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INCOMPLETE CONTACT OF THE ORTHOTROPIC PLATE WITH THE ELLIPTICAL HOLE AND THE CLOSED ELASTIC RIB

Andriy Syasky[1]; Oleg Dejneka[2]; Natalia Shevtsova[3]

[1]Rivne State University of Humanities, Rivne, Ukraine
[2]National University of Water and Environmental Engineering, Rivne, Ukraine
[3]Separated structural subdivision «Rivne college of national university of life and environmental sciences of Ukraine», Rivne, Ukraine

Summary. *In the conditions of a generalized flat stress state created by uniformly distributed tensile forces at infinity, the mixed contact problem for an infinite orthotropic plate with an elliptical hole, the contour of which is strengthened by a closed elastic rib, in the presence two symmetrical interphase incisions with edges which are not in contact during deformation, is considered. The deformation tensor components (relative elongation and normal angle of rotation) at the points of the contour of the plate hole are represented by integral dependences on the contact forces. By modeling strengthening of a closed elastic rod of a constant rectangular cross section and using the basic equations of the linear theory of curvilinear rods, which takes into account the deformations of the transverse shear, the mathematical model of the problem is constructed as a system of singular integral-differential equations with Hilbert cores to find the contact forces between the plate and the rib and the internal forces and moments in the reinforcemen. To determine the initial parameters of a statically indeterminate rod, the conditions of unambiguity of displacements and angles of rotation at points of its axis were used. The structure of the required functions in the areas of interconnection of the plate and the reinforcing rib is established. The approximate solution of the problem is constructed by the combined method of me chanical quadratures and collocations, which investigates the influence of the magnitude of the interphase cuts and the shape of the hole in the plate on the stress state of the plate and the reinforcing rib. It is established that in the vicinity of the ends of the interphase sections, the normal stresses in the longitudinal fibers of the rib have first-order jumps, while remaining limited, and the contact and annular efforts in the plate take unlimited values.*

Key words*: interfacial incisions, orthotropic plate, reinforcing rib, contact forces, singular integral-differential equations.*

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Introduction. Composite plates with curved holes are widely used in the design of modern structures, machines, and constructions; closed elastic ribs of constant or variable cross-section reinforce the contours of curved holes. The stress-strain state of such plates significantly depends on the model of the reinforcing rib.

Nowadays, the model of a closed curved rod of a stable rectangular cross section, whose middle surface does not coincide with the neutral for pure bending the surface of the rib and the surface of the hole in the plate, is the most widely used [1–4].

Based on this model, in [5, 6], a number of problems concerning the contact interaction of an orthotropic plate with an elliptical hole and a closed elastic rib, when they are fully connected by the method of press landing with guaranteed tension or welding, are solved.

During manufacture or operation of plates with a reinforced curvilinear contour at the interface of materials, defects can occur in the form of interfacial cuts (cracks), which will could cause a high concentration of stresses and subsequent interfacial failure under the action of force.

The case of one symmetrical interfacial section, the edges of which do not contact during deformation, between an isotropic (orthotropic) plate with a curved (elliptical) hole and a closed

Corresponding author: Oleg Dejneka; e-mail: dejneka_7@ukr.net

elastic rib, is considered in [7–9]. Based on the method of mechanical squaring and collocation for different loads, the effect on the stress state of the plate structure of the hole is studied, as well as the anisotropy of the plate material, the value of the interphase section and the physical – geometrical parameters of the rib. Providing the presence of two interphase sections between an isotropic plate with a curved hole and a closed elastic rib, a similar problem is considered in paper [10].

A numerical-analytical method for solving a mixed contact problem for an infinite orthotropic plate with a reinforced elliptical contour in the conditions of a generalized planar stress state is proposed, providing the presence of two symmetric sections at the material boundary, the edges of which do not contact during deformation.

Statement of the problem. Assumedly, an infinite orthotropic plate of thickness $2h$ is weakened by an elliptical hole, the contour of which Γ is reinforced by a closed elastic rib of constant cross section. The plate structure is in the conditions of the generalized plane stress state created by evenly distributed at infinity forces p and q, which act in the directions of ellipse axes.

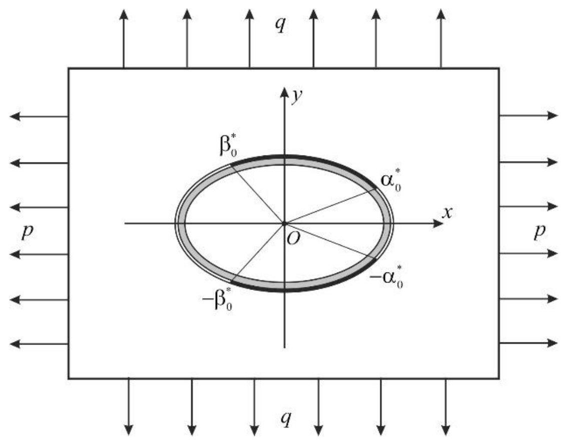

Figure 1. Computational scheme of the plate

The common median plane of the plate and the rib is assigned to a system of rectangular $(x,\, y)$ and polar $(r,\, \delta)$ coordinates with a pole in the centre of the hole. The reference systems are chosen so that the coordinate axes coincide with the axes of symmetry of the ellipse and determine the main directions of orthotropy of the plate material (Fig. 1).

Assumedly, during manufacture or operation of the structure, on the line of the plate and the rib connection outside the sections $[-\beta_0^*,\, -\alpha_0^*]$, $[\alpha_0^*,\, \beta_0^*]$ ($\alpha_0^*,\, \beta_0^*$ – polar angles), two symmetrical relatively the axis Ox interphase sections occur, the edges of which do not contact during deformation.

Objective of the paper. The objective of the paper is to determine the components of the stress state on the contour Γ in the plate and the reinforcing rib and to find their dependence on the magnitude of the interfacial incisions, orthotropy of the plate material, and the type of external load.

Basic equations of the problem. The structure under study is considered as to be conditionally divided into separate elements (an infinite orthotropic plate with an elliptical hole and a closed resilient rib), replacing the action of one body on another by unknown contact forces.

The infinite isotropic plate is in equilibrium under the action of the load at infinity and the contact forces T_ρ, $S_{\rho\lambda}$, which in the areas $[-\beta_0^*,\, -\alpha_0^*]$, $[\alpha_0^*,\, \beta_0^*]$ are transmitted to the contour Γ from the reinforcing rib.

Based on the function [11], a conformal mapping of the appearance of a unit circle γ in a plane $\xi = \tilde{\rho} e^{i\lambda}$ onto the region occupied by the median plane of the plate is considered

$$x + iy = z = \omega(\xi) = R_0 \left(\xi + \frac{\varepsilon}{\xi} \right) \quad (1)$$

if $R_0 = \dfrac{a+b}{2} = 1$; $\varepsilon = \dfrac{a-b}{a+b}$ – eccentricity of the ellipse; a, b – its half-axes; $a = 1+\varepsilon$; $b = 1-\varepsilon$.

Deformations of the contour Γ in an infinite orthotropic plate at a given load are determined by the formulas [8, 9], which with consideration of the symmetry of the stress state relative to the axis Ox can be converted into a formula

$$\varepsilon_\lambda = \frac{1}{2E_x h(\alpha^2+\beta^2)}\Bigg[c_1(\alpha^2+\beta^2)T_\rho(\lambda) + \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[\tilde{\Phi}_1(\lambda,\ t)T_\rho(t) + \tilde{\Phi}_2(\lambda,\ t)S_{\rho\lambda}(t)\right]dt + \alpha(\lambda)\tilde{\varepsilon}_\lambda^0 + \beta(\lambda)\tilde{V}^0\Bigg];$$

$$\tilde{V} = \frac{1}{2E_x h(\alpha^2+\beta^2)}\Bigg[c_1(\alpha^2+\beta^2)S_{\rho\lambda}(\lambda) + \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[\tilde{\Phi}_3(\lambda,\ t)S_{\rho\lambda}(t) + \tilde{\Phi}_4(\lambda,\ t)T_\rho(t)\right]dt + \alpha(\lambda)\tilde{V}^0 - \beta(\lambda)\tilde{\varepsilon}_\lambda^0\Bigg],\ \lambda\in[0,\ \pi], \tag{2}$$

where the notations are entered

$$\tilde{\Phi}_1(\lambda,\ t) = G_1(\lambda,t) - H_1(\lambda,t) - G_2(\lambda,t)\operatorname{ctg}\frac{\lambda+t}{2} + H_2(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2};$$

$$\tilde{\Phi}_2(\lambda,\ t) = G_2(\lambda,t) + H_2(\lambda,t) + G_1(\lambda,t)\operatorname{ctg}\frac{\lambda+t}{2} + H_1(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2};$$

$$\tilde{\Phi}_3(\lambda,\ t) = G_3(\lambda,t) - H_4(\lambda,t) - G_4(\lambda,t)\operatorname{ctg}\frac{\lambda+t}{2} + H_3(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2};$$

$$\tilde{\Phi}_4(\lambda,\ t) = G_4(\lambda,t) + H_3(\lambda,t) + G_3(\lambda,t)\operatorname{ctg}\frac{\lambda+t}{2} + H_4(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2};$$

$$H_1(\lambda,t) = -c_2\alpha(\lambda)\alpha(t) - c_4\beta(\lambda)\beta(t) + c_3(\alpha(\lambda)\beta(t) + \beta(\lambda)\alpha(t)); \tag{3}$$

$$H_2(\lambda,t) = -c_2\alpha(\lambda)\beta(t) + c_4\beta(\lambda)\alpha(t) - c_3(\alpha(\lambda)\alpha(t) - \beta(\lambda)\beta(t));$$

$$H_3(\lambda,t) = c_2\beta(\lambda)\beta(t) + c_4\alpha(\lambda)\alpha(t) + c_3(\alpha(\lambda)\beta(t) + \beta(\lambda)\alpha(t));$$

$$H_4(\lambda,t) = c_2\beta(\lambda)\alpha(t) - c_4\alpha(\lambda)\beta(t) + c_3(\alpha(\lambda)\alpha(t) - \beta(\lambda)\beta(t));$$

$$G_1(\lambda,t) = c_2\alpha(\lambda)\alpha(t) - c_4\beta(\lambda)\beta(t) + c_3(\alpha(\lambda)\beta(t) - \beta(\lambda)\alpha(t));$$

$$G_2(\lambda,t) = -c_2\alpha(\lambda)\beta(t) - c_4\beta(\lambda)\alpha(t) + c_3(\alpha(\lambda)\alpha(t) + \beta(\lambda)\beta(t));$$

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$$G_4(\lambda, t) = c_2\beta(\lambda)\beta(t) - c_4\alpha(\lambda)\alpha(t) + c_3(\alpha(\lambda)\beta(t) - \beta(\lambda)\alpha(t)).$$

$$c_1 = \beta_1\beta_2 - \nu_x;\ c_2 = \frac{\beta_1+\beta_2}{2}\left[(1-\beta_1\beta_2)\cos^2\lambda - 1\right];$$

$$c_3 = \frac{\beta_1+\beta_2}{2}(1-\beta_1\beta_2)\sin\lambda\cos\lambda;\ c_4 = \frac{\beta_1+\beta_2}{2}\left[(1-\beta_1\beta_2)\sin^2\lambda - 1\right];$$

$$\tilde{\varepsilon}_\lambda^0 + i\tilde{V}^0 = \frac{p}{2}\left[a + b(\beta_1+\beta_2-\beta_1\beta_2) - (a + b(\beta_1+\beta_2+\beta_1\beta_2))e^{-2i\lambda}\right] +$$

$$+\frac{q}{2}\beta_1\beta_2\left[a(\beta_1+\beta_2-1) + b\beta_1\beta_2 + (a(1+\beta_1+\beta_2) + b\beta_1\beta_2)e^{-2i\lambda}\right];$$

$E_x,\ \nu_x$ – Young's modulus and the Poisson's ratio of the plate material in the axial direction; $\beta_1,\ \beta_2$ – roots of the characteristic equation [11].

The reinforcing rib is simulated by a closed curved rod of constant rectangular cross section $2h_0 \times 2\eta$, which is deformed by contact forces $T_\rho,\ S_{\rho\lambda}$ transmitted to its outer side surface from the plate.

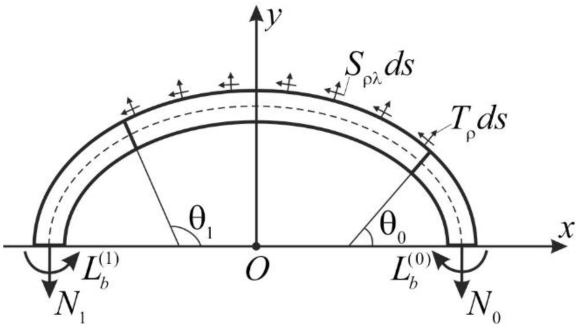

Figure 2. Computational scheme of the reinforcing rib

The lower part of the rib is conditionally discarded, its action is replaced with longitudinal forces $N_0,\ N_1$ and bending moments $L_b^{(0)},\ L_b^{(1)}$ accordingly applied to the ends $\lambda = 0$ and $\lambda = \pi$. As a result, an open statically defined rod deformed under the action of contact forces on the areas $[\alpha_0^*,\ \beta_0^*]$ $([\theta_0;\ \theta_1])$ and the load on the ends is obtained (Fig. 2).

Based on [3, 4], its stress-strain state is determined by equations of the one-dimensional linear theory of curved rods, which take into account the hypothesis of flat sections and the deformation of the transverse shear:

– differential equations of equilibrium of the rod element

$$T_\rho(\lambda) = \frac{N(\lambda)}{\rho} - \frac{dQ(\lambda)}{ds};\ S_{\rho\lambda}(\lambda) = -\frac{Q(\lambda)}{\rho} - \frac{dN(\lambda)}{ds};\ \frac{dL_b(\lambda)}{ds} = \eta\frac{dN(\lambda)}{ds} + Q(\lambda); \qquad (4)$$

– physical dependences for the outer longitudinal fiber of the rod in contact with the plate

$$\varepsilon_{\lambda}^{(c)} = \frac{1}{E_0 F_0}\left[N(\lambda) + \frac{\eta+\eta_c}{\rho\eta_c} L_b(\lambda)\right];$$
$$\frac{d\theta_b}{d\theta} = \frac{1}{E_0 F_0}\left[N(\lambda) + \frac{L_b(\lambda)}{\eta_c} - 2(1+\nu_0)\mu\frac{dQ(\lambda)}{d\theta}\right]; \qquad (5)$$

– conditions of unambiguity of the rotation angle of the normal and horizontal displacement at the points of the rib axis

$$\int_0^{\pi}\left[N(\lambda) + \frac{L_b(\lambda)}{\eta_c}\right]d\theta = 0 ;$$

$$\int_0^{\pi}\left[N(\lambda)(x-\rho\cos\theta) + L_b(\lambda)\frac{x-(\eta+\eta_c)\cos\theta}{\eta_c} + 2(1+\nu_0)\mu\frac{dQ(\lambda)}{d\theta}\right]d\theta = 0 , \qquad (6)$$

and the equilibrium conditions of the open rod shown in Fig. 2

$$\int_{\theta_0}^{\theta_1}\left[T_{\rho}(\lambda)(y\cos\theta - x\sin\theta) - S_{\rho\lambda}(\lambda)(y\sin\theta + x\cos\theta)\right]ds = \qquad (7)$$

$$= L_b^{(0)} - L_b^{(1)} + (x(0)-\eta)N_0 + (x(\pi)+\eta)N_1 . ; \quad \int_{\theta_0}^{\theta_1}\left(T_{\rho}(\lambda)\sin\theta + S_{\rho\lambda}(\lambda)\cos\theta\right)ds = N_0 + N_1$$

In relations (4)–(7) the notations are introduced: $N,\ Q,\ L_b$ – longitudinal and transverse forces and bending moment, which occur in the cross sections of the rod and are related to its axis; $2h_0,\ 2\eta$ – height and width of the reinforcing rib; $E_0,\ \nu_0$ – Young's modulus and Poisson's ratio of the rib material; E_0F_0 – tensile strength of the rod (compression); $F_0 = 2h_0\cdot 2\eta$ – cross-sectional area; η_c – the distance from the axis of the rib to the neutral longitudinal fiber for pure bending; μ – constant (for rectangular section $\mu = 1.2$); r_0 – the radius of curvature of the neutral rib fiber; $\varepsilon_{\lambda}^{(c)}$, θ_b – relative elongation and angle of rotation of the normal in the longitudinal fiber of the rib; ρ – radius of curvature of the contour Γ.

To establish the structure of the functions $N,\ Q,\ L_b$ in different areas $[0;\ \pi]$, the cross section of the open rod, which is inclined to the axis Ox at an angle θ, should be considered. It divides the rod into two parts, each of which is in equilibrium under the action of forces N, Q, bending moment L_b, contact forces and load on the end face. According to the conditions of equilibrium of each part, the expressions for internal force factors in characteristic sections of the rod are found.

Area $\theta \in [0;\ \theta_0]$

$$N(\theta) = N_0\cos\theta;\ Q(\theta) = N_0\sin\theta;\ L_b(\theta) = L_b^{*(0)} + \left[x(\theta_0) - x(\theta) - \eta(\cos\theta_0 - \cos\theta)\right]N_0 .$$

$$Area\ \theta \in [\theta_0;\ \theta_1]$$

$$N(\theta) = \tilde{N}(\theta) + \cos\theta \left(N_0 \frac{\theta_1 - \theta}{\theta_1 - \theta_0} - N_1 \frac{\theta - \theta_0}{\theta_1 - \theta_0} \right);$$

$$Q(\theta) = \tilde{Q}(\theta) + \sin\theta \left(N_0 \frac{\theta_1 - \theta}{\theta_1 - \theta_0} - N_1 \frac{\theta - \theta_0}{\theta_1 - \theta_0} \right);$$

$$L_b(\theta) = \tilde{L}_b(\theta) + L_b^{*(0)} \frac{\theta_1 - \theta}{\theta_1 - \theta_0} + L_b^{*(1)} \frac{\theta - \theta_0}{\theta_1 - \theta_0}. \tag{8}$$

$$Area\ \theta \in [\theta_1;\ \pi]$$

$$N(\theta) = -N_1 \cos\theta;\ Q(\theta) = -N_1 \sin\theta;\ L_b(\theta) = L_b^{*(1)} + \left[x(\theta) - x(\theta_1) - \eta(\cos\theta - \cos\theta_1)\right]N_1.$$

Here

$$L_b^{*(0)} = L_b^{(0)} + \left[x(0) - x(\theta_0) + \eta(\cos\theta_0 - 1)\right]N_0;$$

$$L_b^{*(1)} = L_b^{(1)} + \left[x(\theta_1) - x(\pi) - \eta(\cos\theta_0 + 1)\right]N_1 -$$

bending moments that occur in cross sections $\theta = \theta_0$ and $\theta = \theta_1$, respectively; $\tilde{N}(\theta)$, $\tilde{Q}(\theta)$, $\tilde{L}_b(\theta)$ – are limited and continuous functions on $[\theta_0;\ \theta_1]$, for which conditions are fulfilled at the ends of a site of connection of a plate and a rib

$$\tilde{N}(\theta_0) = \tilde{N}(\theta_1) = \tilde{Q}(\theta_0) = \tilde{Q}(\theta_1) = \tilde{L}_b(\theta_0) = \tilde{L}_b(\theta_1) = 0. \tag{9}$$

Mathematical model of the problem. The boundary conditions of the problem are formulated in the form of conditions of joint deformation of the plate and the reinforcing rib in the areas of their connection.

$$\begin{gathered} \varepsilon_\lambda(\lambda) = \varepsilon_\lambda^{(c)}(\lambda); \quad \tilde{V}(\lambda) = \theta_b(\lambda), \quad \lambda \in [\alpha_0; \beta_0]; \\ T_\rho = S_{\rho\lambda} = 0, \quad \lambda \in [0,\ \alpha_0] \cup [\beta_0;\ \pi] \end{gathered} \tag{10}$$

Substitution (2), (5) with consideration of (4)–(8) in conditions (10) leads to the system of singular integral – differential equations with Hilbert kernels for determining functions T_ρ, $S_{\rho\lambda}$, $\tilde{N}$, $\tilde{Q}$, $\tilde{L}_b$ and constants N_0, N_1, $L_b^{*(0)}$, $L_b^{*(1)}$. Providing these values are known, the annular effort T_λ on the contour of the hole in the plate and the maximum normal stresses in the extreme fibers of the rib and the largest tangential stresses in the axial fiber are deduced from the relations [1]

$$\sigma^{(1)} = \frac{1}{F_0}\left[N + \frac{\eta + \eta_c}{\eta_c} \cdot \frac{L_b}{\rho}\right]; \quad \sigma^{(2)} = \frac{1}{F_0}\left[N + \frac{\eta_c - \eta}{\eta_c} \cdot \frac{L_b}{\rho - 2\eta}\right]; \ \tau_{\max} = \frac{3}{2}\frac{Q}{F_0}. \tag{11}$$

On the basis of (10), the system (2), (5)–(9) possesses the same structure as the corresponding system for an isotropic plate with a curved hole [10], so the method of mechanical quadratures and collocation of its approximate solution is transferred without changes.

Analysis of numerical calculations. For an infinite orthotropic plate with an elliptical $(\varepsilon = \pm 0.2)$ hole and a reinforcing rib with parameters

$$h_0/h = 4/3; \quad \eta / R_0 = 0.1; \quad \alpha_0 = 10^\circ; \quad \beta_0 = 150^\circ; \quad \sqrt{E_x E_y} / E_0 = 0.5$$

the influence on the stress state of the plate and the orthotropic ribs of the plate material, and the value of the interphase sections and the shape of the hole is studied.

The results of numerical calculation of the values T_ρ, $S_{\rho\lambda}$, T_λ on the contour Γ in the plate and $F_0\sigma^{(1)}$, $F_0\sigma^{(2)}$, $F_0\tau_{\max}$ – in the rib are shown in Fig. 3–8. The characteristics of orthotropic materials and the lines that correspond to these materials in the figures are shown in Table 1.

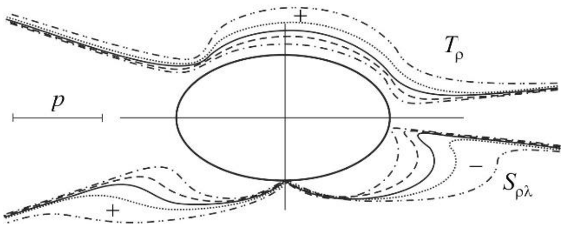

Figure 3. Diagrams of contact forces on the contour Γ in the plate

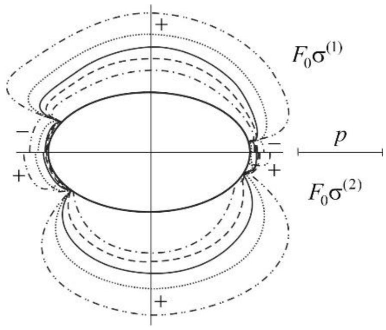

Figure 4. Diagrams of normal stresses in the longitudinal fibers of the rib

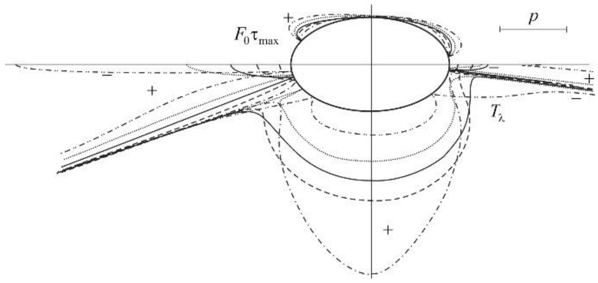

Figure 5. Diagrams of tangential stresses in the rib and annular forces on the contour Γ in the plate

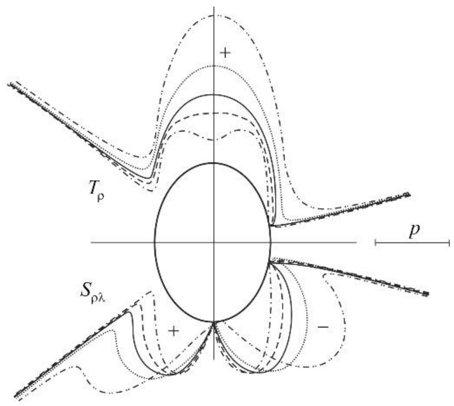

Figure 6. Diagrams of contact forces on the contour Γ in the plate

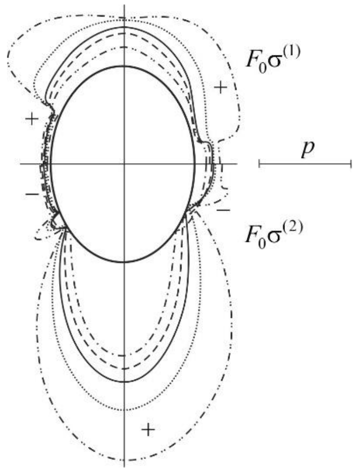

Figure 7. Diagrams of normal stresses in the longitudinal fibers of the rib

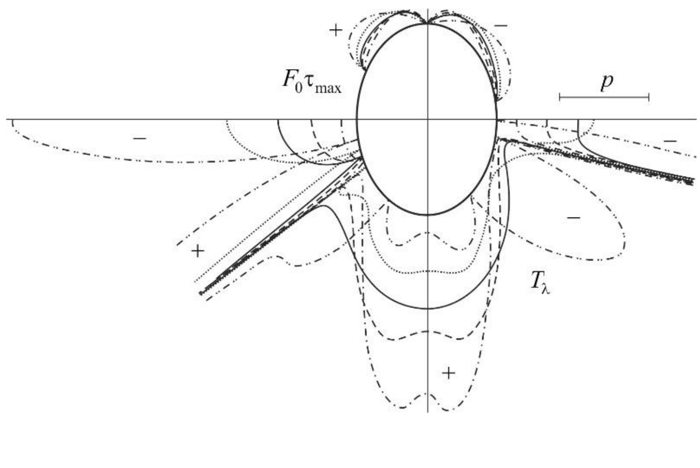

Figure 8. Diagrams of tangential stresses in the rib and annular forces on the contour Γ in the plate

Table 1

The characteristics of orthotropic materials and the lines that correspond to these materials

Plate material	β_1	β_1	ν_x	E_x / E_y	Lines
isotropic material	1	1	0,300	1	———
glass epoxy	2,2712	0,7626	0,250	3	– – – –
graphite epoxy	6,9992	0,7144	0,250	25	– · – · –
epoxy glass	0,4400	1,3100	0,083	1/3	………………
epoxy graphite	0,1430	1,4010	0,010	1/25	– · · – · ·

Based on the obtained results, the authors conclude:

- the presence of interphase sections leads to a significant redistribution of the components of the stress state in the plate and the reinforcing rib, especially neighbourhood their ends. At the joints of the plate and the ribs at a distance from their ends, all components of the stress state are almost independent of the presence of incisions;
- for all considered plate materials, the transverse forces are significantly smaller than other force factors in the reinforcement, so they can be neglected in engineering calculations;
- orthotropy of the plate material significantly affects the distribution of components of the stress state in the plate and the reinforcing rib. As the ratio E_x / E_y increases, the maximum values of the annular efforts T_λ decrease sharply, while for the others the component of the stress state increases;
- in the vicinity of the ends of the interfacial sections normal stresses in the longitudinal fibers of the ribs have jumps of the first type, while remaining limited, and the contact and annular efforts in the plate take unlimited values.

References

1. Pysarenko H. S., Kvitka O. L., Umanskyi E. S. Opir materialiv. K.: Vyshcha shkola, 2004. 655 p.
2. Martynovych T. L. K reshenyyu zadach o napryazhennom sostoyanyy v anyzotropnykh plastynkakh s podkreplennym kraem. Prykl. matematyka y mekhanyka. 1971. T. 35. No. 1. P. 172–178.
3. Fylyn A. P., Tananayko O. D., Cherneva Y. M., Shvarts M. A. Alhorytmy postroenyya razreshayushchykh uravnenyy mekhanyky sterzhnevyykh system. L.: Stroyizdat, 1983. 232 p.
4. Syaskyi A., Shevtsova N. Zastosuvannya metodu syl dlya statychnoho rozrakhunku zamknenykh kryvoliniynykh stryzhniv. Visnyk TNTU. 2015. No. 3 (79). P. 24–30.
5. Bozhydarnik V. V., Andreykiv O. E., Sulym H. T. Mekhanika ruynuvannya, mitsnist i dovhovichnist neperervno armovanykh kompozytiv. Monohrafiya. U 2–kh t. T. 2. Matematychni metody v zadachakh mekhaniky ruynuvannya neperervno armovanykh kompozytiv. Lutsk: Nadstyya, 2007. 424 p.
6. Martynovych T. L., Yurynets V. E. Kontaktnye vzaymodeystvyya plastyn s upruhymy élementamy. Lvov : Vyshcha shkola. Yzd-vo Lvov. un-ta, 1984. 160 p.
7. Syaskyi A. O., Shevtsova N. V., Deyneka O. Yu. Mizhfaznyy rozriz v ortotropniy plastyntsi z pidsylenym kruhovym konturom. Visnyk Khmelnytskoho natsionalnoho universytetu. Tekhnichni nauky. Khmelnytskyi. 2018. No. 5. P. 176–181.
8. Syaskyi A. O., Shevtsova N. V., Deyneka O. Yu. Mizhfaznyy rozriz v ortotropniy plastyntsi z eliptychnym konturom, pidsylenym zamknenym pruzhnym rebrom. Visnyk Khmelnytskoho natsional′noho universytetu. Tekhnichni nauky. Khmelnytskyi. 2019. No. 1. P. 31–39.
9. Syasky A., Dejneka O. Interphase incision on the limit welding of orthotropic plate with elliptical hole and closed elastic rib. Scientific Journal of TNTU (Ternopil). 2019. Vol. 95. No. 3. P. 26–33. https://doi.org/10.33108/visnyk_tntu2019.03.026
10. Syasky A., Dejneka O., Shevtsova N. Interphase gross – sections in the infinity plate with curvilinear contour reinforced by closed rib. Scientific Journal of TNTU (Ternopil). 2019. Vol. 96. No. 4. P. 23–31.
11. Lekhnytskyi S. H. Anyzotropnye plastynky. M.: Gostekhyzdat, 1957. 464 p. https://doi.org/10.33108/visnyk_tntu2019.04.023

Список використаної літератури

1. Писаренко Г. С., Квітка О. Л., Уманський Е. С. Опір матеріалів. К.: Вища школа, 2004. 655 с.
2. Мартынович Т. Л. К решению задач о напряженном состоянии в анизотропных пластинках с подкрепленным краем. Прикл. математика и механика. 1971. Т. 35. № 1. С. 172–178.

3. Филин А. П. Тананайко О. Д. , Чернева И. М., Шварц М. А. Алгоритмы построения разрешающих уравнений механики стержневых систем. Л.: Стройиздат, 1983. 232 с.
4. Сяський А., Шевцов Н. Застосування методу сил для статичного розрахунку замкнених криволінійних стрижнів. Вісник ТНТУ. 2015. № 3 (79). С. 24–30.
5. Божидарнік В. В., Андрейків О. Е., Сулим Г. Т. Механіка руйнування, міцність і довговічність неперервно армованих композитів. Монографія. У 2–х т., Т. 2. Математичні методи в задачах механіки руйнування неперервно армованих композитів. Луцьк: Надстир'я, 2007. 424 с.
6. Мартынович Т. Л., Юринец В. Е. Контактные взаимодействия пластин с упругими элементами. Львов : Вища школа. Изд-во Львов. ун-та, 1984. 160 с.
7. Сяський А. О., Шевцова Н. В., Дейнека О. Ю. Міжфазний розріз в ортотропній пластинці з підсиленим круговим контуром. Вісник Хмельницького національного університету. Технічні науки. Хмельницький. 2018. № 5. С. 176–181.
8. Сяський А. О. Шевцова Н. В., Дейнека О. Ю. Міжфазний розріз в ортотропній пластинці з еліптичним контуром, підсиленим замкненим пружним ребром. Вісник Хмельницького національного університету. Технічні науки. Хмельницький. 2019. № 1. С. 31–39.
9. Syasky A., Dejneka O. Interphase incision on the limit welding of orthotropic plate with elliptical hole and closed elastic rib. Scientific Journal of TNTU (Ternopil). 2019. Vol. 95. № 3. P. 26–33. https://doi.org/10.33108/visnyk_tntu2019.03.026
10. Syasky A., Dejneka O., Shevtsova N. Interphase gross – sections in the infinity plate with curvilinear contour reinforced by closed rib. Scientific Journal of TNTU (Ternopil). 2019. Vol. 96. №4. P. 23–31.
11. Лехницкий С. Г. Анизотропные пластинки. М.: Гостехиздат, 1957. 464 с. https://doi.org/10.33108/visnyk_tntu2019.04.023

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НЕПОВНИЙ КОНТАКТ ОРТОТРОПНОЇ ПЛАСТИНКИ З ЕЛІПТИЧНИМ ОТВОРОМ І ЗАМКНЕНОГО ПРУЖНОГО РЕБРА

Андрій Сяський[1]; Олег Дейнека[2]; Наталія Шевцова[3]

[1]Рівненський державний гуманітарний університет, Рівне, Україна;
[2]Національний університет водного господарства та природокористування, Рівне, Україна[2];
[3]Відокремлений структурний підрозділ «Рівненський коледж Національного університету біотехнологій і природокористування України», Рівне, Україна

Резюме. *В умовах узагальненого плоского напруженого стану, створеного рівномірно розподіленими зусиллями розтягу на нескінченності, розглянуто мішану контактну задачу для нескінченної ортотропної пластинки з еліптичним отвором, контур якого підсилено замкненим пружним ребром, за наявності на межі їх сполучення двох симетричних міжфазних розрізів, береги яких у процесі деформації не контактують. Компоненти тензора деформації (відносне видовження і кут повороту нормалі) в точках контуру отвору пластинки представлені інтегральними залежностями від контактних зусиль. Моделюючи підсилення замкненим пружним стрижнем сталого прямокутного поперечного перерізу та використовуючи основні рівняння лінійної теорії криволінійних стрижнів, що враховує деформації поперечного зсуву, математичну модель задачі побудовано у вигляді системи сингулярних інтегрально-диференціальних рівнянь з ядрами Гільберта для знаходження контактних зусиль між пластинкою і ребром та внутрішніх сил і моментів у підсиленні. При визначенні початкових параметрів замкненого статично невизначеного стрижня використано умови однозначності зміщень і кутів повороту в точках його осі. З'ясовано структуру шуканих функцій на ділянках сполучення пластинки й підсилювального ребра. Наближений розв'язок задачі побудовано комбінованим методом механічних квадратур і колокації, яким досліджено вплив на напружений стан пластинки й підсилювального ребра величини міжфазних розрізів та форми отвору в пластинці. Встановлено, що в околі торців міжфазних розрізів нормальні напруження в поздовжніх волокнах ребра мають стрибки першого роду, залишаючись при цьому обмеженими, а контактні й кільцеві зусилля в пластинці набувають необмежених значень.*

Ключові слова: *міжфазні розрізи, ортотропна пластинка, підсилювальне ребро, контактні зусилля, сингулярні інтегрально-диференціальні рівняння.*

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