.

The problem of thermal conductivity for the desired non-stationary thermal process includes the equation of thermal conductivity, the initial condition of temperature distribution

T_0 and the boundary condition of convective heat transfer through part Γ_{02} of the surface Γ_0 [17].

For the SSS problem, the self-equilibrium of stresses $\{\hat{\sigma}^{(0)}\}$ in the initial reference undeformed state means [18] that

$$[B]'\{\hat{\sigma}^{(0)}\} = 0, \quad (1)$$

$$[n]'\{\hat{\sigma}^{(0)}\}\Big|_{\Gamma_{0\sigma}} = 0, \quad (2)$$

where $\{\hat{\sigma}^{(0)}\}$ is the first Piola-Kirchhoff initial stress vector [18], $[B]$ is matrix of differential operators of geometric relationships [19], $[n]$ is matrix of directing cosines of the external normal $\{n\}$ to the surface Γ_0, $\Gamma_{0\sigma} \subset \Gamma_0$. The problem about stresses with the volumetric forces neglected is formulated as a system of equations with respect to unknown displacements $\{u\}$, strains $\{\varepsilon\}$ and second Piola-Kirchhoff stresses $\{\sigma\}$ with corresponding boundary conditions. This system of equations includes equilibrium equation [18], geometric linear relationship [18] and state equation of plastic nonisothermal yield theory [20]

$$\{d\sigma\} = \left([D]^{t+dt} - \frac{9}{4\left(\bar{\sigma}_i^{\,t}\right)^2} \cdot \frac{[D]^{t+dt}\{\bar{s}\}^t \{\bar{s}\}^{t\,\prime} [D]^{t+dt}}{H^t + 3G^{t+dt}}\right)\left(\{d\varepsilon\} - \{d\varepsilon^T\}\right) +$$
$$+\left([dD] - \frac{9}{4\left(\bar{\sigma}_i^{\,t}\right)^2} \cdot \frac{[D]^{t+dt}\{\bar{s}\}^t \{\bar{s}\}^{t\,\prime} [dD]}{H^t + 3G^{t+dt}}\right)\left(\{\varepsilon\}^t - \{\varepsilon^p\}^t - \{\varepsilon^T\}^t\right) + \quad (3)$$
$$+ \frac{3}{2\bar{\sigma}_i^{\,t}} \cdot \frac{[D]^{t+dt}\{\bar{s}\}^t \dfrac{\partial \tilde{\tilde{\sigma}}_i^{\,t}}{\partial T}}{H^t + 3G^{t+dt}} dT .$$

In the case of static mechanical load, the boundary conditions are set on the part $\Gamma_{0\sigma} \subset \Gamma_0$, to which these forces are applied, and on the part $\Gamma_{0u} \subset \Gamma_0$, where the solid is fixed in a certain way ($\Gamma_{0u} \cup \Gamma_{0\sigma} = \Gamma_0$, $\Gamma_{0u} \cap \Gamma_{0\sigma} = \varnothing$). The state equation (3) obtained in [20] describes the behavior of a material with an isotropic-kinematic nature of hardening during plastic deformation and is based on the correspondingly modified von Mises plasticity condition [20], the specific form of which

$$\sqrt{\frac{3}{2}\{\bar{s}\}^{t\,\prime}\{\bar{s}\}^t} = \sigma_Y + \beta^* b\left(\varepsilon_i^{p\,t}\right)^m \quad \left(0 \le \beta^* \le 1\right) \quad (4)$$
$$\{\bar{s}\}^t = \{\bar{\sigma}\}^t - \{1,1,1,0,0,0\}'\,\bar{\sigma}_0^{\,t}, \ (\{\bar{\sigma}\}^t = \{\sigma\}^t - \{\gamma\}^t,\ \bar{\sigma}_0^{\,t} = 1/3\ \{1,1,1,0,0,0\}\left(\{\sigma\}^t - \{\gamma\}^t\right))$$

is obtained in the work [21] by presenting its right part as a single expression. In the relationships (3), (4), the matrix of elastic constants and the matrix of increments of elastic constants due to changes in temperature are values correspondingly indicated $[D]$ and $[dD]$; $\{\sigma\}$ is second Piola-Kirchhoff stress vector; $\{\varepsilon\}$, $\{\varepsilon^p\}$, $\{\varepsilon^T\}$ are vectors of total, plastic and temperature strains respectively; H is instantaneous value of the tangent of slope angle of isothermal deformation curve «stress intensity σ_i – strain intensity ε_i»; G is shear modulus of elasticity; $\bar{\sigma}_i$ is intensity of stresses related to the center $\{\gamma\}$ of the yield surface in the stress space; ε_i^p is intensity of plastic strains; $\tilde{\tilde{\sigma}}_i$ is intensity of Cauchy stresses; β^*, b, m are parameters of isotropic expansion of the yield surface. The upper indices «t» and «$t+dt$» refer to the values at the deformation time t and $t+dt$ respectively. With the parameters β^* and $\{\gamma\}$ different model approximations of the hardening properties of materials are described, namely: $\{\gamma\}\neq 0$ ($t>0$), $0<\beta^*\leq 1$ for isotropic-kinematic hardening; $\{\gamma\}=0$, $0<\beta^*\leq 1$ for isotropic hardening; $\{\gamma\}\neq 0$ ($t>0$), $\beta^*=0$ for kinematic hardening; $\{\gamma\}=0$, $\beta^*=0$ for ideal material.

The removing of the yield surface center $\{\gamma\}$ occurs according to the Ziegler rule [22]

$$\{d\gamma\} = d\mu\left(\{\sigma\}^t - \{\gamma\}^t\right) \tag{5}$$

with the multiplier $d\mu$ in the explicit form obtained in [21] in accordance with the explicit representation (4) of the yield function. This explicit form is obtained on the basis of implicit functional dependence from the work [23].

A compact form of the equation of state (3)

$$\{d\sigma\} = \left[D^{ep}\right]\left(\{d\varepsilon\} - \{d\varepsilon^T\}\right) + \left[\Delta D^{ep}\right]\left(\{\varepsilon\}^t - \{\varepsilon^p\}^t - \{\varepsilon^T\}^t\right) + \{Q\}dT \tag{6}$$

is convenient for designing FEM calculation schemes.

In the case where the cooling process at high temperature annealing is considered, the initial temperature is the holding temperature (Figure 1)

$$T_0(\{x\}) = T_{hold.}, \tag{7}$$

and the initial stresses are residual stresses $\{\sigma^{(0)}\} = \{\sigma_{hold.}\}$ (Figure 1) known at the end of the holding. The desired result of solving the above problem of thermomechanics for the cooling stage during annealing is a non-stationary thermal process $T = T_{cool.}(\{x\}, t)$ and the time-varying stress distribution $\{\sigma\} = \{\sigma_{cool.}\}$ caused by it. The obtained residual stress state at the end of annealing $\{\sigma_{anneal.}\}$ (Figure 1) is also the initial state for the problem of determining stresses under the influence of thermomechanical conditions that simulate the operating conditions of annealed products.

For the problem of the required stress state after annealing of the product, the influence of either temperature loads, or static mechanical forces, or the combined action of thermal and mechanical factors are assumed. With possible subsequent thermal effects after annealing, the non-stationary temperature distribution is the solution of the thermal conductivity problem [17]. In this paper, we consider cooling due to convective heat transfer through a certain part of the surface Γ_0 from the known initial temperature $T(\{x\},t)\big|_{t=t_0} = T_0(\{x\},t)$, where this temperature in the general case is inhomogeneously distributed in the domain Ω_0. In the case of mechanical effects on the product with pre-relaxed to the level $\{\sigma_{anneal.}\}$ (Figure 1) stresses as a result of annealing, the value and nature of the distribution of applied forces as well as the conditions of fixation are known. The problem of the stress state in the product after annealing is formulated as the problem of yield theory based on the equation of state (3) [20] or its compact representation (6). In the case of only the thermal load influence known from the solution of the thermal conductivity problem, zero boundary conditions for stresses on the part of the surface $\Gamma_{0\sigma} \subset \Gamma_0$ and conditions of fixing

$$\{u\}\big|_{\Gamma_{0u}} = \{u^*\} \tag{8}$$

with the given displacements distributions $\{u^*\}$ on the part of the surface $\Gamma_{0u} \subset \Gamma_0$ are set. At mechanical loading boundary condition (8) and boundary condition

$$[n]'\left(\{\hat{\sigma}^{(0)}\} + \{\hat{\sigma}\}\right)\big|_{\Gamma_{0\sigma}} = \{P_n\} \tag{9}$$

on the part of the surface $\Gamma_{0\sigma} \subset \Gamma_0$ are set, where P_n is quantity of applied pressure. Under the combined influence of temperature and mechanical forces on the stress state formation, the conditions (8), (9) will also be the boundary conditions.

FEM computational approach to solving the problem. Let us name the main difficulties which are fundamental and practically do not allow to solve the formulated problems in analytical form. Solids can have boundaries of non-canonical shape. The nature of known initial residual stresses distribution may not be described by exact functional dependences. The temperature dependence of the thermomechanical properties of the material can be given in tables, in particular obtained as a result of experiments, and also cannot be described functionally. Non-stationary thermal process in the non-canonical domains considered in the general case as a factor influencing the stress state, is practically impossible to obtain in analytical form. Static mechanical forces can have arbitrary distribution on the part of the domain boundary. These and other difficulties are not essential and paramount in the approximate FEM solving of the formulated problems. FEM calculation algorithms are based on equivalent variational formulations for the thermal conductivity problem [19] and the SSS problem [18]. Obtained in [21] the FEM equation with respect to displacement increments is based on equation of state (3) (or (6)) of the plastic nonisothermal yield theory [20] and on the equivalent variational formulation of the problem, the basic relations of which are relations (1)–(5). In this case, the expression of the principle of virtual work [18], which is the basis of the variational formulation of the SSS problem, does not depend on the initial self-balanced stresses in the case of geometrically linear theories [18], and for the used variant of geometrically linear theory it is:

$$\iiint_{\Omega_0} \{\delta\Delta u\}_N'[B]'\left[D^{ep}\right]_{N+1}[B]\{\Delta u\}_N\, d\Omega =$$
$$= \iint_{\Gamma_{0\sigma}} \{\delta\Delta u\}_N'\{\Delta P_n\}_N\, d\Gamma + \iiint_{\Omega_0} \{\delta\Delta u\}_N'[B]'\left[D^{ep}\right]_{N+1}\left\{\Delta\varepsilon^T\right\}_{N+1} d\Omega -$$
$$- \iiint_{\Omega_0} \{\delta\Delta u\}_N'[B]'\left[\Delta D^{ep}\right]_N\left(\{\varepsilon\}_N - \left\{\varepsilon^P\right\}_N - \left\{\varepsilon^T\right\}_N\right) d\Omega - \tag{10}$$
$$- \iiint_{\Omega_0} \{\delta\Delta u\}_N'[B]'\{Q\}_{N+1}(\Delta T)_N\, d\Omega + \{\varphi\}_{N+1}.$$

Equation (10) is solved at each step $\Delta t_N = [t_N, t_{N+1}]$ ($N = 0,1,2,......, N^* - 1$, N^* is number of steps) of tracking the deformation process and within each step on every j - iteration of linearizing iterative process [21]. In this equation, matrix $[\Delta D]^{ep}$ and vector $\{Q\}$ are dependent on the temperature change in the elastic constants of the material and yield limit σ_Y respectively [20], $\{\varphi\}_{N+1}$ is residual of the variational equation of the virtual work principle, accumulated by the end of the load step Δt_N.

Computational aspects of the developed software, obtaining linearizing iterative algorithms, step-by-step approximations for the relations of the basic SSS problem and for the equivalent variational one are presented in the works [10–12, 21].

The proposed approach to estimating the evolution of residual stresses in the annealing process at the cooling stage and known loading conditions after annealing based on the formulated problem of thermomechanics and the approximate method of its solution using the developed software enables solving a number of specific two-dimensional problems of mathematical physics.

Stress state of a thin round disk with a coaxial concentric hole under cooling conditions during annealing and in the conditions of force loading after annealing. As an example, SSS in a thin round disk with a coaxial concentric hole of radius R_1 and with outer radius R_2 and heat-insulated front surfaces is studied. The distribution in radial direction of the residual before annealing radial $\sigma_r^{(A)}$ and circular $\sigma_\theta^{(A)}$ stresses is given in Figure 2 [1] ($R_1 = 50$ mm, $R_2 = 100$ mm). This stress distribution is characteristic for the disk obtained by welding a circular butt weld of two round disks along the arc with the radius $r = \dfrac{R_1 + R_2}{2}$ [1]. The holding temperature (7) is initial one for the cooling stage. Stress $\{\sigma\}^{(0)} = \{\sigma_{hold.}\} = \left\{\sigma_r^{(B)}, \sigma_\theta^{(B)}\right\}$ (Figure 2) before the cooling start is the result of preliminary relaxation during the stages of heating and holding (Figure 1).

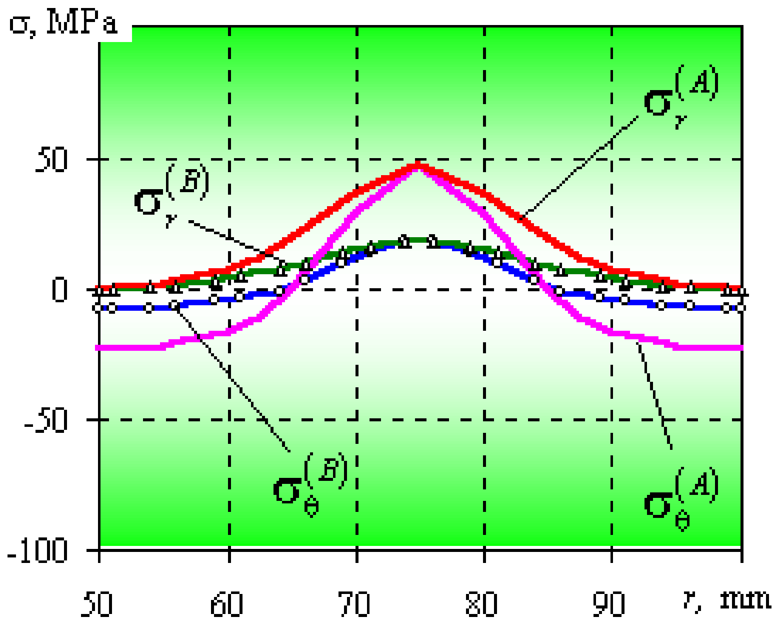


Figure 2. Stress $\{\sigma^{(0)}\}^{*} = \{\sigma_{techno\log.}\} = \{\sigma_r{}^{(A)}, \sigma_\theta{}^{(A)}\}$ distribution in disk before annealing start and stress $\{\sigma^{(0)}\} = \{\sigma_r{}^{(B)}, \sigma_\theta{}^{(B)}\}$ distribution before cooling start in annealing process

The phenomena of creep and phase transformations at the holding stage during annealing, complicate the theoretical prediction of stresses at the end of holding. In this paper, the estimation of the stress $\{\sigma_{hold.}\} = \{\sigma_r{}^{(B)}, \sigma_\theta{}^{(B)}\}$ distribution at the end of holding is based on the following experimental observations and theoretical facts. The intensity of the maximum stresses does not exceed a value approximately equal to the yield limit σ_Y at the appropriate temperatures [1] under the assumption of an ideal elastic-plastic material. There is [1] a decrease of 2.5–3 times the intensity of the maximum stresses at the end of holding comparatively to the residual stresses before annealing. There is also a similarity of uniaxial and biaxial distributions of residual stresses before annealing and after holding [1]. The known temperature distribution $T_0 = T_{hold.}$ (7) and the stress distribution $\{\sigma\}^{(0)} = \{\sigma_{hold.}\}$ estimated in this way are the initial state for the formulation and solving of the problem of thermal conductivity during cooling and the problem of temperature stresses caused by cooling, respectively.

The desired final residual stress state at the end of annealing $\{\sigma_{anneal.}\}$ (Figure 1) is the initial stress state for the subsequent study of the redistribution of these residual stresses in the disk, which is subjected to the action of static load P_n on the hole boundary.

Due to the symmetry of the geometric configuration and load conditions, the calculations were performed in the domain Ω_0 which is the fourth part of the horizontal section of the disk (Figure 3).

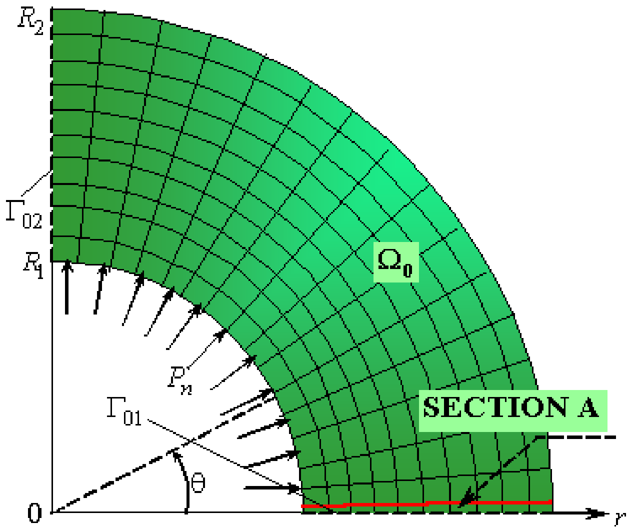


Figure 3. Calculated domain $\varOmega_0$, its finite element discretization scheme of the mechanical loading of disk after annealing

The considered thin round disk with a concentric hole at the initial homogenous distribution of the holding temperature $T_0 = T_{hold.} = 600^\circ C$ is cooled through the hole boundary $r = R_1$ and the outer boundary $r = R_2$ of the disk due to convective heat transfer with a heat transfer coefficient of $\beta = 5{,}6$ W/(m^2·K) (air cooling). At thermal insulation of front surfaces and absence of heat flows through boundaries $\varGamma_{01}$ and $\varGamma_{02}$ of domain $\varOmega_0$ (Figure 3) boundary conditions of a problem of thermal conductivity are:

$$-\lambda_q \left.\frac{\partial T}{\partial r}\right|_{r=R_1} = -\beta\left(T - T_A\right),\; -\lambda_q \left.\frac{\partial T}{\partial r}\right|_{r=R_2} = \beta\left(T - T_A\right),$$
$$\left.\frac{\partial T}{\partial \theta}\right|_{\theta=0} = \left.\frac{\partial T}{\partial \theta}\right|_{\theta=\frac{\pi}{2}} = 0, \tag{11}$$

where λ_q is coefficient of thermal conductivity of the material, T_A is ambient temperature.

Temperature stresses caused by a non-stationary thermal cooling process are the desired solution of the problem based on equation of state (3) (or (6)). When the sections $\varGamma_{01}$ and $\varGamma_{02}$ (Figure 3) are not displaceable in the circular directions, the boundary conditions on the sections $\varGamma_{01}$, $\varGamma_{02}$ for the problem of determining the SSS are:

$$u_\theta \Big|_{\theta=0} = u_\theta \Big|_{\theta=\frac{\pi}{2}} = 0, \tag{12}$$

and boundary conditions, when $r = R_1$ and $r = R_2$ are

$$\sigma_r\Big|_{r=R_1} = \sigma_r\Big|_{r=R_2} = 0\,, \tag{13}$$

which means the absence of mechanical loads on the contour of the hole and the outer boundary of the disk at the stage of cooling during annealing.

Therefore, for the cooling stage during annealing, the problem of thermal conductivity is solved with the initial condition (7) and boundary conditions (11). The initial condition for the SSS task for the cooling stage is the condition $\{\sigma\}^{(0)} = \{\sigma_r^{(B)}, \sigma_\theta^{(B)}\}$ (Figure 2), the boundary conditions are (12), (13).

The final residual stress state $\{\sigma_{anneal.}\}$ obtained at the end of annealing is the initial stress state for the subsequent investigation of stresses in the annealed disk under the influence of static load $P_n = 60$ MPa. In this case the boundary conditions of the problem will be the conditions (12) and conditions

$$\sigma_r\Big|_{r=R_1} = P_n,\ \sigma_r\Big|_{r=R_2} = 0\,. \tag{14}$$

The calculations are performed for a disk with dimensions $R_1 = 50$ mm, $R_2 = 100$ mm and made of steel 0X13. The thermal sensitivity of the properties of steel in the considered temperature range is indicated by the reference data of Table 1 [24] and a linear decrease in the yield limit σ_Y from 96 MPa at $20°C$ [20] to 1 MPa at $1000°C$. The specific volumetric heat capacity $C = 3592{,}44$ kJ/(m^3·K) and the coefficient of linear thermal expansion $\alpha_T = 11{,}0\cdot10^{-6}$·K [24] are constant. In Table 1 ν is Poisson's ratio. The values of the isotropic hardening parameters included in the right part of the yield condition (4), are $\beta^* = 0{,}515$; $b = 2208$ MPa, $m = 0{,}435$ [20].

Table 1

Temperature dependence of thermomechanical properties of 0X13 steel [24]

T, °C	λ_q, W/(m·K)	T, °C	E, GPa	ν
20	26,7	20	224,78	0,268
100	27,7	100	224,78	0,268
200	27,7	149	211,68	0,268
300	28,0	260	204,09	0,268
400	27,7	427	190,99	0,272
500	27,2	482	184,77	0,276
600	26,4	538	177,20	0,282

Solving of the problems of both annealing and mechanical load after annealing was performed for a disk sampled by a grid of 150 finite elements shown in Figure 3.

Cooling stage at high temperature annealing. The non-stationary process of thermal cooling to almost homogeneous temperature $T \approx 154^\circ C$ obtained at the initial time step $(\Delta\ t)_0 = 1$ sec and the subsequent agglomeration of time steps, according to calculations continues for the time $t_{cool.} = 20000$ sec $\approx 5{,}5$ hours. The total number of time steps in solving the thermal conductivity problem is $N_T = 4801$. Information about the obtained thermal regime in the form of appropriate temperature distributions for discrete time moments is the input information for the problem of temperature stresses. In the calculation analysis of SSS to provide higher speed of calculations as well as to ensure the accuracy, the agglomeration of time intervals has been maintained. The calculation of temperature stresses was obtained during $N_{SSS} = 33$ steps. It is established that under these cooling conditions there occurs elastic deformation of the material, which does not cause undesirable redistribution of stresses at the stage of cooling during annealing.

Mechanical load after annealing. The following studies are related to the estimation of the stress state that take place under the influence of a pressure of $P = 60$ MPa applied to the hole boundary. Graphs of stress σ_r, σ_θ distributions and stress intensity σ_i distribution are given below for the section A (Figure 3) that crosses the points of finite elements integration in the radial direction. These distributions coincide with the distributions of the corresponding stresses in other radial sections of the disk. To compare, in Figure 4, distributions of radial σ_r and circular σ_θ stresses as well as stress intensity σ_i in the disk free from residual stresses are obtained and presented. The calculations are based on the model of isotropic-kinematic hardening of the material.

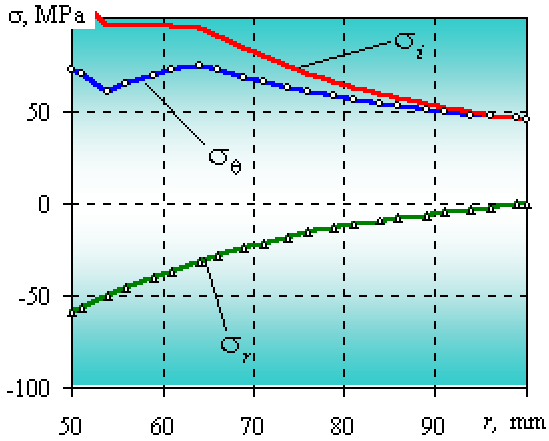


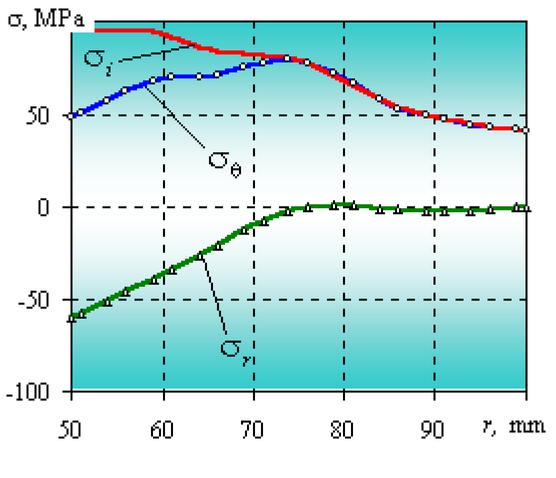
Figure 4. Stress distribution in the disk from residual stresses free under pressure $P = 60$ MPa applied to hole boundary $r = R_1 = 50$ mm (isotropic-kinematic hardening)

The results presented in Figure 5 are found for an ideal material and three variants for its hardening and illustrate the effect of this pressure on the disk with known pre-obtained for the end of annealing residual stresses.

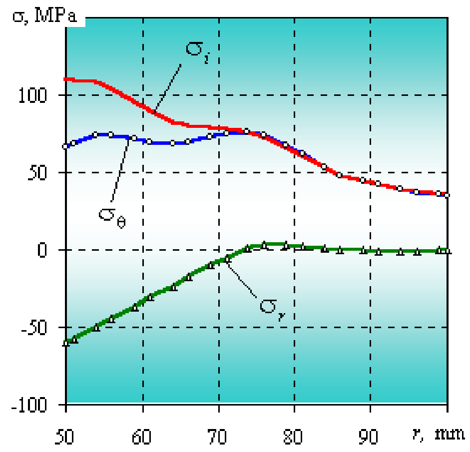
According to the analysis of the results obtained on the basis of the model of isotropic-kinematic hardening of the material, significant differences in the nature of stress distribution and their magnitude for the disk in the initial unstressed state (Figure 4) and in the disk with residual stresses after annealing (Figure 5 *b*) are established. In addition, in the disk with the previously known stresses after annealing, the pressure applied to the hole causes stresses which are significantly different for different variants of hardening the material (Figure 5). Therewith, the calculated stress state for an ideal material and kinematically hardened one almost coincide (Figure 5 *a*).

Consideration of isotropic-kinematic hardening is substantiated [20] by good consistency of the experimental deformation curve of this material and its approximation using the above values of the parameters β^*, b, m, γ of isotropic-kinematic hardening, which are included in the criterion of plasticity (4). Therefore, the calculated estimation of the corresponding stress distributions (Figure 5 *b*) is physically more reasonable. Calculations within the model of isotropic hardening of the material (Figure 5 *c*) give overrated values of stresses relative to the results of Figure 5 *b* with a similar nature of their distribution.

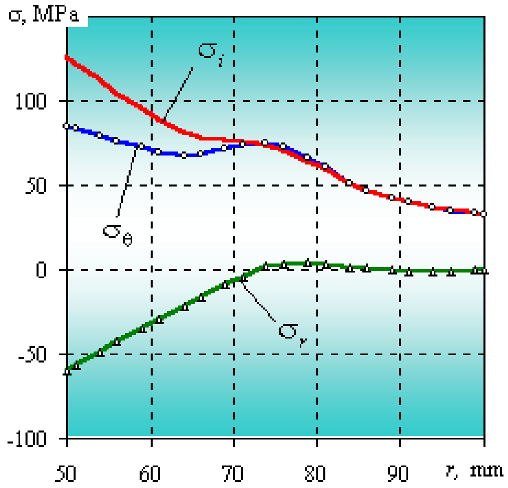
Conclusions. Thus, a sequence of problems for estimating thermal and stress states in products subject to heat treatment by annealing in order to relax technologically undesirable residual stresses and the subsequent after annealing effect of thermomechanical conditions



a)



b)



c)

Figure 5. Stress distribution in the disk with residual after annealing stresses under pressure $P =$ 60 MPa applied to hole boundary $r = 50$ mm:
a) ideal and kinematic hardenable material;
b) isotropic-kinematic hardening;
c) isotropic hardenable material

are formulated. For each of these considered stages with different thermomechanical factors of influence on the product, the problem of thermal conductivity for non-stationary thermal process in thermosensitive solids and the problem of stress state for solids from plastically deformable thermosensitive hardenable materials are formulated. FEM software is generalized for solving mechanics problems within the geometrically linear theory variant for solids with initial stresses. The ability of this software package to work with initial stresses is an important difference from the known FEM software systems. The proposed problem statement and developed software in connection with the complex consideration of physically observed phenomena and initial stresses allow to investigate the thermomechanical state of products caused by the above factors and insufficiently predicted in the theoretical and experimental approaches to solving a number of scientific and applied thermoplasticity problems. The developed software is focused on solving two-dimensional problems. Based on the calculated results for the disk in the conditions of cooling during annealing, and the influence of pressure of a given value applied to the hole after annealing, the patterns of relationship in redistribution of residual stresses in the considered sequentially implemented conditions are established for the ideal material model and three variants for its hardening.

The following studies may focus on modeling thermal and mechanical states caused by the technological operation of quenching products, annealing operation of quenched products and the subsequent influence of mechanical factors and (or) in the general case of non-stationary thermal regimes in quenched and annealed products.

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УДК 539.3

ДОСЛІДЖЕННЯ ТЕРМОМЕХАНІЧНИХ ПРОЦЕСІВ У ВИРОБАХ ПРИ ВІДПАЛІ ТА В ЕКСПЛУАТАЦІЙНИХ УМОВАХ ПІСЛЯ ВІДПАЛУ: ТЕОРІЯ, РОЗРАХУНКИ

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Резюме. *Дана робота орієнтована на послідовне вирішення двох проблем: а) оцінювання перерозподілу напружень у результаті охолодження при високотемпературному відпалі, метою якого є релаксація технологічно набутих залишкових напружень; б) оцінювання напружень, спричинених експлуатаційними умовами у відпалених виробах та конструктивних вузлах. Запропоновано підхід до прогнозування напружень, викликаних відповідними технологічними умовами відпалювання та експлуатаційними тепловими і (або) механічними умовами після відпалювання. Теоретичною основою цього підходу є теорія нестаціонарної теплопровідності й теорія пластичного неізотермічного течіння. Розв'язування сформульованих задач математичної фізики базується на методі скінченних елементів (FEM) та розробленому програмному забезпеченні. Програмне забезпечення орієнтоване на розв'язування двовимірних задач і дозволяє досліджувати еволюцію термомеханічних станів в однорідних і кусково-однорідних, у загальному випадку термочутливих зміцнюваних у процесі деформування ізотропних тілах та елементах конструкцій канонічної й неканонічної форми під впливом теплових та (або) механічних факторів. Специфікою цих задач є наявність початкових напружень. Відомі програмні системи FEM не працюють з початковими напруженнями й тому не дозволяють розв'язувати окреслене коло задач. Важливість досліджень у цьому напрямку пов'язана з обмеженістю теоретичних підходів і, як наслідок, недостатньо вивченими термомеханічними явищами. Розроблене програмне забезпечення алгоритмічно узагальнено на випадок існування початкових напружень, якщо деформування розглядуваних об'єктів описується геометрично лінійною теорією.*

Як приклад, проаналізовано перерозподіл у процесі відпалювання попередньо набутих зварних залишкових напружень у тонкому диску з коаксіальним коловим отвором та наступний вплив прикладеного до отвору тиску. Обґрунтовано характер розподілу та рівень напружень у диску на стадії охолодження при відпалюванні після проходження стадій нагрівання та витримування. Ці напруження є початковими для вивчення механічних процесів, спричинених наступним впливом прикладеного до отвору статичного навантаження. Встановлено закономірності формування напружень у розглядуваних послідовно реалізованих умовах для ідеального матеріалу й трьох модельних наближень його зміцнюючих властивостей.

Ключові слова: *відпалювання, охолодження, термомеханічні умови, залишкові напруження, релаксація, пластичне деформування, термочутливість, зміцнення, метод скінченних елементів, диск.*

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